# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER NO.</th>
<th>TITLE</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td>LIST OF TABLES</td>
<td>xv</td>
</tr>
<tr>
<td></td>
<td>LIST OF FIGURES</td>
<td>xvi</td>
</tr>
<tr>
<td></td>
<td>LIST OF SYMBOLS AND ABBREVIATIONS</td>
<td>xvii</td>
</tr>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>AN OVERVIEW</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>PATHWAY MODEL</td>
<td>2</td>
</tr>
<tr>
<td>1.3</td>
<td>SPECIAL FUNCTIONS</td>
<td>10</td>
</tr>
<tr>
<td>1.3.1</td>
<td>Gamma Function</td>
<td>11</td>
</tr>
<tr>
<td>1.3.2</td>
<td>Residue Calculus for Gamma Functions</td>
<td>12</td>
</tr>
<tr>
<td>1.3.3</td>
<td>Mellin-Barnes Integrals</td>
<td>13</td>
</tr>
<tr>
<td>1.3.4</td>
<td>Beta Integrals</td>
<td>13</td>
</tr>
<tr>
<td>1.3.5</td>
<td>Generalized Hypergeometric Series</td>
<td>14</td>
</tr>
<tr>
<td>1.3.6</td>
<td>Wright Hypergeometric Function</td>
<td>15</td>
</tr>
<tr>
<td>1.3.7</td>
<td>Meijer G-Function</td>
<td>15</td>
</tr>
<tr>
<td>1.3.8</td>
<td>Fox’s H-Function</td>
<td>16</td>
</tr>
<tr>
<td>1.4</td>
<td>INTEGRAL TRANSFORMS</td>
<td>19</td>
</tr>
<tr>
<td>1.4.1</td>
<td>Laplace Transform</td>
<td>20</td>
</tr>
<tr>
<td>1.4.2</td>
<td>Mellin Transform</td>
<td>20</td>
</tr>
<tr>
<td>1.5</td>
<td>FRACTIONAL CALCULUS</td>
<td>21</td>
</tr>
</tbody>
</table>
CHAPTER NO.  TITLE  PAGE NO.

1.5.1  Riemann-Liouville Fractional Operators  22
1.5.2  Erdély- Kober Fractional Operators  23
1.5.3  Saigo Fractional Operators  23
1.6  SUMMARY OF THE THESIS  24

2  GENERALIZED KRÄTZEL FUNCTION  28

2.1  INTRODUCTION  28
2.2  TYPE-1 AND TYPE-2 GENERALIZED KRÄTZEL FUNCTIONS  30
2.3  GENERALIZED KRÄTZEL FUNCTION AND FRACTIONAL OPERATORS  31
2.4  INEQUALITIES INVOLVING GENERALIZED KRÄTZEL FUNCTION
2.4.1  Complete Monotonicity and Log-Convexity  34
2.4.2  Laguerre Type Inequality  37
2.4.3  Turán Type Inequality  38
2.5  GENERALIZED KRÄTZEL FUNCTION AS AN $\psi$-FUNCTION  40
2.6  ASYMPTOTIC ESTIMATES OF GENERALIZED KRÄTZEL FUNCTION  46
2.7  GENERALIZED KRÄTZEL FUNCTION IN COMPUTABLE SERIES FORMS  48
2.8  INTEGRALS INVOLVING GENERALIZED KRÄTZEL FUNCTION  52
<table>
<thead>
<tr>
<th>CHAPTER NO.</th>
<th>TITLE</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \mathcal{P} )-TRANSFORM AND ( P_{\alpha} )-TRANSFORM</td>
<td>55</td>
</tr>
<tr>
<td>3.1</td>
<td>INTRODUCTION</td>
<td>55</td>
</tr>
<tr>
<td>3.2</td>
<td>( \mathcal{P} )-TRANSFORM AND ITS PROPERTIES</td>
<td>56</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Behaviour of Kernel Function of ( \mathcal{P} )-Transform</td>
<td>57</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Linearity Property</td>
<td>59</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Shifting Property</td>
<td>60</td>
</tr>
<tr>
<td>3.2.4</td>
<td>Differential Operator and ( \mathcal{P} )-Transform</td>
<td>61</td>
</tr>
<tr>
<td>3.3</td>
<td>CONNECTION OF ( \mathcal{P} )-TRANSFORM WITH ( \mathcal{I} )-TRANSFORM</td>
<td>65</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Laplace Transform of a ( \mathcal{P} )-Transform</td>
<td>70</td>
</tr>
<tr>
<td>3.4</td>
<td>FRACTIONAL CALCULUS OF ( \mathcal{P} )-TRANSFORM</td>
<td>73</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Saigo Fractional Operator and ( \mathcal{P} )-Transform</td>
<td>73</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Riemann-Liouville Fractional Operator and ( \mathcal{P} )-Transform</td>
<td>79</td>
</tr>
<tr>
<td>3.5</td>
<td>( \mathcal{P} )-TRANSFORM OF SOME GENERALIZED SPECIAL FUNCTIONS</td>
<td>82</td>
</tr>
<tr>
<td>3.5.1</td>
<td>( \mathcal{P} )-Transform of a Power Function</td>
<td>82</td>
</tr>
<tr>
<td>3.5.2</td>
<td>( \mathcal{P} )-Transform of an ( \mathcal{I} )-Function</td>
<td>84</td>
</tr>
<tr>
<td>3.5.3</td>
<td>( \mathcal{P} )-Transform of a Generalized Hypergeometric Series</td>
<td>86</td>
</tr>
<tr>
<td>CHAPTER NO.</td>
<td>TITLE</td>
<td>PAGE NO.</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>3.6</td>
<td>$\mathcal{P}_\alpha$-TRANSFORM AND ITS PROPERTIES</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>3.6.1 Composition of $\mathcal{P}_\alpha$-Transform with Differentiability</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>3.6.2 Composition of $\mathcal{P}_\alpha$-Transform With Integrability</td>
<td>93</td>
</tr>
<tr>
<td>3.7</td>
<td>$\mathcal{P}_\alpha$-TRANSFORM OF SOME ELEMENTARY FUNCTIONS</td>
<td>95</td>
</tr>
<tr>
<td>3.8</td>
<td>$\mathcal{P}_\alpha$-TRANSFORM OF SOME GENERALIZED SPECIAL FUNCTIONS</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>3.8.1 $\mathcal{P}_\alpha$-Transform of a Power Function</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>3.8.2 $\mathcal{P}_\alpha$-Transform of a Mittag-Leffler Function</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>3.8.3 $\mathcal{P}_\alpha$-Transform of a Generalized Hypergeometric Series</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>3.8.4 $\mathcal{P}_\alpha$-Transform of a Wright Hypergeometric Function</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>3.8.5 $\mathcal{P}_\alpha$-Transform of an $H$-Function</td>
<td>99</td>
</tr>
<tr>
<td>3.9</td>
<td>APPLICATIONS OF $\mathcal{P}_\alpha$-TRANSFORM</td>
<td>101</td>
</tr>
</tbody>
</table>

4 EXTENSION OF THERMONUCLEAR FUNCTIONS AND FUSION YIELD 106

4.1 INTRODUCTION 106

4.2 EXTENDED REACTION RATES IN CLOSED FORMS 113

4.2.1 Evaluation of $I_{3\alpha}(\nu, a, \delta, x, \rho)$ 115
<table>
<thead>
<tr>
<th>CHAPTER NO.</th>
<th>TITLE</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.2</td>
<td>Evaluation of $I_{2\gamma}^{(d)}(\nu, \alpha, \delta, x, \rho)$</td>
<td>119</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Behaviour of $I_{1\alpha}$, $I_{2\gamma}^{(d)}$ and $I_{3\gamma}$</td>
<td>121</td>
</tr>
<tr>
<td>4.3</td>
<td>SERIES REPRESENTATIONS</td>
<td>123</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Series Representation for Extended</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>Maxwell-Boltzmann Case</td>
<td></td>
</tr>
<tr>
<td>4.3.2</td>
<td>Series Representation for</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>Extended Cut-off Case</td>
<td></td>
</tr>
<tr>
<td>4.4</td>
<td>FUSION YIELD INTEGRAL FOR SHOCK-COMPRESSED</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>HEATED PLASMA</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>FUSION ENERGY INTEGRAL BY</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>EXTENDED REACTION RATES</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>EXTENDED STELLAR NUCLEAR</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>ENERGY GENERATION RATES</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>ANALYSIS AND COMPARISON</td>
<td>151</td>
</tr>
<tr>
<td>4.7.1</td>
<td>Comparison of Maxwell-Boltzmann</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>and Pathway Densities</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>CONCLUSION AND FUTURE SCOPE</td>
<td>157</td>
</tr>
<tr>
<td>5.1</td>
<td>CONCLUSION</td>
<td>157</td>
</tr>
<tr>
<td>5.2</td>
<td>FUTURE SCOPE</td>
<td>159</td>
</tr>
</tbody>
</table>

REFERENCES                                                                 | 161     |

LIST OF PUBLICATIONS                                                       | 176     |
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Special Cases of Generalized Type-1 Beta Form of Pathway Model</td>
<td>4</td>
</tr>
<tr>
<td>1.2</td>
<td>Special Cases of Generalized Type-2 Beta Form of Pathway Model</td>
<td>5</td>
</tr>
<tr>
<td>1.3</td>
<td>Special Cases of Generalized Gamma Form of Pathway Model</td>
<td>7</td>
</tr>
<tr>
<td>3.1</td>
<td>Table of $I_{\gamma}$-Transform</td>
<td>95</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NO.</th>
<th>TITLE</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Behaviour of $h_1(y)$ for Various Values of $\alpha &lt; 1$</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>Behaviour of $h_2(y)$ for Various Values of $\alpha &gt; 1$</td>
<td>6</td>
</tr>
<tr>
<td>3.1</td>
<td>Behaviour of $Z^{\mu,\alpha-}(x)$ for Various Values of $\alpha &lt; 1$</td>
<td>58</td>
</tr>
<tr>
<td>3.2</td>
<td>Behaviour of $Z^{\mu,\alpha+}(x)$ for Various Values of $\alpha &gt; 1$</td>
<td>58</td>
</tr>
<tr>
<td>4.1</td>
<td>Behaviour of $I_{1,\alpha}$ for Various Values of $\alpha &gt; 1$</td>
<td>121</td>
</tr>
<tr>
<td>4.2</td>
<td>Behaviour of $I_{2,\alpha}^{(d)}$ for Various Values of $\alpha &lt; 1$</td>
<td>122</td>
</tr>
<tr>
<td>4.3</td>
<td>Maxwell-Boltzmann Case or the Limiting Situation $\alpha = 1$</td>
<td>122</td>
</tr>
<tr>
<td>4.4</td>
<td>Depletion for $\delta = 1, 2, 3$ and $\alpha = 1, 1.2, 1.3, 1.4$</td>
<td>123</td>
</tr>
<tr>
<td>4.5</td>
<td>Behaviour of $\alpha$ for Various Values of $E \left( \frac{E}{kT} \right)$</td>
<td>153</td>
</tr>
<tr>
<td>4.6</td>
<td>$f_{PD}(E)$ for Various Values of $\alpha$ at $k=1$ and Temperature (a) 373K and (b) 1000K</td>
<td>153</td>
</tr>
<tr>
<td>4.7</td>
<td>Pathway Energy Density for $\alpha = 1, 1.2, 1.3, 1.4$ and for $k=1, T = 100K$</td>
<td>154</td>
</tr>
<tr>
<td>4.8</td>
<td>Maxwell-Boltzmann Energy Density for $k = 1$, $T = 100K$</td>
<td>154</td>
</tr>
<tr>
<td>4.9</td>
<td>Schematic Plot of the Energy-Dependent Factors for the Standard Reaction Rate Probability Integral</td>
<td>156</td>
</tr>
<tr>
<td>4.10</td>
<td>Schematic Plot of the Energy-Dependent Factors for the Extended Reaction Rate Probability Integral</td>
<td>156</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS AND ABBREVIATIONS

\[ |(\cdot)| \quad - \quad \text{Absolute value of } (\cdot)
\]
\[ \sim \quad - \quad \text{Asymptotically equivalent to}
\]
\[ N_A \quad - \quad \text{Avagadro’s constant}
\]
\[ k \quad - \quad \text{Boltzmann’s constant}
\]
\[ g_c \quad - \quad \text{Central density of the Sun}
\]
\[ P_c \quad - \quad \text{Central pressure of the Sun}
\]
\[ T_c \quad - \quad \text{Central temperature of the Sun}
\]
\[ M_{\alpha}(g) \quad - \quad \text{Continuous analogue of Mathai’s entropy of order } \alpha
\]
\[ S(E) \quad - \quad \text{Cross section factor}
\]
\[ \tilde{\alpha} \quad - \quad \text{Electromagnetic fine structure constant}
\]
\[ E_{ij} \quad - \quad \text{Energy release by the fusion reaction}
\]
\[ \gamma \quad - \quad \text{Euler’s constant}
\]
\[ \mathcal{E}(\cdot) \quad - \quad \text{Expected value of } (\cdot)
\]
\[ \langle \sigma v \rangle_{ij} \quad - \quad \text{Expected value of } \sigma v
\]
\[ \tilde{E}_{\text{fusion}} \quad - \quad \text{Extended fusion yield}
\]
\[ \tilde{r}_{ij} \quad - \quad \text{Extended reaction rates}
\]
\[ I_{p,q}^{m,n}(\cdot) \quad - \quad \text{Fox’s } I/I\text{-function}
\]
\[ \Gamma(z) \quad - \quad \text{Gamma function}
\]
\[ E_G \quad - \quad \text{Gamow energy}
\]
\[ pF_q(\cdot) \quad - \quad \text{Generalized hypergeometric series}
\]
\[ E_{\beta,\rho}^{\beta}(z) \quad - \quad \text{Generalized Mittag-Leffler function with three parameters}
\]
\[ E_{\beta,\rho}(z) \quad - \quad \text{Generalized Mittag-Leffler function with two parameters}
\]
\[ G \quad - \quad \text{Gravitational constant}
\]
\[ \mathcal{D} \quad - \quad \text{Integer order differential operator} \]
\[ ([\cdot]) \quad - \quad \text{Integer part of } (\cdot) \]

\[ E \quad - \quad \text{Kinetic energy of the particles} \]

\[ Z_p^{(n)}(x) \quad - \quad \text{Krätzel function} \]

\[ (K^d_{\beta} f)(x) \quad - \quad \text{Krätzel transform} \]

\[ \delta_{ij} \quad - \quad \text{Kronecker delta} \]

\[ L_f(s), (L_f)(s) \quad - \quad \text{Laplace transform of } f \]

\[ \mathbb{L}_{\alpha}(0, \infty) \quad - \quad \text{Lebesgue measurable complex valued functions} \]

\[ (T^{-\gamma}_{\alpha}, f)(x) \quad - \quad \text{Left-sided Erdélyi-Kober fractional operator} \]

\[ (T^{\gamma}_{\alpha}, f)(x) \quad - \quad \text{Left-sided Riemann-Liouville fractional integral} \]

\[ (I^{-\gamma}_{0+}, f)(x) \quad - \quad \text{Left-sided Saigo fractional operator} \]

\[ M^{\alpha}_{k,\alpha}(P) \quad - \quad \text{Mathai's additive entropy of order } \alpha \]

\[ M^{\alpha}_{k,\alpha}(P) \quad - \quad \text{Mathai's entropy of order } \alpha \]

\[ f_{MBD} \quad - \quad \text{Maxwell-Boltzmann energy density} \]

\[ G_{p,q}^{m,n}(z) \quad - \quad \text{Meijer's } G\text{-function} \]

\[ \mathfrak{L} \quad - \quad \text{Mellin-Barnes contour} \]

\[ \mathcal{M}_f(s), (\mathcal{M}_f)(s) \quad - \quad \text{Mellin transform of } f \]

\[ E_\beta(z) \quad - \quad \text{Mittag-Leffler function with one parameter} \]

\[ D^m f(\cdot) \quad - \quad m\text{th integer order derivative of } f(\cdot) \]

\[ f^{(m)}(\cdot) \quad - \quad m\text{th integer order derivative of } f(\cdot) \]

\[ \ln(\cdot) \quad - \quad \text{Natural logarithm of } (\cdot) \]

\[ \sigma(E) \quad - \quad \text{Nuclear reaction cross section} \]

\[ f_{PD} \quad - \quad \text{Pathway energy density} \]

\[ \alpha \quad - \quad \text{Pathway parameter} \]

\[ h \quad - \quad \text{Planck's quantum of action} \]

\[ (a)_p \quad - \quad \text{Pochhammer symbol} \]
$\psi(z)$  - Psi function

$P_\alpha[f(t),s]$  - $P_\alpha$-transform

$\Re(\cdot)$  - Real part of $(\cdot)$

$\text{res}_{z=a} f(z)$  - Residue of $f(z)$ at $z = a$

$(D_{-}^{-\lambda}f)x$  - Riemann-Liouville fractional derivative

$(K_{\gamma}^{-\alpha}, f)x, (I_{-1,1}^{-\alpha}, f)x$  - Right-sided Erdélyi-Kober fractional operator

$(\mathcal{I}_{-}^{-\lambda}f)x$  - Right-sided Riemann-Liouville fractional integral

$(I_{-\alpha}^{-\alpha}, f)x$  - Right-sided Saigo fractional operator

$\mathbb{C}$  - Set of complex numbers

$\mathbb{Q}^+$  - Set of positive rational numbers

$\mathbb{R}^+$  - Set of positive real numbers

$\mathbb{R}$  - Set of real numbers

$\rho(r)$  - Solar density distribution

$\eta(E)$  - Sommerfeld parameter

$r_{ij}$  - Standard reaction rates

$L_r$  - Stellar luminosity

$M(r)$  - Stellar mass

$T(r)$  - Stellar temperature

$\rightarrow$  - Tends to

$l_{2,\alpha}^{(d)}$  - Thermonuclear function in the extended cut-off case

$l_{3,\alpha}^{(d)}$  - Thermonuclear function in the extended depleted case

$l_{1,\alpha}$  - Thermonuclear function in the extended Maxwell-Boltzmann case

$l_{2}^{(d)}$  - Thermonuclear function in the standard cut-off case

$l_3$  - Thermonuclear function in the standard depleted case
\( I_1 \) - Thermonuclear function in the standard Maxwell-Boltzmann case

\( I_4 \) - Thermonuclear function in the standard screened case

\( Z_\rho^{\mu,\alpha} (x) \) - Type-1 generalized Krätzel function

\( (\mathcal{P}_{\nu}^{\lambda,\beta,\alpha} f)x \) - Type-1 \( \mathcal{P} \)-transform

\( Z_\rho^{\mu,\alpha-} (x) \) - Type-1 \( \mathcal{P} \)-transform kernel

\( Z_\rho^{\mu,\alpha+} (x) \) - Type-2 generalized Krätzel function

\( (\mathcal{P}_{\nu}^{\lambda,\beta,\alpha+} f)x \) - Type-2 \( \mathcal{P} \)-transform

\( Z_\rho^{\mu,\alpha} (x) \) - Type-2 \( \mathcal{P} \)-transform kernel

\( \wedge \) - Wedge product

\( p \Psi_q(\cdot) \) - Wright hypergeometric function