Chapter: 1

Review of Relevant Literature and Orientation to the Research Topics Considered in the Thesis

1.1 Introduction

In every branch related with business and industry, there is a competition. Different manufacturers put the same type of product in the market as per the demand of the consumers. The manufacturer has to ensure that his products always compete with other products from the viewpoint of quality and price and so the manufacturer has to strive continuously to maintain quality standards of his products. Hence one of the primary needs of the top management in any organization is to ensure that the product manufactured by them is of optimum quality. To achieve the optimum quality of a product at desired level and at as low the cost of production as possible, the technique of quality control is used. The technique of quality control is a continuous process in which the need for action is identified at the earliest possible stage.

There are so many new techniques which have been developed to control the quality of the products. In the field of manufacturing products the statistical methods are also applied for maintaining the standards of quality of the products. The technique of using the statistical methods for maintaining the standards of quality and improving it by implementing modifications there too, is known as a Statistical Quality Control.(S.Q.C).
If any two products are examined from a production process, they cannot be identical in all respects. Some sort of variation in the two items is bound to be there. In fact it is an integral part of any manufacturing process. This difference in the characteristic of a product is known as variation. The variations in the quality are of two types. (I) Variations due to chance cause (II) Variations due to assignable causes. The chance causes of variations result from many minor causes and behave in a random manner. There is no way in which they can be completely eliminated. Even if these variations are present, in the process, the process is said to be under the state of statistical control. The assignable causes of variations may be due to some non-random causes like improper machine setting, inexperienced operators, and inferior raw materials etc. The variations due to assignable causes are serious in nature and they can be overlooked. The causes of these variations should be detected and should be removed from the production process. If the variations in the products are due to assignable causes they are regarded significant and the process is said to be out of statistical control. These two types of variations can be separated by control charts.

Hence the role of Statistical Quality Control (S.Q.C.) is to collect and analyze relevant data for the purpose of detecting whether the process is under control or not. If, not what can possibly be the reason for the lack of control.

To control a process using data on a variable of the product, it is necessary to keep a watch on the current state of the accuracy (central tendency) and variation (spread) in the data of the variable. This may be achieved with the aid
of the control chart known as variable control charts. A control chart is a graphical device of the representation of the data for knowing the extent of variations from the expected standard. The use of control charts can provide more information about a process than workers with years of experience. A powerful combination is formed when experienced workers use the control charts, because the charts indicate when a source of variation occurs, the chart patterns provide hints to the causes of variation, and the workers can use the hints along with their experience to identify the source of the variation problem and eliminate it.

Shewhart developed a method to do this and he presented his findings in 1931 in his book ‘Economic Control of Quality of Manufactured Product’. Shewhart’s methods evolved from the concept that more information can be obtained from data in a timely manner by using rational sub-grouping. Small samples of consecutive pieces with sizes n = 4, 5, or 6, taken over time, will provide the three basic keys of data interpretation: position, variation, and shape.

The average value of the small samples will show the position of the measurements relative to the target value. The range of the sample will give an immediate measure of the variation, and, as more samples are taken, any changes in the sample variation may be noted as well as changes in position. When enough samples have been taken, the individual measurements can be plotted on a histogram and the shape of the distribution can be determined.
1.2 Brief History of Statistical Quality Control

Quality always has been an integral part of virtually all products and services. However, our awareness of its importance of formal methods for quality control and improvement has been an evolutionary development. During 1700-1900 quality was largely determined by the efforts of an individual craftsman. Eli Whitney introduced standardized, interchangeable parts to simplify assembly. In 1875, Taylor introduced “Scientific Management” principles to divide work into smaller, more easily accomplished units- the first approach to dealing with more complex products and processes. Gilbreth and others extended this concept to the study of motion and work design. Much of this had a positive impact on productivity, but it often de-emphasized the quality aspect of application of work.

The techniques of quality control have a very long history. These techniques were first introduced by Shewhart in 1924 during Second World War. Shewhart was a scientist in the Bell Telephone company of America. He developed the Statistical Control Chart concept which is considered to be the beginning use of techniques of Statistical Quality Control. By the middle of 1930s Statistical Quality Control methods were widely used at Western Electric, the manufacturing arm of Bell system. In 1931, Shewhart presented a book entitled ‘Economic control of quality of manufactured product’. This book is considered to be the first book on Statistical Quality Control.

World War II saw greatly expanded use and acceptance of Statistical Quality Control (SQC) concept in manufacturing industries. War time experience made
it apparent that statistical techniques were very useful to control and improve product quality.

During 1940 to 1946 training courses on SQC were given in industry and more than 50 quality societies were formed in North America. In 1944 publication of “Industrial Quality Control” began. The American Society for Quality Control (ASQC) was formed in 1946. In 1948; professor Taguchi began study and application of experimental design. In, 1950, classic texts on statistical quality control by Grant and Duncan appeared. Also, professor Ishikawa introduced the cause and effect diagram. In 1951, Box and Willson published fundamental work on using design experiments and response surface methodology of process optimization for chemical industry. In 1954, British statistician Page introduced the Cumulative Sum (CUSUM) control charts. In 1957, Juran and Gryna’s ‘Quality Control Handbook’ is published for the first time. In 1959, Technometrics (a journal of statistics for physical and engineering sciences) is established. Also, Roberts introduced the exponentially weighted moving average (EWMA) control charts. In 1960, the quality control circle concept was introduced in Japan by Ishikawa. In 1960, zero defects (ZD) programs were introduced in certain U.S. Industries. In 1969, Industrial Quality Control ceased its publication and replaced it by Quality Progress and Journal of Quality Technology. During 1975 to 1978, interest in quality circles began in North America and grew into the Total Quality Management (TQM) movement. In 1980s, experimental design methods were introduced too and adopted by a wider group of organizations, including electronics, aerospace,
semiconductor and automotive industries. Also, the work of professor Taguchi on designed experiment appeared in the United States for the first time. In 1984, the American Statistical Association (ASA) established the adhoc committee on quality and productivity which later on became a full section of the ASA. In 1989, the Journal of Quality Engineering appeared and Motorola’s six-sigma approach got spread to other industries. In 1998, the American Society for Quality Control became the American Society for Quality, attempting to indicate the broader aspects of the Quality improvement field.

1.3 A Brief Review of Literature on SQC Charts for variables

After Shewhart (1931), the basic ideas of control charts for nonsymmetrical distribution were supplied by Nilsson (1944). Winterhalter (1945) had developed Reject Limits for measurement. Enrick (1945) has provided operating characteristics of rejects limits for measurement. Hammer and Probity Engineers (1945) have shown why the quality control engineer must not uncritically apply limit formulas and have suggested what may be done to obtain limits for satisfactory levels of significance. For that moving range technique has been successfully applied to a variety of charts. Rice (1946) provided good review of Statistical Quality Control and Major Grubbs (1946) introduced special type of control chart known as the difference chart when strict reproducibility of measurements against a standard is costly and time-consuming with production schedules. In such instances difference chart may prove to be of considerable value in judging quality. Croxton and Cowden (1946) gave tables to facilitate computation of sampling limits of s and Fiducial
Limits of sigma. Cratg (1947) gave detail discussion on control chart versus the analysis of variance in process control by variables. In his paper he tried to show that analysis of variance has same purpose that of Shewhart control chart. Burr (1949) setup new methods for approving a machine or process setting. Burr proposed simple sequential methods for a second type of machine checking and present a battery of criteria tables providing a variety of possibilities for practical application. Westman and Lloyd (1949) proposed Quality Control charts for $\bar{X}$ and R adjusted for within subgroup pattern. Subgroup pattern include methods based on moving range and analysis of variance techniques. This method is found effective in particular instances. Burr and Weaver (1949) have introduced stratification control charts. They proposed construction of control charts to properly analyzed data which are obtained in stratified samples. They also, proved that this technique will be desirable to collect data in the form of stratified samples and in some cases, “Stratification Control Charts” and Quality Control Charts for $\bar{X}$ and R adjusted for within subgroup pattern represent two independent and similar solution of a problem. Eisenhart (1949) set up probability control chart limits for standard deviation and Range chart and observed that when the process is under control, for chart for standard deviation $\sigma$ or for range $R$, most of the times more points lie below rather than above the central line. For that he observed that positive skewness of the distribution of $\bar{\sigma}$ and $\hat{R}$ in random samples from a normal population, values less than the average ($C_2 \sigma'$ for $R$, where $\sigma'$ is the population standard deviation.) have a probability of
occurrences slightly in excess of 0.5. Smith (1949) established the relationship between specification limits and control limits. Boyd (1950) studied the group chart for \( \bar{X} \) and R. He has found the group chart for \( \bar{X} \) and R an effective and economical device for charting machining operations in which the use of conventional \( \bar{X} \) and R charts would impose a prohibitive burden of paper work as well as obstacle to rapid overall evaluation of “group-process” of the type described. Springer (1951) developed a method for determining the optimum position of a process mean. Wilson (1952) illustrated how advantage can be taken of available data by applying statistical methods to establish realistic specification and to trace causes of significantly large variation in data. When control charts were first used, they were charts of the averages and standard deviations of small samples or subgroups. But the standard deviation soon gave way to the range, because it is easier to compute. Also, range chart, it is believed, does a better job of locating common types of troubles. Ferrell (1953) proposed a further step in that direction through the use of mid range and the median more specially; it proposed the use of mid range within subgroups and of the median between subgroups. Rott and Mundel (1954) discuss some of their experiences with the use of Narrow limit (NL) gaping. King (1959) presented probability limits for the average chart when process standard are unspecified. Liinghtstone (1954) explained use of Band Chart in quality control. Band chart developed by Maccauley and Meroz. Lightstone gave important advantages of Band Chart in relation to dimensional inspection. Behnken (1957) worked on moving trend control charts. The moving trend approach can
be used either in conjunction with control charts or through a direct comparison with calculated limits. Youden, Connor and Severo (1959) have discussed the problem of choosing the proper size and number of intervals in establishing a standard method for determining the standard deviation of the matching process. Roberts (1959) described a graphical procedure for generating geometric moving average. He also concluded that the graphical procedure for applying geometric weight pattern may prove useful in wide variety of application. Jacjson (1959) introduced quality control methods for several related variables. For this he introduced concept of matrix notation. He also discussed approximate multivariate techniques, designed to simplify the administration of these control programs. Cumulative Sum Control charts were first proposed by Page (1954) and have been studied by many authors. A major disadvantage of any Shewhart control chart is that it only uses the information about the process contained in the last plotted points. This feature makes the Shewhart control chart relatively insensitive to small shift in the process say, on the order of about $1.5\sigma$ or less. Of course, other criteria can be applied to Shewhart charts, such as tests for runs and the use of warning limits which attempt to incorporate information from the entire set of points into the decision procedure. However, the use of these supplemental sensitizing rules reduces the simplicity and ease of interpretation of the Shewhart control chart. The use of these sensitizing rules can dramatically reduce the average rules can dramatically reduce the average run length of the control chart when the process is in control, and this may be undesirable. Page (1951) gave brief
discussion on cumulative sum chart. In (1962), Page described CUSUM schemes using gauging which are developed for controlling the mean and standard deviation of normal distribution. He observed that CUSUM gauged scheme for means seems less affected by changes in standard deviation than Shewhart mean chart. Ewan (1963) outlined the various types of continuous graphical control schemes which are most appropriate. Johnson (1963) developed methods of construction of cumulative sum control charts for folded normal variates. Page (1963) has shown how to control the standard deviation by CUSUMs and warning lines.

Mitten and Sanoh (1961) have developed method of construction of warning limit chart of optimal efficient and have shown how the characteristics of such a chart vary with the variation in maximum permissible number of auxiliary samples. He concluded that \( \bar{X} \) charts with optimally placed warning limits compared to conventional \( \bar{X} \)-charts or non optimal warning charts, they permit very substantial reductions in the amount of inspection required while maintaining the desired level of protection against undetected shifts in the process average.

Enrick (1962) described variation flow analysis for process improvement which is based on modifications of range methods for analysis of variance. Osinski (1962) discussed use of median chart in Rubber industry. Freund (1962) discussed the cumulative sum chart, the geometric moving average chart, and the acceptance control chart. He also concluded that the cumulative sum or geometric moving average chart should be used where greater
sensitivity in detecting small process shifts is required or where the estimation and interpretive properties are useful in data analysis. The acceptance control system is valuable in many batch type operations or wherever there are potential problems connected with over control. The Shewhart chart should be used more often in process development work to determine whether or not a “state of statistical control” has been achieved. Johnson and Leone (1962) have provided much of the mathematical development towards cumulative sum charts. Burr (1967) has discussed the effects of non-normality on constants for $\bar{X}$ and R-charts. He found that there are serious implications in using the ordinary normal curve control chart constants unless the population is marked non normal. Yang and Hiller (1970) have considered two measures, the average subgroup variance and the sample variance and has shown for each case to obtain statistically sound control limits for mean and variance based on a small number of subgroup. Hahn (1971) worked on normality condition. He also examined conditions under which one might or might not expect some variable to be normally distributed and suggested procedures which may be used to check the normality assumption. He also suggested transformations to achieve a better approximation to normality and gave the comments on the consequences of incorrect assumptions of normality. He observed that for many situations, the normal distribution provides a reasonable working approximation. In other situations it does not. The context of the problem and the data itself should help to determine when the assumption of normality can be reasonably used. This article has attempted to provide information which
can serve as a guide in making appropriate decision. Goel and Wu (1971) determined average run length and a contour monogram for CUSUM charts to control normal mean. Chiu and Wetherill (1974) have given sample semi-economic scheme for the design of a control plan using $\bar{X}$-chart for practical application, where the principle for the choice of parameters is to minimize the average loss-cost subject to constraints on the O.C.curve. Weindling Littauer and Olivera (1970) have modified the Shewhart control chart in order to increase its sensitivity to small shifts in process mean for a specified or control values. Also, the effects of action and warning limits on the mean action time were discussed. Stralkowski, Devor, Wu (1974) have studied the chart for the interpretation and estimation of the second order moving average and mixed first order autoregressive moving average models. He has used two charts illustrating the patterns in the theoretical and partial correlation functions which are presented to facilitate the interpretation and identification of the moving average (MA) (2) and auto regressive moving average (ARMA) (1,1) models. He has also given (a) charts and table to provide initial estimates for parameters of MA (2) and ARMA (1, 1) models and (b) a chart and a table for the construction of approximate confidence regions for the models were also examined. A chart to aid this examination for MA (1) models was presented. Duncan (1975) has studied in detail the development of the theory and practice of acceptance sampling by variables to control the fraction defective. Schilling and Nelson (1976) discussed the effect of non-normality on the control limits of $\bar{X}$-charts. The central limit theorem essentially states that under general
conditions the distribution of sample means will approach normally for large sample sizes. By numerically inverting the appropriate characteristic functions, tables are provided which show manner of approach to normality for various underlying distributions and sample sizes. Sample sizes are also given such that, at selected points, the sum of the tail areas of the distribution of sample means will be within given values. Lucas and Crosier (1982) have presented the average run length and the distribution of run length for cumulative sum (CUSUM) schemes with fast initial response feature and have compared fast initial response CUSUM schemes of standard CUSUM scheme. The comparisons show that if the process starts out in control, the fast initial response feature has little effect, however, if the process mean is not at the desired level, an out of control signal will be given faster when the FIR feature is used. Taguchi (1976, 1977) has prompted the use of statistical design of experiment methods for product design improvement. He has also given an excellent starting point for further research in statistical methods for product design improvement.

Off line quality control methods are the quality and cost control activities conducted at the product and process design stages to improve product manufacturability and reliability and to reduce product development and life time costs. Parameter design is an off line quality control methods. At the product design stage the goal of parameter design is to identify settings of product design characteristics that make the product’s performance less sensitive to the effect of environmental variables deterioration, and
manufacturing variations. Because parameter design reduces performance variation by reducing the influence of the sources of variations rather than controlling them, it is a very cost effective technique for improving product quality. Kacker (1985) introduced the concept of off line quality control and parameter design and then discussed the Taguchi method for conducting design experiment. A modified approach to the computation of control limits $\bar{X}$ and R chart is introduced by Langenberg and Iglewicz (1986). This procedure consists of replacing $\bar{X}$ with the trimmed mean of the subgroup ranges where standard table of control limits may continue to be used, requiring some adjustment by a constant multiplier, values of which are tabulated. The proposed control chart limits have been shown to be less influenced by extreme observations than their classical counter parts, and to lead to tighter limits in the presence of outline control observations. This method is simple to use and is a viable refinement of the Shewhart control chart.

Mee, Shah and Lefante (1987) tried to compare k independent sample means with a known standard. Methods are presented for testing the null hypothesis $H_0: \mu_1 = \ldots = \mu_k = \mu_0$ where $\mu_0$ is a specified standard value. Both one sided and two sided alternative hypothesis are considered for each case, an overall test procedure is initially discussed and then followed by multiple comparison procedure. The methods are simple to use and should be widely applicable in quality control inspection. The specified standard value $\mu_0$ is the established long-term mean performance of some standard process. There
will generally be some possibility that due to current conditions, the mean for
the standard process will differ from the assumed mean $\mu_0$.

Crowder (1987) has presented numerical procedure for the tabulation of
average run length (ARLs) of a control chart for individual measurements in
combination with a moving range chart based on two consecutive
measurements. An exact expression for the ARLs is given in terms of an
integral equation approximated using numerical quadrature. ARLs are given for
various setting of the control limits and shifts in the nominal level of the
process mean and standard deviation. A control chart design strategy is also
presented. Standard numerical integration techniques have been used to
evaluate ARLs for various settings of the control limits. The table of ARLs
presented indicates the overall ARL of the procedure. Also, ARLs for a
combination of a control chart for individuals and a moving range chart could
be approximated using similar methods even for non normal process
distributions.

When it is neither easy nor desirable to form rational subgroups a control chart
for individual measurements may provide useful information about a process.
A control chart for individual measurements is used whenever it is desirable to
examine each individual value from the process immediately. This should be
done only when all individual observations are available. Nelson (1982) used
moving range of two for calculating the control limits for individual
measurements.
Rational subgroups for control chart have three properties: their rationality, their size and the frequency with which they are taken. Cost consideration is almost always paramount when the size of a rational subgroup is chosen. Average run length to detect a given change (a function of subgroup size) is traded off with cost of labor and material, when change in the size of the subgroup results in a change in the positions of the control limits. Nelson (1988) calculated new limits for $\bar{X}$ and R-charts when subgroup size based on the exiting data associated with the old subgroup size.

The usual practice in using a control chart to monitor a process is to take samples from the process with fixed sampling intervals. Reynolds, Amin, Arnold and Nachias (1988) considered the properties of the $\bar{X}$-chart when the sampling interval between each pair of samples is not fixed but rather depends on what is observed in the first sample. Reynolds, Amin, Arnold and Nachias (1988) compared between fixed sampling interval (FSI) and variable sampling interval (VSL) for $\bar{X}$-charts. It is also concluded that VSI $\bar{X}$-chart is more efficient. If $\bar{X}$ is actually outside the control limits, then the chart signals in the same way as the standard Fixed Sampling Interval (FSI) $\bar{X}$-chart. Properties such as the average time to signal and an average number of samples to signal are evaluated. Comparisons between fixed sampling interval and variable sampling interval $\bar{X}$-charts indicate that the VSI chart is substantially more efficient.

Since 1950s, the problem of determining economically optimal control chart designs has received considerable attention in the academic literature—see for
example Montgomery (1980) and (1985) Saniga and Antoniuk (1981) and Saniga (1978) (1979) and (1984) have treated the joint economic design of $\bar{X}$ and R control charts. Apart from such works, however, the design of procedures to monitor control process variability appears to have commanded very little attention.

Collani and Sheil (1989) specifically address this gap in the literature by examining control procedure based on economically optimal chart designs. Such charts, the use of which is favored in Shewhart (1931), employ the sample standard deviation as the Statistics to be plotted. A comparison between the approximate and exact design shows that whereas there may be small difference between the design parameters, the approximate design are never more than marginally suboptimal with respect to the optimality criterion of maximizing profit. Saniga (1989) introduces economic statistical control chart designs with an application to $\bar{X}$ and R-charts. He placed statistical constraints on economical models to provide designs that meet industry’s demand for low-process variability and long-term product quality. This constrained economic model yields a design known as an economic statistical design. This model can be readily adopted to design any Shewhart-type control chart. This method has been developed to find the most economical statistical design for Shewhart-type control charts and have been applied for joint determination of the parameters of $\bar{X}$ and R charts. These designs are as good as statistical designs in terms of Statistical properties but are also generally less costly and never more costly under the assumptions of the economical model. These designs are
more costly than economic designs but gives protection over a wider range of shifts, and they have some other advantage as well.

A standard cumulative sum (CUSUM) chart for controlling the process mean takes samples from the process at fixed length sampling intervals and uses a control statistic based on a cumulative sum of differences between the sample means and the target value. Reynolds, Amin and Arnoldes (1990) proposed a modification of the standard CUSUM scheme that varies intervals between samples depending on the value of the CUSUM control statistic. The Variable Sampling Interval (VSI) CUSUM chart uses short sampling intervals if there is an indication that the process mean may have shifted and long sampling intervals if there is no indication of a change in the mean. If the CUSUM statistic actually enters the signal region, then the VSI CUSUM-chart signals in the same manner as the standard CUSUM-chart. A Markov-chain approach is used to evaluate properties such as the average time to signal and the average number of samples to signal. Results show that the proposed VSI CUSUM-chart is considerably more efficient than standard CUSUM-chart.

Crowder (1987) computed average run length (ARL) for combined individual measurements and moving range chart.

Amin and Miller (1993) have studied $\bar{X}$-charts with variable sampling intervals (VSI). They considered the properties of the VSI $\bar{X}$-chart in an environment where the process data are not normally distributed but are contaminated. They evaluated the behaviors of VSI charts where a trimmed mean, a winsorized mean or the median is used as the chart statistic. In conclusion VSI charting
approach, reported by Reynolds (1988) to be useful for normally distributed data, continues to be efficient in detecting process shifts when the data are distributed as a contaminated normal. The trimmed mean performed better than the mean when the normal distribution was contaminated and yet did not suffer significantly when the distribution was normal. When the contamination level was high the median performed best.

The Exponentially Weighted Moving Average (EWMA) control chart is also a good alternative to the Shewhart chart, when we are interested in detecting small shifts. The performance of the EWMA control chart is approximately equivalent to that of the cumulative sum control chart, and in some way it is easier to setup and operate. The EWMA control chart is very effective against small process shifts. Statistical process control charts such as the Shewhart, CUSUM and EWMA have been used extensively to monitor product quality and detect special events that may be indicators of out of control situations. In the discrete parts manufacturing industries the most common charts are $\bar{X}$ and R-charts. Alternatively a CUSUM or EWMA chart on $\bar{X}$ could replace the Shewhart chart. However, since in the continuous process industries the sampling and analysis costs are often quite high, it is extremely common to plot charts on individual observations. Exponentially Weighted Moving Variance (EWMV) and Exponentially Weighted Mean Square Deviation (EWMS) charts are proposed as ways of monitoring various types of continuous process variation. They are particularly useful for augmenting control charts on individual observations where no estimate of variability is available from
replicates and for providing measures of process variance when the observations are auto correlated.

The use of exponentially weighted mean square to monitor variations was suggested by Wortham and Ringer (1971) and Wortham (1972). Sweet (1986) proposed the exponentially weighted mean absolute deviation and exponentially weighted moving variance control limits of them were derived in case of independent observations by Wortham (1972) and Sweet (1986). Others such as Montgomery and Mastrangelo (1991) have advocated the use of exponentially weighted moving variance type statistics. Bauer and Hackl (1978, 1980) investigated moving sums of statistics, again for the case of independent observations. Exponentially weighted mean square deviation charts are also a special case of the omnibus EWMA schemes studied by Domangue and Patch (1991). Ng and Case (1989) investigated some Average Run Length (ARL) Properties for various exponentially weighted moving average and moving ranges and Crowder and Hamilton (1992) used exponentially weighting to Smooth \( \ln(s^2) \) obtained from the range statistic in the conventional \( \bar{X} \) and R-charts. Macgregor and Harris (1993) investigate the ability of the EWMS and EWMV to monitor process with changing mean and variances.

In practice, situations arise that require a charting procedure for individual measurements. Several options in designing a Shewhart-type control chart for individual observations are discussed. Charting of individual observations has received extensive attention in the literature using moving range. Ducan (1974)
proposes a chart for individual observation. Other such as Nelson (1982), Grant and Levenworth (1980), Wadsworth, Stephens and Godfrey (1986), and Wheeler and Chambers (1986) have also worked in same area. Later on Ryan (1989) discussed the limits using standard deviation of the individual observation control chart. Also, Harding, Lee and Mullins (1992) Whetherill and Brown (1991) have worked on this chart. A number of possible estimators of standard deviation are considered. A two stage procedure is suggested for retrospective testing. Roes, Does and Schurink (1993) have studied estimators of spread for use with Shewhart-type control charts for individual measurements. They have shown that displaying a moving range chart provides no real added value and therefore it is ill-advised.

The usual approach to setting up $\bar{X}$-chart entails collecting $m$ samples of size $n$ and using these values to compute an estimate $\bar{X}$ of the process mean $\mu$ and an estimate $\frac{\overline{s}}{c_4}$ of the process standard deviation $\sigma$. Many researchers have recommended the number of samples $m$ and sample size $n$ needed in order to estimate the parameters and control limits for classical Shewhart $3\sigma$ control charts. Many of these recommendations suggest that 20 to 30 samples of size 4 or 5 taken while the process is stable are enough data to treat the estimated control limits as though they are the correct limits. Some authors consider these temporary or “trial” limits until more data are obtained. Quesenberry (1993) studied the effect of sample size on estimated limits for $\bar{X}$ and X control chart. The result of this study indicate that $\bar{X}$-chart with subgroup of size $n$ require
about 400/(n-1) samples and $\bar{X}$-chart require about 300 values to estimate control limits that perform like known limits.

Many innovations have been proposed to improve standard Shewhart $\bar{X}$-chart. Champ and Woodall (1987) determined the properties of control charts with supplementary runs rules. Reynolds, Amin, Arnold and Nachlas (1988) considered the properties of the $\bar{X}$-chart with variable sampling interval (VSI). Cui and Reynolds (1988) considered the $\bar{X}$-chart with runs rules and sampling intervals. Sawalapurkar, Reynolds and Arnold (1990) used the Markov Chain approach to evaluate the properties of $\bar{X}$-charts with variable sampling intervals and sample sizes. Rendtel (1990) considered CUSUM schemes with variable sampling intervals and sample sizes and Prabhu, Montgomery and Runger (1994) considered $\bar{X}$-charts with variable sampling intervals and sample sizes. The economic design of $\bar{X}$ control charts with Variable Sample Size (VSS) was studied by Park and Choi (1992). The VSS $\bar{X}$-chart is more economical than the $\bar{X}$-chart with fixed sample size in terms of the expected cost per hour when only two different sample sizes are used. Daudin (1992) considered the $\bar{X}$-chart with Double Sampling (DS), that is two samples are taken from the process every h hour, but the second sample is analyzed only if the first is not enough to decide if the process is in control. However, in practice the effort required to decide if the process is in control may discourage the use of the $\bar{X}$-chart with double sample. Costa (1994) compared the standard $\bar{X}$-chart with the VSS, VSI and DS $\bar{X}$-charts in terms of the speed of detecting changes in the process. The speeds of
VSS $\bar{X}$-chart was also compared with the speeds of EWMA charts, CUSUM charts and $\bar{X}$-charts with supplementary runs rules. It has been concluded that VSS $\bar{X}$-chart is substantially quicker than the traditional $\bar{X}$-chart in detecting moderate shifts in the process.

The most commonly used techniques in statistical process control are parametric and so they require assumptions regarding the statistical properties of the underlying process. For example, Shewhart control chart assumed that the observations are independent and that the variable of interest is normally distributed. These assumptions are often violated in practice, for example, the distribution of the variable being measured may be strongly skewed or may fail a test for normality. In such cases the control limits, especially for small subgroup sample, may not be accurate. The bootstrap is a computer intensive re-sampling procedure that does not require a priori distribution assumptions. It was developed to find the distribution of a statistic when the distribution is not known. Bootstrap method offer relatively new and generally powerful techniques for statistical inference, the concept of bootstrapping were theoretically developed by Efron (1979, 1982, and 1987) and Efron and Tibshisani (1986) to reflect the uncertainty from sampling without requiring the rigorous methods based on assumptions of normality. Considerable research on and application of the bootstrap has accumulated in 1980’s and 1990’s. Swanepoel (1990), as well as Leger, Politis, and Romano (1992) have provide excellent survey on the bootstrap methods. Bone, Sharma, and Shimp (1987) have applied bootstrap to evaluate goodness of fit indices. Fisher and Hall
(1989) have studied the bootstrap confidence regions. Cooil, Winer, and Rados
(1987) have used the bootstrap for cross validation. Kinsella (1989) have used
the bootstrap to assess bioequivalence measures. Alemyehu and Doksum
(1990) have used the bootstrap in correlation analysis. Biddle, Bruton, and
Siegel (1990) have determined confidence limits for difference and ratio
have used the bootstrap method to estimate confidence intervals for audit
values from skewed population and small samples. A number of articles
discussing the use of the bootstrap with respect to quality have appeared since
Efron’s (1979, 1987, and 1990) has used percentile method and considered the
application of the bootstrap to the assessment of process capability for hole-
drilling errors and life testing of a compressor. The use of bootstrap for
assessing lower limits on process capability has been discussed by Franklin and
for \( \bar{X} \)-chart has been proposed by Bajgier (1992). Seppala (1995) has discussed
the bootstrap percentile method to include a series of subgroup, which are
typically used in assessing process control limits and control charts for
correlated process. Alwan and Roberts (1988), Moskowitz, Plante, and Wardell
(1992), Montgomery and Mastrangelo (1991) and Wardell, Moskowitz, and
Plante (1994) have discussed bootstrap methods for dependent data. Politis
(1991), Lorlstein (1992) have suggested the potential usefulness of bootstrap in
designing control chart for correlated process outputs.
Alwan and Roberts (1995) have discussed the problem of misplaced control limits. Hinckley and Barkan (1995) have studied the role of variation, mistakes and complexity in producing non-conformities.

Quessenberry (1991) has defined Q statistics that can be computed in a number of situations, assuming a normal process distribution and either one or both parameters unknown. Under the assumption that the normal process distribution is stable, (i.e. the mean $\mu$ and standard deviation $\sigma$ are constant), these statistics have distribution that are either exactly or approximately independent. Quessenberry has suggested that these statistics can be plotted on Shewhart charts with control limits at $\pm 3$, and pointed out those tests such as those suggested by Nelson (1984) can be made on these charts. It is also apparent that Q statistics can be used as the input data to plot EWMA and CUSUM charts. For a stable normal process distribution, the several types of Q statistics given in Quesenberry (1991) all have either exactly or approximately standard normal distributions, we know exactly the type of point pattern that represents a stable process on a Shewhart Q chart of these statistics. In order to study the effectiveness of Q charts to detect parameter variation, it is necessary to focus on a particular type of parameter variation. These are many possible tests that can be made on a Shewhart Q chart to detect a shift in $\mu$ or $\sigma$. The sensitivity of four tests on Shewhart type Q charts and of specially designed EWMA and CUSUM Q charts to detect one step permanent shift of either a normal mean or standard deviation has been studied by Quessenberry (1995). Three of these tests have been discussed by Nelson (1984).
Multivariate statistical process control uses the relationships between variables to improve the detection of assignable cause in processes. Process-monitoring problems in which several related variables are of interest are sometimes called multivariate quality control (or process monitoring) problems. The original work in multivariate quality control was done by Hotelling (1947). Subsequent papers dealing with control procedures for several related variables include Hicks (1955), Jackson (1956, 1959, and 1985), Crosier (1958), Hawkins (1991, 1993), Lowry (1992), Lowry and Montgomery (1985), Pignatiello and Runger (1993), Tracy, Young, and Mason (1992), Montgomery and Wadsworth (1972) and Alt (1985). Multivariate control charts such as Chi-square Charts, multivariate CUSUM charts or multivariate EWMA charts are also used to detect a shift in the mean vector of several variables. Runger (1996) used simple projection method for designing multivariate control scheme.

Amin, Wolf, Besenfelder, and Baxley (1999) have proposed maxmin EWMA control chart based on the smallest (min) and largest (max) observation in each sample. Costa (1999) has provided an example to motivate the use of a joint $\bar{X}$ and R-charts with variable samples. Costa (1999) studied the Variable Parameter (VP) $\bar{X}$-chart. Also, concluded that VP $\bar{X}$-chart is more powerful than CUSUM scheme in detecting small shifts in the process mean. Zhang Wu and Spedding (2000) have proposed a synthetic control chart that is an integration of the Shewhart $\bar{X}$-chart and the conforming run length (CRL) chart. Using the proposed synthetic chart may be more complex than using the Shewhart $\bar{X}$-chart, however, performance test indicate that the synthetic chart
has a greater detecting power for shift in the process mean than the Shewhart $\overline{X}$-chart and many other chart. Koning and Does (2000) have proposed cumulative sum (CUSUM) chart for preliminary analysis of individual observation. In this paper they have considered individual observations and focused on the retrospective situation. After introducing standardized recursive residuals and Q-Statistics, they have developed a new CUSUM-type chart for the detection of linear trends based on the uniformly most powerful test.

Misiorek and Barnett (2000) have studied mean selection for filling process under weights and measures requirement where weights and measures requirements of using the optimal mean has been investigated. Costa-Mejia and Pignatiello Jr. (2000) have analyzed several control charts suitable for monitoring process dispersion when subgrouping is not possible or not desirable. They have compared the performance of moving ranges, a CUSUM chart based on an approximate normalizing transformation, a self-stating CUSUM chart, a change point CUSUM chart and an exponentially weighted moving average chart based on the subgroup variance. The average run length performances of these charts are also estimated and compared. They have also, concluded that when monitoring process dispersion with subgroup size one, the Change Point (CP) CUSUM has a better ARL performance than the CUSUM procedures. Except for the CPCUSUM chart, they have found that these other charts can be ARL-biased, which can lessen their ability to detect quality improvement, that is, deceases in the process standard deviation. When process mean is unknown they have suggested modifications to Hawkins’ (1981)
procedure and have introduced a change-point CUSUM control chart for monitoring process dispersion. They have also analyzed the CUSUM based on moving ranges and Hawkins’ self-stating CUSUM. Again they have found that the CP CUSUM is ARL-unbiased and has the best ARL performance in detecting both increases and decreases in process dispersion. Klein (2000) has discussed two alternatives to the Shewhart $\bar{X}$ control chart. Average run length (ARL) values are calculated for two $\bar{X}$ control chart schemes and compared with those of a standard Shewhart chart. Both control charts are based on runs rules and are easy to implement. An out of control condition for one of the chart is a run of two successive points beyond a special control limits. The other chart used a run of two of three successive points beyond a different control limit. Both scheme have been shown to have better, that is lower, ARL values than the standard Shewhart chart for process average shifts as large as $2\sigma$ standard deviations from the mean. It has also been concluded that proposed control charts, in ARL terms are not good as compared to EWMA schemes.

Neduraran and Pignatiello Jr. (2001) have proposed an approach for constructing control limits that attempts to match any specific percentile point of run length distribution of the true limits, even when the limits are estimated using data from only a few subgroups. They have compared the performance of the proposed approach with that of the standard approach through Manto Carlo simulation experiments. The simulation results have shown that the control limits constructed using proposed method perform similarly as the true limits even when they are estimated from a small number of subgroup.
Naik and Desai (2005) have developed mean and root mean square deviation (RMSD) control charts based on deviations taken from targeted process mean (TPM), mean and mean of absolute deviation (MOAD) control chart based on deviations measured about targeted process mean and mean and mean squared deviation (MSD) control charts based on deviation measured about targeted process mean.

1.4 Orientation to the Research Work Considered in this Thesis

A fundamental assumption in the development of $\bar{X}$, R and s charts is that the underlying distribution of the quality characteristic is normal. The normal distribution is one of the most important distributions in the statistical inference in which mean ($\mu$) and standard deviation ($\sigma$) are the pivotal parameters of this distribution.

Several authors have studied the effect of departures from normality on control charts. Burr (1967) has noted that the usual normal theory control limit constants are very robust to the normality assumption and can be employed unless the population is extremely nonnormal. Schilling and Nelson (1976), Chan, Hapuarachchi, and Macpherson (1988), and Yourstone and Zimmer (1992) have also studied the effect of nonnormality on the control limits of the $\bar{X}$ chart. Schilling and Nelson investigated the uniform, right triangular, gamma and two bimodal distributions formed as mixtures of two normal distributions. Their study indicates that, in most cases, samples of size four or five are sufficient to ensure reasonable robustness to the normality assumption.
Desai (2011) have suggested an alternative of normal distribution, which is called moderate distribution. In moderate distribution mean ($\mu$) and mean deviation ($\delta$) are the pivotal parameter and, which has properties similar to normal distribution. Under this statistical model with different values of $\mu$ and $\delta$, one gets different moderate distributions. All moderate distributions are symmetric and bell-shaped. Desai (2011) have also prepared moderate table (Similar to normal table) pertaining to the area under standard moderate curve. With the help of this table, we can check the significant moderation taking place in the distribution of probability in normal distribution when for the fixed value of first degree dispersion; the dispersion parameter $\sigma$ (standard deviation) is replaced by another dispersion parameter, $\delta$ (mean deviation). For example, in standard normal distribution having FDD, $K= \sigma =1$, about 3 observations out of 10 are expected to lie outside the range $\mu \pm K$ whereas in moderate distribution with $K=\delta =1$, about 4 observations out of 10 are expected to lie outside the range $\mu \pm K$. Similarly, for normal distribution with FDD, $K= \sigma =3$, about 27 observations out of 10,000 are expected to lie outside the range $\mu \pm 3K$ whereas in moderate distribution with $K=\delta =3$, about 167 observations out of 10,000 are expected to lie outside the range $\mu \pm 3K$. They have also suggested the central limit theorem based on the mean error of statistic rather than standard error, which is very useful for the construction of the proposed control charts. Keeping these concepts in mind, an attempt has been made to develop the main traditional Shewhart type control charts under the assumption that the underlying distribution of the quality characteristic is moderate.
Control limits are key factor in the quality control charts. Shewhart has suggested $3 \times$ (standard error) (i.e., $3\sigma$) control limits when the production process follows normal distribution. It can be seen that in a normal distribution about $99.73\%$ of the observation lie within the interval $\text{mean} \pm 3\sigma$. Therefore, replacing moderate distribution for normal distribution and dispersion parameter $\delta$ (mean deviation about mean) for $\sigma$ we have proposed conventional control charts for variable under moderateness assumption and $3 \times$ (mean error) (i.e., $3\delta$) control limits in this thesis.

The work of this thesis has originated from such thoughts.

### 1.5 Layout of The Thesis

This thesis is primarily divided into five chapters.

The present chapter-1 contains general introduction, brief history of Statistical Quality Control and review of literature on topics related to the topics of research.

Chapter-2 contains the basic theory of Statistical Quality Control charts for variable. It also involves the theory and the formula of existing control charts for variables under the assumption of normality. Various properties of exiting control charts for variables as well as some merits and demerits of the charts are also discussed in this chapter.

In chapter-3, moderate distribution and its various properties are discussed in brief. It also contains some aspects of general theory of control charts under moderate distribution. In this chapter preferring mean deviation ($\delta$) over
standard deviation ($\sigma$) as dispersion parameter, we have proposed $3\delta$ control limits rather than $3\sigma$ control limits.

After having proposed and discussed $3\delta$ control limits in the chapter-3, control limits for the control charts $\bar{X}$, $R$, $s$ and $d$-charts are derived and general procedure for construction of these charts is discussed in chapter-4. Also an empirical study has been carried out for the data-sets given in appendix-1.

While selecting control charts, it is very important to know how effective they are. There are two commonly used methods for comparing control chart performance. The first method is to determine and study the chart’s operating characteristic (OC) curve. The second is to determine and study the average run length (ARL).

In chapter-5, efficiency analysis has been carried out for the two types of control charts discussed in chapter-2 (i.e. under normality assumption) and chapter-4 (i.e. under moderateness assumption) using these two methods. Also comparisons are made through OC curves and ARL curves in this chapter.

Appendices and References are given at the end of the thesis.