PART II

PRESENT WORK
CHAPTER 1

Model for Calculation of Probability of Division of a Fissioning Nucleus in Different Modes

1. Introduction

A comprehensive model description of binary fission phenomenon, derived on the basis of systematics and stability of fragment nuclei, is attempted in the present work.

The fissioning nucleus splits into two fragments when in the nucleonic rearrangements, the repulsive coulomb interaction between the impending fragments gains ascendancy over the attractive nuclear forces between them. In the process, the nucleus predominantly divides in asymmetric modes. In the fissioning nucleus, then nuclear charges and neutron numbers of the impending fragment pairs are the two very important parameters. Experimental studies give directly yields and many other parameters as functions of fragment mass. This is due to the fact that from experimental point of view, the parameters can be studied more easily as functions of fragment mass rather than fragment charge. But Iyer and Ganguly (1) and also Norenberg (32) preferred primarily to consider the elemental distribution instead of mass distribution in fission. The description
of this phenomenon takes into account the process of charge polarization and mutual repulsion of the charge pairs. Success of the elemental description in ODM\(^{(1)}\) in computing and interpreting a number of experimental results\(^{(1,22,23)}\) encouraged further investigation of the model in the present work.

2. **Order-Disorder Model**

A schematic representation of the Order-Disorder Model description of fission is given in Fig. 1.1\(^{(23)}\). In the model, as stated earlier (cf. p. 8) a fissioning nucleus in the early stages of the fission process undergoes charge polarization into two impending fragments (L & H) of charges \(Z_L\) and \(Z_H\) each with neutrons \(N_{L}^{S}\) and \(N_{H}^{S}\) corresponding to the most stable ground-state nuclides for the two charges. This is followed by random distribution of the balance neutrons \(N_{\text{bal}}^{L} = 144 - N_{L}^{S} - N_{H}^{S}\) in case of \(^{236}\)U between the polarised fragments prior to scission. Thus the process is understood as involving two basic steps, viz:

(i) charge polarization;

(ii) distribution of \(N_{\text{bal}}^{L}\) between the impending fragments.

The second step has earlier been used to study the charge distribution, neutron evaporation and energy distribution in
FIG. 1.1: The Order-Disorder Model (ODM) of Fission (Ref. 23)
The procedure used in the calculation for thermal fission is indicated in the flow chart in Fig. 1.2 and consists of the following:

(i) Calculation of the probability distributions for fission fragments along isotopic lines (fragments isotopic distributions);

(ii) Calculation of total isotopic yields from fragment isotopic distributions and experimental fragment mass yields and of independent yields; charge distribution parameters are obtained therefrom;

(iii) Calculation of the number of neutrons \( \Upsilon(Z_i, N_j) \) evaporated, using the fragment isotopic distributions as given by the model and those of products calculated from published data on product mass yields and charge distribution parameters;

(iv) Calculation of kinetic energy of fragments from the difference between fission energy and the excitation energy calculated from \( \Upsilon(Z_i, N_j) \) values.

These computations thus need experimental mass yield data as input for calculating the independent yield and other fission parameters. This type of semi-empirical approach was
Scheme for calculation of various parameters for thermal fission from the second step in GEM (Ref.23)
necessary since the statistical methodology remained to be
developed for the first step of the model, viz. relative
probability of polarization of the fissioning nucleus into
different pairs of charges.

The mode of elemental division of the fissioning nucleus
are attributed to polarization into charge pairs in the first step
of QDII. Sharma et al.\(^{23}\) in their scheme for distribution of
extra excitation energy (cf. page 9) between the two impending
fragments in binary fission have stipulated that this is shared
between the fragments in proportion to the number of neutrons out
of \(N_{\text{bal}}\) going to either of them. The excitation energy of fragments
in spontaneous fission is then understood to be due to charge
polarization and the subsequent rearrangement of nucleons.

3. Fission as a rate process

3.1 Basic observations

We glean from the review of work done by earlier workers
presented in Part I, the following basic points, in comprehension
of the mathematical formulation developed subsequently:

(1) It has been envisaged\(^{12,53}\) that the fissioning nucleus
proceeds through states of charge polarization at the very early
stages of fission configuration\(^{1}\).
(ii) Neutron deficit fragments or positron emitters are not formed (any significantly) and as such there exists cut off point for the neutron numbers for a fragment of given charge at the beta stable regions\(^{(1,22)}\).

(iii) Cluster formations\(^{(10,18)}\) in compound nucleus have been speculated which remain essentially unaffected throughout the rest of the fission process;

(iv) In fission, the compound nucleus life time\(^{(4,5,13,26)}\) (or the fission process life time) is very long, compared to the nuclear periods and therefore quasi-stationary equilibrium states are obtained in the compound nucleus before scission. Analogies have also been drawn between the rates of chemical reaction and fission reaction\(^{(30-32)}\);

(v) It was stated\(^{(24,28)}\) that the potential energy is the main factor determining the yields and dynamic treatments are intrinsically not dependable due to the limitations of theory and the computed results are not likely to improve only by improvement of mathematical treatment;

(vi) It has also been stated\(^{(4)}\) that at excitation energies, not too far above the fission-threshold, the nucleus while passing
over the threshold is cold, since major part of the energy is bound in the potential energy of deformation, and such ground state properties can be applied to the fissioning nucleus. Further, heavy fissile nuclides have an elongated shape even in their ground state and therefore ground state excitation may resemble that obtained at the saddle point:

(vii) The concepts of deformation and fission barrier are all model-dependent \((27)\) and the discrepancies of various approaches are attributed to the utter complexities of the nuclear many body problem which "has not left much choice for doing better and rules are desperately needed not to get lost in the flood of data" \((27)\).

3.2 Mathematical formulation

We take fission as a rate process and as such apply a conditional stochastic process under the charge polarization constraint. The relationship for the rate of a chemical reaction is given as:

\[
R = f \times \exp \left( -\frac{W(Z_L)}{kT} \right)
\]  \hspace{1cm} \text{...(1.1)}

where 'f' is the frequency factor determined by the stochastic process involved in the reaction and \( \exp \left( -\frac{W(Z_L)}{kT} \right) \) is the energy dependent factor, \( W(Z_L) \) is the activation energy of the chemical
reaction and XT the temperature at which the stochastic distribution for the reaction take place.

In a chemical reaction, say even for a monatomic reaction, the reactants mutually interact and also interact with the ambient. Thus equilibrium conditions are applied to the entire assemblage of interacting chemical species present in the system, whereas in fission reaction a nuclide is an isolated system undergoing changes without interaction with the other nuclides. In this reaction each nuclide is a system of nucleons in itself and hence basic statistical considerations are applied to each of them and no equilibrium conditions are invoked amongst the fissioning nuclides. However, still the equation is applicable in the present form to the nuclear fission situation with the following stipulations:

In nuclear reactions having various possible reaction channels leading to different products, the frequency factor can be taken in relative terms where the relative yields or properties are of interest, and the exponential factor is understood as the probability of at least a quasi equilibrium condition obtained for the given mode of polarization. Only those polarizations are of interest which give rise to coulomb repulsion force greater than the attractive nuclear forces between the two impending fragments and
other polarizations adiabatically change over the ground state. The process of charge polarization in fission is visualized like a thermodynamically reversible adiabatic process at constant volume with energetics of equilibrium condition. The energetics are applicable under the adiabaticity of charge polarization when it is reckoned that there is considerable release of energy (E) into the system arising out of charge polarization and rearrangement of nucleons (increase of entropy, S).

Then the free energy of the system is given by

$$ F = E - TS $$  \hspace{1cm} \ldots\ldots (1.2) $$

Mathematical formulation of the two factors in equation (1.1) for fission reactions has been carried out applying the statistics of random selections of nucleons for the charge polarization. This bears analogy with order-disorder phenomenon \((20,21)\) in a crystal.

### 3.2.1 Frequency factor

The 'f' factor in the random process envisaged, corresponds to the relative number of ways, in which charge
pairs with respective number of neutrons required for beta stable configurations can be chosen from an assemblage of the number of protons and neutrons in the fissioning nucleus. Burnside (64) has given the expression for the chance (i.e., relative frequency) of random pick up of 'p' white and 'q' black balls (N = p+q) from a box containing 'a' white and 'b' black (but otherwise identical) balls as:

\[
\frac{\binom{a}{p} \times \binom{b}{q}}{\binom{a+b}{N}} \quad \cdots (1.3)
\]

or

\[
\frac{a!b!N!(a+b-N)!}{(a+b)!p!q!(a-p)!(b-q)!} \quad \cdots (1.4)
\]

The same scheme is applied to the nucleons of the fissioning nucleus. For example, we take nucleons of \(^{236}\text{U}\), in the fission configuration, as a box containing 92 protons and 144 neutrons in which random choice of nucleons are permitted as in the order-disorder process in binary alloy crystals (20). Then the chance of choosing \(Z_L\) protons and \(N_L\) neutrons from a total of 236 nucleons is given by

\[
\left[ \binom{92}{Z_L} \times \binom{144}{N_L} \right] / \binom{236}{M_L} \quad \cdots (1.5)
\]
where \( M^S_L = Z_L + N^S_L \)

and \( N^S_L \) = neutron number for the most beta stable nuclide of charge \( Z_L \).

When \( Z_L \) protons are chosen, the remaining proton set of \( Z_H \) is also chosen since \( Z_L + Z_H = 92 \). In chance that \( N^S_H \) neutrons together with \( Z_H \) protons are chosen from the balance of \( 144 - N^S_L \) neutrons is given by

\[
\begin{bmatrix}
144 - N^S_L \\
\binom{N^S_H}{s} \\
\binom{236 - M^S_L}{s}
\end{bmatrix}
\]

\[-(1.6)\]

where \( N^S_H = Z_H + N^S_H \) and \( M^S_L + N^S_H \leq 144 \)

Therefore, the chance of choosing \( M^S_L \) and \( M^S_H \) nucleons consisting of \( Z_L + N^S_L \) and \( Z_H + N^S_H \) nucleons from 236 nucleons is given by

\[
\begin{bmatrix}
92 \\
\binom{144}{s} \\
\binom{144 - N^S_L}{s}
\end{bmatrix}
/ \begin{bmatrix}
\binom{236 - M^S_L}{s} \\
\binom{236 - M^S_H}{s}
\end{bmatrix}
\]

\[-(1.7)\]

The expression is related to the number of ways in which polarization of the nucleus into \((Z_L + N^S_L)\) and \((Z_H + N^S_H)\) nucleons can be visualized. Each of the possible ways of
polarization relates to one energy state of the nucleus in its fission mode. The number of eigenstates for each of these energy states of an assembly of nucleons composed of two species of fermi particles is given by the product of the number of ways $Z_L$ and $N^S_L$ can be distributed in $m^S_L$ phase space locations and $Z_H$ and $N^S_H$ can be located in $m^S_H$ phase space locations. The implicit assumption made here is that the total number of elementary wave functions available on $(Z_L, Z_H)$ charge polarization are $m^S_L$ for $(Z_L+N^S_L)$ nucleons, $m^S_H$ for $(Z_H+N^S_H)$ nucleons and $N_{\text{bal}}$ for $N_{\text{bal}}$ neutrons. Thus the frequency factor for $^{236}\text{U}$ case is given by

$$
\left[ \frac{m^S_L \times N^S_L \times C_{Z^L} C_{N^S_L}} {m^S_H \times Z^H \times C_{Z^H}} \right] / \left[ C_{Z^L} C_{N^S_L} \times C_{Z^H} C_{N^S_H} \right] \quad (1.8)
$$

or in general the equation can be written as

$$
f = D \left[ \frac{m^S_L \times N^S_L \times C_{Z^L} C_{N^S_L}} {Z^L \times Z^H \times N^S_L \times N^S_H} \right] \quad (1.9)
$$

where $D$ is a constant for a nuclide.

3.2.2 Exponential energy dependent factor

The free energy of the charge polarization step, $F$, is

$$
F = B - TS \quad (1.10)
$$
where 'E' is the total energy of the system by virtue of the particular distribution on charge polarization, T the thermodynamic temperature and S the increase in the entropy of the system.

The total number of eigenstates 'G' for (Z_L, Z_H) mode of polarization of the system is given by:

\[ G \propto \left[ Z_L^N_L \right] \left[ Z_H^N_H \right] \left[ N_{\text{bal}}^N \right] \]  \hspace{1cm} \ldots \ldots \ldots \ldots (1.11)

Increase in entropy due to polarization is then given by:

\[ S = k \ln G \times \text{Const.} \]  \hspace{1cm} \ldots \ldots \ldots \ldots (1.12)

The minimum in free energy corresponding to an equilibrium for a given charge polarization is obtained by differentiating the equation (1.10) and equating it to zero:

\[ \text{i.e.} \quad \frac{\delta E}{\delta Z_L} - kT \frac{\delta}{\delta Z_L} (\ln G) = 0 \]  \hspace{1cm} \ldots \ldots \ldots \ldots (1.13)

\[ \text{or,} \quad \frac{\delta E}{kT} = \frac{\delta}{\delta Z_L} (\ln G) \]  \hspace{1cm} \ldots \ldots \ldots \ldots (1.14)

Since

\[ \frac{\delta E}{\delta Z_L} = W(Z_L) \]

\[ W(Z_L) / kT = \frac{\delta}{\delta Z_L} (\ln G) \]  \hspace{1cm} \ldots \ldots \ldots \ldots (1.15)
Here \( \frac{\delta}{\delta Z_L} (l_n G_l) \) is the slope of the entropy curve at \( Z_L \) and the probability of equilibrium distribution as depicted for \((Z_L, Z_H)\) polarization is given by

\[
\exp \left( - \frac{W(Z_L)}{kT} \right) = \exp \left( - \frac{\delta}{\delta Z_L} (l_n G_l) \right) \quad (1.16)
\]

From the beta stable neutron numbers \( N_L^S \) and \( N_H^S \) for the two charges \( Z_L \) and \( Z_H \), the energy dependent exponential factor can be calculated.

4. **Estimation of Beta-Stable Neutron Numbers**

In the chart of nuclides with proton number on the \( X\)-axis, and neutron number on the \( Y\)-axis, the positron emitters appear on the left side of the table of nuclides and the negatron emitters appear on the right. Though in general several stable nuclides occur for a given proton number, energetically one particular species can be found corresponding to the minimum beta-decay energy as the most beta-stable nuclide. If the mass parabola is taken to be continuous, this minimum may correspond to a non integral neutron number for a charge. Beta-stable neutron number for the charges can be
derived either by using a suitable mass-formula or directly from the experimental beta-decay energies.

In the earlier work on QDh, stable-neutron number for a charge was derived using Levy's mass-formula by equating the expression for $Q_b = 0$ and solving for the stable neutron number for a given charge. The isobaric stable charge number vs $A$ curve was used to get the stable neutron number for integral charges. These values are given in column 2 of Table 1.

As beta-decay is an isobaric process, the most stable charge for a mass has a definite meaning, indicating the most-stable species in the mass-parabola. A more accurate mass-formula of Zeldes is used to calculate the mass value. The mass values are fitted to parabola, one each for odd $Z$ points and even $Z$ points for every mass. The fitted parabolic equation on differentiation and equating to zero gives the minimum in the mass parabola. It was found that the minimum points coincide well for both even and odd $Z$ for a given $A$. The maximum differences observed have been 0.166 charge units for odd $A$ and 0.199 charge unit for even $A$. The average of the two minima for the odd-$Z$ parabola and even-$Z$ parabola for a
given $A$ is taken as corresponding to the most stable charge for the mass. The most stable neutron number for a charge is derived from this distribution of stable charge vs $A$ curve and are given in column 3 of Table 1.1.

Dewdney (57) has used the experimental beta-decay energies for isobars by fitting them to a straight line and finding the point of intercept of this line with the $Q_b=0$ line to find the stable charge number for various masses. Fig. 1.3 shows typical treatment of this procedure for mass $A = 131 \& 132$. The beta-energies have been plotted at the mid-point of the charges between the parent and daughter. For odd $A$, both odd $Z$ and even $Z$ points lie on the same line and lead to the most stable charge number unequivocally. In case of even masses, the odd $Z$ and even-$Z$ points give two different straight lines parallel to each other. Dewdney has shown that (cf. Fig. 1.3) a central-line between these two lines gives the most stable charge number at the intercept with $Q_b=0$ line. The stable neutron numbers corresponding to integral charges obtained in this way are shown in column 4 of Table 1.1.

Dewdney's beta decay energies are taken from the data available as on 1961. 1971 mass-table of Wapstra (58)
FIG. 1.3: Dewdney's procedure for calculation of stable charge number for a mass from experimental beta decay energies. Figure shows treatment of the procedure for \( A=131 \) & \( 132 \).
gives the most recent compilation of experimental beta decay energies. Dewdney's approach is adopted for finding the most stable charge for a mass using the data from Wapstra's table. The stable neutron numbers for the charges obtained from this are shown in the last column of Table 1.1.

Comparing the different sets of values in Table 1.1, the general agreement between the last three sets of values is good for almost all charges. It has been seen that the values obtained using a mass formula generally give rise to some problems at the shell edges, because of difficulties in getting suitable constants at such locations. Since the 1971 mass-table represents the latest compilation available for the beta-decay energies and since this set is derived from experimental data, the stable neutron numbers for the charges are taken from the last set in the present work. The stable neutron numbers vs charge for the last three sets are given in Fig.1.4. The charge polarization with stable neutron numbers imply that the fragments will be formed with a lower and upper limit of neutron numbers for a charge, whose span is fixed by the value. The-fission fragment formation range for 236 U is given in Fig.1.5.
The stable neutron values vs. Z obtained using Zeldes' mass formula (—■—), Bewdney's data (Ref. 57) (- - -), and Jensen's mass table (Ref. 56) (•-•). The change of Z axis for the curves are shown in the figure.
Table - 1.1

Stable Neutron Number for Charges

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<th>Calculated from mass formula of Levy (Ref 54)</th>
<th>Calculated from mass formula of Zeldes (Ref 54)</th>
<th>Derived from Dewdney's (Ref 55) stable mass value</th>
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Balance Neutrons

An important observation has been that the balance neutrons, \( N_{bal} \) (i.e. \( N_{bal} = N_{S}^L - N_{S}^H \) where \( N_{bal} \) is neutron number of the fissioning nuclide) show asymmetric behaviour, i.e. a higher \( N_{bal} \) in the symmetric region and lower \( N_{bal} \) for the asymmetric region for the fission of the near-actinides. This is illustrated in the Fig. 1.6, where the parameter \( 1/N_{bal} \) as a function of charge for a number of fission reactions are given. This is brought about by the property of stable neutron number function with respect to the charge of the fissioning nucleus. These distributions correlate with the observed effect of the bunching of the higher Z-peaks with staggering of the lower Z-peaks in the yield distributions and also the reduction of asymmetry with increasing mass of the fissile nuclide. This remarkable correlation is highly suggestive of the influence of charge polarization with beta stable configuration on the yields of fission fragments.

The frequency factor in Section 5.2.1 is a function of \( Z_L, Z_H, N^S_L, N^S_H \). It can be shown that:

\[
 f(Z_L, Z_H, N^S_L, N^S_H) \propto \left[1/N_{bal}\right]^\rho \quad \text{...(1.17)}
\]

where \( \rho \) is found to be \( \gg 1 \) for a given charge polarization.
FIG. 1.61 Distribution of $1/N_{_{bot}}$ vs Z for the charge polarization in 236, 240, 252, 256, 260, U, Pu, Cf, Fm & Ku.
Further if $N_{bal}$ is kept constant the function is almost invariant. Thus the frequency factor is very sensitive to the choice of $N_L^S$ & $N_H^S$ values which determine the $N_{bal}$ values for a fissioning nucleus.

6. Computational Procedure

The expression for the reaction rate derived in Section 3 can be written as

$$R = \exp \left[ -\frac{W(ZL)}{kT} \right]$$

$$= D \left[ \frac{M_L^S! \ M_H^S!}{Z_L! \ Z_H! \ N_L^S! \ N_H^S!} \right]^2 \exp \left[ \frac{\delta}{\delta Z_L} (\ln G) \right]$$

Where

$$\ln G = C - \left[ \ln Z_L! + \ln Z_H! + \ln N_L^S! + \ln N_H^S! + \ln N_{bal}! \right]$$

and

$$D \text{ and } C \text{ are Constants.}$$

The above expression has been derived for integral number of neutrons, $N_L^S$ and $N_H^S$. In actual cases however, the neutron numbers obtained for most beta-stable nuclides are non-integral ones. The
factorials for non-integral numbers are calculated using table of \( \Gamma \) functions. A simple procedure which reflects the physics of the situation can also be adopted by taking two consecutive integral neutron numbers on either side of \( \langle N_L^S \rangle \) and \( \langle N_H^S \rangle \) with weights such that, for the L-fragments

\[
\langle N_L^S \rangle = x N_L^S + (1-x)(N_L^S + 1) \quad (1.2a)
\]

and for the H-fragments

\[
\langle N_H^S \rangle = y N_H^S + (1-y)(N_H^S + 1) \quad (1.2b)
\]

"Relative probabilities" of polarization in the four pairs

\((z_L^S, n_L^S; z_H^S, n_H^S), (z_L^S, n_L^S + 1; z_H^S, n_H^S), (z_L^S, n_L^S + 1; z_H^S, n_H^S + 1)\) and

\((z_L^S, n_L^S + 1; z_H^S, n_H^S + 1)\) are given separately by \(xy, x(1-y), (1-x)y, \) and \((1-x)(1-y)\). The frequencies obtained for these four combinations weighted with their relative probabilities give the frequency factor corresponding to a set of non-integral \( N_L^S \) and \( N_H^S \) for a charge pair of \( z_L \) \& \( z_H \).

The frequency factors obtained as described above normalised to two are given in Fig.7 for \( ^{228}\text{Ac}, ^{236}\text{U}, ^{240}\text{Pu}, ^{246}\text{Cm} \) and \( ^{252}\text{Cf} \). The distributions are asymmetric.
FIG. 1-7 - FREQUENCY DISTRIBUTION FOR $^{228}$Ac, $^{236}$U, $^{240}$Pu, $^{246}$Cm AND $^{252}$Cf.
The frequency distributions for $^{256}$Fm and $^{260}$Ku are given in Fig. 1.8, these are also basically asymmetric. It may be observed that the qualitative features of these distributions are similar to those of $1/N_{\text{bal}}$ distributions (cf. Fig. 1.6).

The bunching of higher Z-peaks around Z=56 is evident in the figures with corresponding shift of the lighter Z-peaks towards the higher Z-peak for increasing mass/charge of the fissioning nucleus. The reduction of peak to valley ratio with the increasing mass/charge of the fissioning nucleus is also seen in Fig. 1.7. Disappearance of asymmetry for the nuclide whose symmetric charge happens to be beyond the region of Z=50, can be observed in Fig. 1.8.

The exponent of the energy term is obtained from the differential of entropy of polarization as given in equation (1.15). For the purpose of computation the entropy curve is generated corresponding to the actual non integral values of beta stable neutron numbers adopting the procedure outlined earlier (cf. page 43). The entropy distribution of two typical cases are given in Fig. 1.9. The differential at each charge points are calculated and $\gamma(Z,t)$ are thus obtained. Slope at the symmetric charge is obtained by extrapolation.
FIG. 1-8 - FREQUENCY DISTRIBUTION FOR $256_{Fm}$ & $260_{Ku}$
FIG. 1.9: Relative entropy of polarization for $^{236}\text{U}$ and $^{252}\text{Cf}$.
Results and Discussions

7.1 Total Isotopic Distributions

The rate of reaction $R$, calculated as outlined in earlier section gives the relative probabilities of change polarization. This distribution normalised to two between the ranges of charges, give the total isotopic distribution, $Y_Z(Z_i)$. Since no input of energy from any external source is considered in the formulation of the expression (1.19), all the total isotopic distributions obtained in the present work are those corresponding to spontaneous fission.

The distribution for $^{236}$U obtained in the present work is compared with that computed using the second step of CDM and the experimental fragment mass yield data for $^{235}(n_{th},f)$ in Fig. 1.10. Although, the results are compared with the distribution for thermal fission, general features are reproduced. However, the distributions obtained in the present work are somewhat narrow. There is some shift in the location of the peaks. General peak to valley ratio is also in good agreement with the thermal data, keeping in view that the thermal peak to valley ratio is expected to be less than that for spontaneous fission.
Total isotopic yields obtained in the present work (→) for $^{236}$U compared with the results of Ref. 1. The crosses (x) indicate experimental yields of certain elements (Ref. 34).
The total isotopic yields obtained for $^{236}\text{U}$, $^{240}\text{Pu}$, $^{252}\text{Cf}$, and $^{256}\text{Fm}$ are given in Fig. 1.11 and those obtained for $^{246}\text{Cm}$, $^{257}\text{Fm}$, and $^{260}\text{Ku}$ are given in Fig. 1.12.

In Fig. 1.11 the higher Z-peaks are found to be clustered around $Z=56$. The lower Z-peaks shift towards the higher Z-peaks with increasing mass/charge of the fissioning nucleus. Peak to valley ratio is also found to reduce with increase of mass/charge of the nuclides. For the nuclides whose symmetric charges are beyond $Z=50$ (Fig. 1.12) the ODM shows prominence in symmetric yield and marked reduction in asymmetry as in case of $^{260}\text{Ku}$. In the results some structures have also appeared in the symmetric region of $^{236}\text{U}$ and $^{240}\text{Pu}$. The lower edge of the higher Z-peaks, in all cases are found to be at the magic number $Z=50$. In the final analysis, asymmetry and other characteristics of the distributions are traced in the peculiarities of $1/N_{\text{bal}}$ distribution as pointed out earlier (Sec. 5).

A difference in $N_{\text{bal}}$ (for example, 11 in case of $^{236}\text{U}$), between the asymmetric and symmetric points gives rise to an yield ratio of about a thousand. The distributions are quite sensitive to the variation of stable neutron number estimates as discussed in Section 5. For example, one unit variation of
FIG. 1.11: Total isotopic yield distributions for the fission of $^{236}_U$, $^{240}_{Pu}$, $^{252}_{Cf}$ & $^{256}_{Fm}$
the \( N_{\text{bal}} \) which may arise due to error in determination of most beta stable neutron number for a charge can give rise to a variation of a factor of 12 in the yields and hence care has to be taken to get the best values obtainable. This is specially so in the shell closure regions where computation of stable neutron numbers are subject to greater uncertainties.

Although this approach gives reasonably good results for a wide range of fragment charges up to low yield asymmetric region, but in very highly asymmetric regions the distribution has a tendency to increase. Energetically fission fragments in these regions have unfavourable formation probabilities. This factor is not considered in the present work.

In Fig. 1.13 stable neutron numbers used in the calculations are given. The distribution shows a departure from a straight line at shell edges. Without changes at the shell edges the stable neutron numbers vs charge would have been as indicated by the solid line in the figure. \( N_{\text{bal}} \) obtained using such a straight line show a constant value (see inset for \(^{236}\text{U}\)) and the observed asymmetry in the yield distributions would also disappear. This strongly suggests that shell effects are felt in \( N_{\text{bal}} \) values or in \( 1/N_{\text{bal}} \) distribution which as seen earlier in turn influence the yield distributions. The asymmetry in fission is generally
The solid line in the main figure represents the stable neutron number \( Z \) without any shell effects. Distribution for \( ^{236}U \), \( ^{274}X \) and \( ^{314}Y \) are given in the insets. The dashed line in the inset for \( ^{236}U \) represents the \( 1/\nu_{\text{bal}} \) values obtained using the solid line in the main figure.
believed to be due to shell effects and the ODM approach also leads to the same conclusion. However in this approach, there is no necessity for explicitly assuming any preference for the formation of closed shell species and no weightage for shell effects is given in the mathematical formulation. The possible shell effects are reflected in a natural manner in the stable neutron number chosen for the charge. Stable neutron number for a charge is the only input data needed in the calculations using this model.

The concept of charge polarization with stable neutron numbers seems to have profound consequences in deciding the asymmetric or symmetric yield distribution in a given fission reaction. This is further illustrated in Fig. 1.13 where \( \frac{1}{N_{\text{bal}}} \) distribution for three fissioning nuclides viz. \( ^{236}\text{U} \), \( ^{274}\text{X} \) & \( ^{314}\text{Y} \) are shown. \( ^{256}\text{U} \) and the two hypothetical nuclei (X and Y) have their symmetric charges at the mid-points of the three neutron and proton shell regions. The asymmetry in the actinide region whose symmetric charges are in the first shell region considered above was correlated earlier with \( \frac{1}{N_{\text{bal}}} \) distribution. The decrease in asymmetry with increasing charge of the fissioning nuclide (e.g., \( ^{256}\text{Fm} \) and \( ^{260}\text{Ku} \)) was also related to \( \frac{1}{N_{\text{bal}}} \) distribution. When the symmetric charge increases
further and lies in the middle of the second shell region $(Z > 50)$ asymmetry in $1/N_{\text{bal}}$ almost disappears as illustrated for the hypothetical nuclide $^{274}_{106}X$. For nuclei with symmetric charge in the third shell region $(Z > 56)$, the asymmetry in $1/N_{\text{bal}}$ re-appears (see inset for $^{314}_{124}Y$). Judging from the systematics of correlation between the $1/N_{\text{bal}}$ and yield distributions noticed for the first two shell regions (Figs. 1.11 & 1.12), the ODM approach predicts asymmetric fission for super heavy nuclei whose symmetric charge lies in the third shell region.

In Chapter II the results derived here are used to calculate a number of fission parameters.