Appendix-2:

One of the basic plasma fluid equation that are used to generate the evolution of plasma density with time is continuity equation. In the present problem where quasi charge neutrality is assumed, the electron continuity equation is solved as the other equation for ions becomes redundant. The following continuity equation, wherein the two dimensional transport due to perturbation potential $\phi_i(x,y)$ and the loss due to chemical recombinations are included, is solved to obtain evolution of electron densities $n(x,y)$.

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x} \left( \frac{nc}{B} \frac{\partial \phi_i}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{nc}{B} \frac{\partial \phi_i}{\partial x} \right) = -\nu \n$$  (1)

Where $\nu$ represents recombination rate which is given as input. Here the constants $c$ and $B$ are the velocity of light and strength of the earth magnetic field respectively. The perturbation potential $\phi_i^o(x,y)$ is obtained by solving its differential equation and by knowing $n^o(x,y)$, the $t_o + \Delta t$ equation (1) is solved for $n^o(x,y)$.

The equation (1) is solved numerically over the region 100Km east to 100Km west in zonal direction and 252 Km to 534Km in vertical direction. As mentioned in the appendix 1, the region is divided into $N-1 \times M-1$ grid points denoted by $J$ and $I$ with the grid sizes $\Delta x = 5Km$ and $\Delta y = 2Km$ in $x$ and $y$ directions respectively. Let $n(I,J)$ be the value of the electron density at the grid point $(I,J)$. The following boundary conditions are assumed.

$$n(I,1) = n(I,N) \quad \text{for } I=1,M$$
\[
\frac{\partial n_{i,j}}{\partial t} = 0 \quad \text{for } j=1,N 
\]

\[
\frac{\partial n_{m,j}}{\partial t} = 0 \quad \text{for } j=1,N
\]

Let \( V_{x(i,j)} \) and \( V_{y(i,j)} \) are the drift velocities of the electrons at the grid point \((i,j)\) in zonal(x) and vertical(y) directions respectively due to the perturbation potential \( \xi \). They are obtained by the following difference equations

\[
V_{x(i,j)} = -c \frac{\partial \xi}{\partial y} \approx -c \frac{\xi_{i+1,j} - \xi_{i-1,j}}{2 \Delta y} 
\]

\[
V_{y(i,j)} = c \frac{\partial \xi}{\partial x} \approx c \frac{\xi_{i,j+1} - \xi_{i,j-1}}{2 \Delta x}
\]

The time step \( \Delta t \) is chosen in such way that both the stability criterion and positivity of \( n(x,y) \) are satisfied.

\[
\Delta t \leq \frac{1}{2V_{x(i,j)}V_{y(i,j)}} \left( \frac{\Delta x}{\Delta y} \right)^{1/2}
\]

In the present case, \( \Delta t \) varies from 20 sec. to 1 sec at different temporal stages of the solution.

The equation (1) can be written as

\[
\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}(f) + \frac{\partial}{\partial y}(g) = -n \eta
\]

where \( f \) and \( g \) are fluxes in \( x \) and \( y \) directions respectively. They are defined as \( f= -V_x n \) and \( g= V_y n \).

The equation (4) is solved by explicit finite difference method. The finite difference approximation for \( \frac{\partial f}{\partial x} \) and \( \frac{\partial g}{\partial y} \) can be of second or fourth order. When the electron densities \( n(x,y) \) and the velocities \( (V_{x}(x,y) \) and \( V_{y}(x,y) \)) are
smooth, second order difference schemes can be used satisfactorily. When there are steep gradients in \( n \), the truncation error by the second order difference scheme may be as large as that of the solution. Even by using higher order difference scheme, the truncation error may be reduced but the solution obtained may contain spurious oscillations wherever steep gradients are present. In order to circumvent these problems, Flux Corrected Transport (F.C.T) techniques were developed by Boris and Book (1973, 1976) and Book et al (1975). This method was modified for fully multidimensional problems by Zalesak (1979). The F.C.T technique constructs the net transportive flux, every point (non linearly) as a weighted average of flux computed by a lower order and flux computed by a higher order scheme. The weighting is done in a manner which insures that the higher order flux is used to the extent possible without introducing spurious ripples. The following method as described in Zalesak (1979) is used to solve equation 4. The procedure adopted is as follows:

If equation (4) is written as

\[
 \frac{t+\Delta t}{\Delta x}(I,J) = \frac{F(I,J+1) - F(I,J)}{\Delta x} + \frac{G(I+1,J) - G(I,J)}{\Delta y} - \nu \frac{t}{\Delta t}(I,J) \Delta t
\]

(5)

where \( F_{I,j} \) and \( G_{I,j} \) are called transportive fluxes and are functions of \( t \) at one or two time levels.

1) Low order transportive fluxes \( F_{I,j}^L \) and \( G_{I,j}^L \) are computed using Lax-Friedrichs scheme.

2) High order transportive fluxes \( F_{I,j}^H \) and \( G_{I,j}^H \) are computed
using Lax-Wendroff scheme with fourth order differences for space derivative.

3) Low order time advanced solution of $n_{i,j}^{t+\Delta t}$ is obtained by substituting $F_{i,j}^L$ and $G_{i,j}^L$ in equation (5) in place of $F_{i,j}$ and $G_{i,j}$.

4) The antidiffusive fluxes are defined as:

$$AF_{i,j} = F_{i,j}^H - F_{i,j}^L$$

$$AG_{i,j} = G_{i,j}^H - G_{i,j}^L$$

5) Antidiffusive fluxes are limited by the correction factors $C_{1,i,j}$ and $C_{2,i,j}$ which lie between 0 and 1. The corrected antidiffusive fluxes ($AF_{i,j}^C$ and $AG_{i,j}^C$) are obtained as follows:

$$AF_{i,j}^C = C_{1,i,j} AF_{i,j}$$

$$AG_{i,j}^C = C_{2,i,j} AG_{i,j}$$

The fully multidimensional algorithm developed by Zalesak (1979) gives the procedure to obtain $C_{1,i,j}$ and $C_{2,i,j}$.

6) Using the corrected antidiffusive fluxes the final time advanced solution of $n_{i,j}^{t+\Delta t}$ is obtained as follows:

$$n_{i,j}^{t+\Delta t} = n_{i,j}^{td} - (AF_{i,j}^C - AF_{i,j-1}^C + AG_{i,j}^C - AG_{i,j-1}^C)$$

Procedure for the evaluation of correction factors:

The algorithm developed by Zalesak (1979) is used to obtain the correction factors. The following quantities are calculated to determine the correction factors $C_{1,i,j}$ and $C_{2,i,j}$.
\[ W_{ij} = \text{Max} \left( n_{lo}^{i+1, j}, n_{lo}^{i-1, j} \right) \]  

(9)

\[ W_{ij} = \text{Max} \left( W_{i-1, j}^{a}, W_{i, j-1}^{a}, W_{i, j+1}^{a}, W_{i+1, j}^{a} \right) \]

\[ W_{ij} = \text{Min} \left( n_{lo}^{i+1, j}, n_{lo}^{i-1, j} \right) \]

(10)

\[ W_{ij} = \text{Min} \left( W_{i-1, j}^{b}, W_{i, j-1}^{b}, W_{i, j+1}^{b}, W_{i+1, j}^{b} \right) \]

Then the following conditions are used

\[ AG_{ij} = 0 \text{ if } AG_{ij} \left( n_{ld}^{I+2, J} - n_{ld}^{I, J} \right) < 0 \]

and either

\[ AG_{ij} \left( n_{ld}^{I+2, J} - n_{ld}^{I, J} \right) < 0 \]

\[ \text{or} \]

\[ AG_{ij} \left( n_{ld}^{I, J+2} - n_{ld}^{I, J} \right) < 0 \]

(11)

\[ AF_{ij} = 0 \text{ if } AF_{ij} \left( n_{ld}^{I+1, J+1} - n_{ld}^{I, J} \right) < 0 \]

and either

\[ AF_{ij} \left( n_{ld}^{I+1, J+1} - n_{ld}^{I, J} \right) < 0 \]

\[ \text{or} \]

\[ AF_{ij} \left( n_{ld}^{I, J-1} - n_{ld}^{I, J} \right) < 0 \]

Then the following 6 parameters are defined as:

\[ P_{ij} = \text{the sum of all antidiffusive fluxes into grid point}(I, J) \]

\[ = \text{Max}(0, AG_{I-1, J}) - \text{Min}(0, AG_{IJ}) + \text{Max}(0, AF_{I, J-1}) - \text{Min}(0, AF_{IJ}) \]

\[ Q_{ij} = \left( W_{ij}^{\text{Max}} - n_{lo}^{i+1, j} \right) \times \frac{1}{At} \]

(12)
\[ R_{ij}^+ = \min (1, Q_{ij}^+/P_{ij}^+) \quad \text{if } P_{ij}^+ > 0 \]

\[ R_{ij}^- = 0 \quad \text{if } P_{ij}^- = 0 \]

\( P_{ij}^- \) = the sum of all antidiffusive fluxes away from the grid point \((i, j)\)

\[ = \max (0, A_{ij}) - \min (0, A_{i,j-1}) + \max (0, A_{i,j}) - \min (0, A_{i,j-1}) \]

\[ Q_{ij}^- = (n_{ij}^d - \min_{ij}) \times \frac{1}{\Delta t} \]

\[ R_{ij}^- = \min (1, Q_{ij}^-/P_{ij}^-) \quad \text{if } P_{ij}^- > 0 \quad (13) \]

\[ R_{ij}^- = 0 \quad \text{if } P_{ij}^- = 0 \]

Using set of equations in (12) and (13) the correction factors are calculated by the following equations.

\[ C_{ij}^1 = \min (R_{ij}^+, R_{ij,j-1}^-) \quad \text{if } A_{ij} > 0 \]

\[ = \min (R_{ij,j-1}^+, R_{ij}^-) \quad \text{if } A_{ij} < 0 \quad (14) \]

\[ C_{ij}^2 = \min (R_{ij}^+, R_{ij,j-1}^-) \quad \text{if } A_{ij} > 0 \]

\[ = \min (R_{ij-1,j}^+, R_{ij,j}^-) \quad \text{if } A_{ij} < 0 \]