CHAPTER - III

SCENARIOS AND OPTIMAL POLICY IN ACQUISITION OF POLLUTION CONTROL EQUIPMENT UNDER UNCERTAINTY

3.1 INTRODUCTION :

The commitment of Government on abatement of pollution for preventing deterioration of the environment is stated here. The policy elements seek to shift emphasis from defining objective for each problem area towards actual implementation but the focus is on the long term. The complexities are considerable given the number of industries, organisations and government bodies involved. To achieve the objectives maximum use will be made of mix of instruments in the form of legislation and regulation, fiscal incentives, voluntary agreements, educational programmes and information campaigns. The emphasis will be on increased use of regulations and an increase in the development and application of financial incentives.

A comprehensive approach should be taken to integrate environmental and economic aspects in developmental planning; stress is laid on preventive aspects for pollution abatement and promotion of technological inputs to reduce industrial pollutants; and through reliance upon public cooperation in securing a clean environment to respond to the coming challenges. For a country like India, suitable vegetative cover and resource recover technologies cannot only be attractive alternative but also economical, safe and socially acceptable. Standards will not merely be a regulatory tool but will be mechanism to promote technological upgradation to prevent
pollution, conserve resources and regulate waste. A system of 'Environmental Audit' would be introduced for local bodies statutory authorities and public limited companies by requiring them to prepare an annual environmental statement to evaluate the effect of their policies operation and activities on the environment, particularly compliance with standards and the generation and recycling of wastes.

This paper presents a firm's pending environmental regulation i.e. the firm will have to acquire a certain stock of pollution control equipment when the target stock and compliance date both are unknown. A concrete example of this issue is provided by the Indian thermal power plants being faced with possible acid rain control legislation since the early 1985's.

A deterministic case of this problem was considered by Beavis and Dobbs(1986) with fixed target stock and compliance date. A stochastic model was considered by Forster(1988) with fixed target stock and stochastic compliance date. Recently a complete model was considered by Richard(1992) with announcement date compliance date and target stock to be unknown.

In this paper (a) we consider convex-concave adjustment cost function instead of concave-convex adjustment cost function as considered by Richard (1992), (b) tax reduction and other benefits is dropped and (c) we allow for the compliance date to be a random variable.

In section 3.2, we pose the deterministic model when target stock and compliance date are assumed to be known after the announcement date whereas in section 3.3, we develop a stochastic model when
the target stock is unknown. Using these results the uncertain model is formulated in section 3.4, when target stock and compliance date both are unknown. In the end useful scenarios is analyzed for obtaining the optimal investment policy.

3.2 DETERMINISTIC MODEL:

Let us denote

- \( a \) : announcement date
- \( c \) : compliance date
- \( S \) : stock of abatement equipment acquired earlier
- \( S \) : target stock of pollution control capital
- \( I \) : rate of investment
- \( C(I) \) : adjustment cost function
- \( n \) : rate of decay, i.e., 100n% replacement of equipment per year
- \( u \) : rate of technological progress, i.e., 100u% reduction of the adjustment cost per unit time.
- \( r \) : rate of discount.

We assume a convex-concave adjustment cost function, i.e., the cost function is strictly convex for small investment, i.e., before the announcement date and strictly concave for large investment as shown in figure (1).

Symbolically, investment is the rate of increase of capital over time, i.e.,

\[
I = \frac{dC(I)}{dt} \quad \ldots(3.2.1)
\]

The problem of minimizing the total discounted cost of procuring \( S \) stock over interval \((0,a)\) and \((a,c)\) (see figure 1) when target stock and compliance date are known is given by
\[ j(a,c, S, \bar{S}) = \min_{I} \left\{ \int_{0}^{a} e^{-(r+u)t} C(I)dt + \int_{a}^{c} e^{-(r+u)t} C(I)dt \right\} \]

s.t. \( S = I - nS \), \( S(a) = \bar{S} \), \( S(c) \geq \bar{S} \) \hspace{1cm} (3.2.2)

The value function for the control problem (3.2.2) is given by

\[ v(c, \bar{S} - \bar{S} e^{-nc}) = j(a,c, \bar{S}, \bar{S}) \] \hspace{1cm} (3.2.3)

where \( v(c,z) = \min_{I} \left\{ \int_{0}^{c} e^{-(r+u)t} C(I)dt \right\} \]

s.t. \( S = I - nS \), \( S(0) = 0 \), \( S(c) \geq z \) \hspace{1cm} (3.2.4)

i.e., \( z \) stock has to be acquired beginning at \( t=0 \) to \( t=c \), this means that

\[ (\bar{S} - S) + (S - S e^{-nc}) \] \hspace{1cm} (3.2.5)

i.e., part first of (3.2.5), \( (\bar{S} - S) \) has to be acquired and part 2nd of (3.2.5), \( (S - S e^{-nc}) \) is to be replaced in the interval \((0, c)\).

Since the cost function \( C(I) \) is positive and convex, \( v(c,z) \) is also non-decreasing and convex function of \( z \).

For \( z \geq 0 \), \( v(c,z) \) is convex in \( z \) (this can be seen by observing that \( \frac{d^2 v(c,z)}{dz^2} \geq 0 \) ) and minimizing \( z \) can be determined by necessary condition that

\[ \frac{dv(c,z)}{dz} = 0 \] \hspace{1cm} (3.2.6)

Let \( z \equiv z_1 \) be determined by this condition. Evidently for \( z \geq 0 \), \( v(c,z_1) \leq v(c,z) \) for any \( z \geq 0 \).

We note that \( I \) is monotone increasing by (3.2.2) and (3.2.4). If \( I \) is concave and positive (see also figure 2a) then \( v(c,z_1) \) is pseudoconvex on \( z \geq 0 \).(A real valued differentiable function \( q(x) \) is said to be pseudoconvex on an open convex set, if
Every local minimum of a pseudoconvex function is also global, i.e., if $I$ is concave then the necessary condition $v'(c,z_1) = 0$, $z_1 \geq 0$ is also sufficient for a global minimum of $v(c,z_1)$ on $z_1 \geq 0$.

Thus the optimal investment rate is decreasing and increasing respectively for $z$ small and large enough. The firm will delay to invest in procuring the abatement equipment.

### 3.3 Stochastic Model 1

Let us realize that the announcement date and compliance date are known but the target stock is not known in advance and subject to some probability distribution. Forster (1988) considers that there is usually more uncertainty about the compliance date than the amount of stock. As an example we refer to the fact that most of bills that have been introduced in the Indian congress have called for similar (aggregate reduction of 8-10 million tons of SO$_2$ annually. However it seems more realistic to assume, e.g., a triangular distribution between the values of $\bar{S}$ induced by the 8-10 million ton limits. In other cases when the uncertainty is high a uniform distribution is advisable.

Let us denote

- $h(S)$: probability density function for $\bar{S}$
- $S^\star$: maximum possible value of $\bar{S}$

Using (3.2.3), we define

$$f(c, \bar{S}) = \int_0^{S^\star} \int_0^{\bar{S}} v(c, \bar{S} - S e^{-nc}) h(S) dS d\bar{S}$$

... (3.3.1)
If we change the order of integration, i.e.,
\[ S = 0, \quad \bar{S} = 0, \quad \tilde{S} = s^* \]
the integration becomes
\[
f(c, S) = \int_0^{s^*} h(\bar{S}) d\bar{S} \int_0^{s^*} \nu(c, \bar{S} - \tilde{S} e^{-nc}) d\tilde{S}
\]
\[
= \int_0^{s^*} h(\bar{S}) \nu(c, \bar{S} - \tilde{S} e^{-nc}) d\tilde{S}
\]
\[....(3.3.2)\]
\[
\frac{df(c, S)}{d\bar{S}} = -\int_0^{s^*} \frac{h(\bar{S})}{\nu(c, \bar{S} - \tilde{S} e^{-nc})} e^{-nc} d\tilde{S}
\]
\[....(3.3.3)\]
\[
\frac{d^2f(c, S)}{d\bar{S}^2} = \int_0^{s^*} \frac{h(\bar{S})}{\nu(c, \bar{S} - \tilde{S} e^{-nc})} e^{-2nc} d\tilde{S}
\]
\[....(3.3.4)\]
i.e., from (3.3.3) and (3.3.4), we have
\[
f' < 0, \quad \text{if } s^* < \tilde{S} e^{-nc}
\]
\[
f'' > 0, \quad \text{if } s^* > \tilde{S} e^{-nc}
\]
Hence from the convexity of \( \nu \), \( f \) is a nondecreasing and convex function of \( S \).

From (3.3.2), we can write
\[
f(c, S) = P(S) \]
\[....(3.3.5)\]
where
\[
P(S) = \int_0^{s^*} h(\bar{S}) \nu(c, \bar{S} - \tilde{S} e^{-nc}) d\tilde{S}
\]
\[....(3.3.6)\]
Due to insufficient stock at time \( c \), the term \( P(S) \) in (3.3.6) is defined as penalty function.

The expected cost of requirement of abatement equipment is given by (3.3.5) and that \( f \) and \( P \) are non-decreasing and convex as shown in figure (2b).
3.4 UNCERTAIN MODEL:

Let us consider a general problem in which a firm has to acquire a certain piece of abatement equipment. Letting announcement date a known because unknown date of announcement does not effect on requirement of abatement equipment because some of the stock would be stored prior to announcement date and the rest will be stored before compliance date c. And considering this compliance date c and target stock S to be unknown. Taking this compliance date as a random variable subject to some probability distribution.

To acquire the abatement equipment earlier than required the firm is usually offered tax reduction and other benefits as considered by Richard. But here we consider that there is no tax reduction and other benefits.

Let us denote

\[ T^* : \text{maximum possible date for compliance date} \]
\[ x(t) : \text{probability density function for } c, \]
\[ y(t) : \text{probability that the compliance date will not occur prior to } t \text{ s.t.} \]
\[ y(t) = \int_c^{T*} x(t) \, dt \quad \ldots (3.4.1) \]

It is clear from above equation that if compliance date will not occur before a given date the probability decreases as the time passes and if occur, the probability increases with time. The conditional density of occurrence of compliance date is given by

\[ q(t) = \frac{x(t)}{y(t)} \quad 0 < t < c < T^* \quad \ldots (3.4.2) \]

The firm is supposed to encounter with a penalty for not sufficient stock at c (defined as P in (3.3.6)).
Using (3.2.2) and (3.3.5), we can define
\[
\begin{align*}
\min_{I} \int_{0}^{T^{*}} x(t) \left\{ \int_{0}^{C(I)} e^{-(r+u)t} \, dt + e^{-rt} \, P(S) \, dc \right\} \quad \cdots(3.4.3)
\end{align*}
\]
Integrating by parts, we get
\[
\begin{align*}
\min_{I} \int_{0}^{T^{*}} \left[ y(t) \, C(I) \, e^{-ut} + x(t) \, P(S) \right] \, e^{-rt} \, dc \\
\text{s.t.} \quad S = I - nS, \quad S(0) = 0, \quad I \geq 0
\end{align*}
\]
\[
\cdots(3.4.4)
\]
We can write the above problem as Hamiltonian
\[
H = -y(t) \, e^{-ut} \, C(I) - x(t) \, P(S) + \phi(I-nS) \quad \cdots(3.4.5)
\]
Where \( \phi \) stands for social opportunity cost or shadow price by Feichtinger and Hartl(1986) or Seierstad and Sydsæter(1987).

The maximum principle conditions (necessary conditions) are
\[
\frac{dH}{dI} = -y \, e^{-ut} \, C'(I) + \phi > 0
\]
\[
\Rightarrow \phi = y \, e^{-ut} \, C'(I), \quad \text{if} \quad \phi > y \, \frac{C(I^*) - C(0)}{I^*} \, e^{-ut} \cdots(3.4.6)
\]
and
\[
\frac{dH}{dS} = -x(t)P(S) - \phi n \quad \cdots(3.4.7)
\]
\[
\phi = (r+u)\phi - H_s = (r+u+n)\phi + x(t)P(S) \quad \cdots(3.4.8)
\]
\[
\phi(T^*) = 0 \quad \cdots(3.4.9)
\]
The conditions (3.4.6)-(3.4.9) are also sufficient.
Therefore at \( I = 0 \) there is no capital stock, i.e., investment is not advisable if shadow price is less.
If \( I > 0 \), expected adjustment cost will be equal to shadow price at time \( t \).
With \( I > 0 \), combining (3.4.6) and (3.4.8) we obtained the firm's investment equation.
\[
\dot{I} = \frac{1}{C(I)} \left[ (u+n+r+q)C'(I) + qP(S) \right] \quad \cdots(3.4.10)
\]
Due to \( q(t) \), (3.4.10) defines a non-autonomous differential equation. Similar uncertain time problem is considered by Richard with benefit function \( B(K) \) and also by Forster with \( n=r=0 \).

### 3.5. OPTIMAL INVESTMENT SCENARIOS:

1. The first case when the compliance date \( c \) is known, i.e., \( q=0 \), (Richard special case) the problem (3.4.10) becomes

\[
\dot{I} = \frac{1}{C(I)} \left[ (u+n+r)C'(I) \right] 
\]

Since \( \dot{I} \) is independent of time, \( \dot{I} > 0 \).

In this case \( I \) is rising as shown by arrow in figure(3), i.e., the firm will delay to invest more.

2. The second case when the compliance date \( c \) is unknown and \( u=r=n=0 \) (Forster special case), the problem (3.4.10) reduces

\[
\dot{I} = \frac{q}{C(I)} \left[ C'(I) + P'(S) \right]
\]

which is still a non-autonomous differential equation. The locus of points for which \( \dot{I} = 0 \) is defined by

\[
C'(I) = -P'(S)
\]

i.e., the locus is constant and does not depend on time.

As \( S \) increases, penalty term \( P' \) decreases and investment rises. The figure (3) shows the monotonic behaviour of investment policy.

3. The third case, when \( u=r=n=0 \) and \( P(S)=0 \),

\[
\dot{I} = \frac{qC'(I)}{C'(I)}
\]

Which is similar to the investment behaviour as in previous case with \( u=r=n=0 \).

If \( I = 0 \), then
\[ C'(I) = 0 \quad \ldots \quad (3.5.5) \]

The investment is falling to the left of \( I = 0 \) locus and rising to the right of \( I = 0 \) locus as shown in figure(3).

(4) The last case, when \( q > 0 \), i.e., the compliance date \( c \) occur prior to earlier date. In this case monotonicity property of investment diminishes.

If \( I = 0 \) (3.4.10) becomes

\[ q = - \frac{(u+r+n)C'(I)}{C(I) + P(S)} \quad \ldots \quad (3.5.6) \]

i.e., \( C'(I) > 0 \), if \( q > 0 \), this means that under uncertainty the firm's optimal investment policy demands high investment initially followed by falling investment in the early phase, then increases in the latter phase.

Appendix

The objective function in (3.2.4) is defined as

\[ v(z) = \min_{I} \int_{0}^{C} e^{-\lambda t} C(I) dt \]

s.t. \( S = I - nS, \ S(0) = 0, \ S(c) = z \),

with \( v(z) \) is an increasing and convex function of \( z \), i.e., \( v' > 0 \) and \( v'' > 0 \).

The optimal investment rate \( I \) is always monotone increasing.

Now let us show that \( v \) is pseudoconvex on \( z \), since \( v' > 0 \) and

\[ v(z_2)(z_1 - z_2) \geq 0 \]

\[ \Rightarrow z_1 - z_2 \geq 0 \]

\[ \Rightarrow S_1 \geq S_2 \]

\[ \Rightarrow v(z_1) - v(z_2) \]

\[ = \int_{0}^{c} e^{-(r+\mu)t} C(I_1) dt - \int_{0}^{c} e^{-(r+\mu)t} C(I_2) dt \geq 0 \]
FIGURE 1: convex-concave investment cost function

FIGURE 2a: The value function $v(c, z)$
FIGURE 2b: Penalty function

FIGURE 3: Investment behaviour