Part V

Summary
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Summary and Scope

In this thesis we have studied two applications of AdS/CFT conjecture. In the first part of the thesis we have used the conjecture to understand the IR cut-off appearing in the calculation of partition function of matrix model for strings. This apparently ad hoc introduction of IR cut-off is crucial to investigate phase transition in matrix model, and hence the Hagedorn transition. In the second part, we have used AdS/CFT conjecture to analyze the duality between Charged BTZ black hole and Luttinger liquids. We will here summarize the results and discuss some future directions.

13.1 Hagedorn transition in Matrix Model for Strings

In this part of the thesis (Chapter 4-6), we have studied 1-loop partition function of DLCQ M-theory for two phases: Long D-strings and Clustered. By comparison of free energy we found two possible phase transitions. As we increase the temperature from zero, String phase dominates, at a temperature $T_H$ this Long D-strings “Cluster”, which we identify as the Hagedorn transition. As we increase the temperature, there is a Gregory-Laflamme kind of transition to again string phase. This may be the reminiscent of $T \rightarrow \frac{1}{T}$ symmetry discussed in [45, 39], as the “second Hagedorn transition” where the thermal tachyon vanishes. It was also recently found in analysis of thermodynamics of p-adic string theory in [142, 143]. A proper understanding for this duality symmetry in temperature is still lacking.
We have found that \( T_H \sim \frac{1}{L_0} \), where \( L_0 \) is the IR cutoff of the Yang-Mills theory which needs to be introduced to make the calculations well defined. This can be an artifact of the perturbation theory. We can in principle use AdS/CFT correspondence for \( D0 \)-branes, to construct supergravity duals for the matrix model or the SYM theory configurations we have considered, which may throw light on the origin of this IR cut-off. In this thesis, we have demonstrated for simple supergravity model that such an IR cut-off has to be introduced in the dual theory if we ignore finite \( g_s \) corrections. But an exact analysis for the supergravity models for the configurations considered in this thesis is yet to be done. Also simple parameter counting shows that the BFSS matrix model needs one more dimensionful parameter if it is to be compared with string theory at finite \( g_s \), so the IR cutoff \( L_0 \) can be thought of as one choice for this extra parameter. Higher loop calculations for the partition function may also resolve this issue.

In our analysis we have considered the background of time component of the gauge field to zero. Introduction of a finite value for the background (chemical potential) removes the necessity of IR cut-off, by introduction of another scale given by chemical potential. Our result is the special case of zero chemical potential. The effect of non-zero background of \( A_0 \) was studied in a different context in [21]. The effect of non-zero chemical potential needs to be analyzed for our case. Also we have considered only two possible configurations, a complete analysis of the phase structure for matrix model is lacking.

13.2 Duality between Charged BTZ black hole and Luttinger liquid

In the second part of the thesis (Chapter 9-12) we have studied the behavior of Green’s functions of a \( 1+1 \) field theory with some background charge density using the AdS/CFT correspondence. We have studied Green’s functions of scalars, fermions and currents in the boundary theory have been obtained at zero as well as non-zero temperatures.

So far the use of AdS/CFT in Condensed matter was limited to the study of
boundary theory where calculations can not be done, but in $1 + 1$ dimension due to presence of various analytic tools like Bosonization, our aim was to explore the duality from both sides.

The fermion Green’s function shows interesting behavior of Fermi-Luttinger liquid, which is Luttinger liquid with a non-linear term in dispersion. Our hope is that this Green’s may be equivalent to a strong coupling limit of this Fermi-Luttinger liquid.

We were also able to suggest a possible explanation for the $k$-independent non-analyticity at $\omega = 0$ in the fermion (or scalar) Green’s function from the point of view of the boundary theory, by constructing a toy model. It was seen in this toy model, the non-analyticity can be explained if there are modes with almost zero velocity (i.e. non propagating) that interact with these fermions. These could be impurities for instance. Further progress can be made by incorporating more features of the Green’s function in our toy model.

Another immediate extension of our work, will be to construct Green’s function corresponding to massive fermions, which we were unable to presently due to some technical problems in constructing proper numerics. Bulk mass corresponds to scaling dimension of boundary operators, so massive fermion analysis will provide an opportunity to study operator with scaling dimension identical to free electrons.

As a test of our numerics we have compared it with various limits where analytical results can be obtained, and it shows a very reasonable match. In particular the intriguing log periodicity found in [73, 75] is also seen.

The conductivity behavior is seem to qualitatively match with doped Mott insulator, but a detailed comparison needs to explored.

Finally there is the obvious question of understanding what experimental setup would correspond to these theories. There is an ample amount of experimental systems available in $1 + 1$ dimensions, like edge modes of quantum Hall liquid, carbon nanotubes, cleaved edge semiconductor wires, antiferromagnetic spin chains, and
cold atoms in 1D optical traps. Connections with these experiments may provide “experimental” verification of AdS/CFT conjecture!