CHAPTER II

APPLICATION OF LEXICOGRAPHIC GOAL PROGRAMMING IN DOWNSTREAM OIL INDUSTRIES

2.1 INTRODUCTION

The petroleum industry is usually divided into three major components: Upstream, midstream and downstream. Midstream operations are usually included in the downstream category. The downstream oil sector is a term commonly used to refer to the refining of crude oil and the selling and distribution of natural gas and products derived from crude oil. Such products include liquefied petroleum, jet fuel, diesel oil, other fuel oil. The downstream sector includes oil refineries, petrochemical plants, petroleum product distribution, retail outlets and natural gas distribution companies. The downstream industry touches consumers through thousands of products such as petrol, diesel, jet fuel, heating oil, asphalt, lubricants, synthetic rubber, plastics, fertilizers, antifreeze, pesticides, pharmaceuticals, natural gas and propane.

In this chapter, we present a lexicographic goal programming (LGP) model for management decision-making in petroleum refinery industry, involving the distribution of oil to the various depots. The model is designed to illustrate how LGP can be used as an aid for solving transportation problems with multiple objectives. The
results of the study have utilized in decision making process at a petroleum refinery industry in India.

The classical single objective transportation problems are a special type of linear programming (LP) problems. The sources may include plants and warehouses and destinations may include sales outlets and customers. The coefficients of the objective function represent transportation cost, delivery time, number of goods transported, unfulfilled demand, and many others. In operations research, several quantitative techniques have been used for solving transportation problems. The most commonly used techniques are linear programming (LP) and generalized minimum cost network (Hadley [1972], Hemaida and Kwak [1994]). The decision-maker, public or private, may not have a utility function with a single argument (usually profits). The single objective optimization techniques presented by Romero in his article are examples (Romero [1991]). Businesses and industries are practically faced with both economic optimization such as cost minimization and non-economic items that are vital to the existence of their firms (Lee [1972]). Transportation problems involve multiple and conflicting goals such as the cost minimization, balancing work among the plants, transportation fleets, and many others. These multiple and conflicting goals can be achieved by using goal programming (GP) technique.

The goal programming (GP) technique has become a widely used approach in Operations Research (OR). GP model and its variants have been applied to solve large-scale multi-criteria decision-making problems. The GP technique was first used
by Charnes and Cooper in 1960s. This solution approach has been extended by Ijiri [1965], Lee [1972], and others. For detailed research survey on GP, see Lee [1972], Ignizio [1976], Romero [1991], Romero [1986], Tamiz and Jones [1995], and Sharma, Alade and Vasishta [1999]. Lee and Moore [1973] used GP model for solving transportation problem with multiple and conflicting objectives. Arthur and Lawrence [1982] designed a GP model for production and shipping patterns in chemical and pharmaceutical industries. Kwak and Schniederjans [1985] applied GP to transportation problem with variable supply and demand requirements. Several other researchers (Sharma et al. [1999]) have also used the GP model for solving the transportation problem.

2.2 **GENERIC LEXICOGRAPHIC GOAL PROGRAMMING MODEL**

To formulate a generic lexicographic goal programme algebraically we define the number of priority levels as $L$ with corresponding index $i = 1, 2, \ldots, L$. Each priority level is now a function of a subset of unwanted deviational variables which we define as $h_l(n, p)$. This leads to the following formulation:

\[ \text{Lex Min: } a = \{h_1(n, p), h_2(n, p), \ldots, h_L(n, p)\} \]

Subject to

\[ f_q(x) + n_q - p_q = b_q \quad (q = 1, 2, \ldots, Q) \]

\[ x \in F \]

\[ n_q, p_q \geq 0 \quad (q = 1, 2, \ldots, Q) \]
where $F$ is the feasible region made up of points in decision space that satisfy all of the constraints and sign restrictions. Each $h_l(n, p)$ contains a number of unwanted deviational variables. The exact nature of $h_l(n, p)$ depends on the nature of the goal programme to be formulated, but if we assume that it is linear and separable then it will assume the form

$$h_l(n, p) = \sum_{q=1}^{Q} \left( n_q^l K_a \cdot \frac{p_a}{k_q} \right)$$

where $n_q^l$ is the preferential weight associated with the minimization of $n_q$ in the $l^{th}$ priority level. $p_a$ is the preferential weight associated with the maximization of $p_a$ in the $l^{th}$ priority level. $k_q$ is the normalization constant associated with the $q^{th}$ goal. These constants are necessary in order to scale all the goals onto the same units of measurement. The meaning of the lexicographic minimization of the achievement function is that the minimization of deviational variables placed in a higher priority level is regarded as infinitely more important than that of deviational variables placed in a lower priority level.

### 2.3 FORMULATION OF GOAL PROGRAMMING MODEL

Let

- $S_i =$ Supply from the $i^{th}$ site
- $C_i =$ Demand of the $i^{th}$ customer
- $S_{ij} =$ Quantity of fuel transported from $i^{th}$ site to $j^{th}$ refinery
- $R_{jk} =$ Quantity of fuel transported from $j^{th}$ refinery to $k^{th}$ Depot
\( D_{kl} = \) Quantity of fuel transported from \( k^{th} \) Depot to \( l^{th} \) customer

\( n_i = \) Underachievement of depot capacity level of 50%

\( p_i = \) Overachievement of depot capacity level of 50%

\( nn_i = \) Underachievement of depot capacity level of 90%

\( pp_i = \) Overachievement of depot capacity level of 90%

\( nr_i = \) Underachievement of refinery operating capacity of 75%

\( pr_i = \) Overachievement of refinery operating capacity of 75%

\( n_c = \) Underachievement of cost to be £ 40,000 per day

\( p_c = \) Overachievement of cost to be £ 40,000 per day

**Table 2.1: Cost of Refining and Transportation from Site to Refinery**

<table>
<thead>
<tr>
<th>From ( \rightarrow )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>27</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>19</td>
<td>16</td>
<td>11</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>22</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

**Table 2.2: Cost of Transportation from Refinery to Depot**

<table>
<thead>
<tr>
<th>From ( \rightarrow )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>6</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
</tr>
<tr>
<td>------</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>D₁</td>
<td>16</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>D₂</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>D₃</td>
<td>9</td>
<td>14</td>
<td>6</td>
</tr>
</tbody>
</table>

**Table 2.3: Cost of Transportation from Depot to Customer**

**Supply constraint:**

Total supply from the three sites is being restricted to 1000, 500, 800 (in 1000l per day) respectively.

Therefore,

\[ S_{11} + S_{12} + S_{13} \leq 1000 \]

\[ S_{21} + S_{22} + S_{23} \]

\[ S_{31} + S_{32} + S_{33} \leq 800 \]

**Demand constraint:**

Total demand for the three customers must be fulfilled i.e. 500, 800, 900 (in 1000l per day) respectively.

Therefore,
\[D_{11} + D_{12} + D_{13} = 500\]
\[D_{21} + D_{22} + D_{23} = 800\]
\[D_{31} + D_{32} + D_{33} = 900\]

**GOAL 1**: Keep costs to be below £40,000 per day.

Therefore,

\[
\text{Total Cost} = 27S_{11} + 13S_{12} + 13S_{13} + 19S_{21} + 16S_{22} + 11S_{23} + 22S_{31}
+ 14S_{32} + 16S_{33} + 6R_{11} + 8R_{12} + 5R_{13} + 4R_{21} + 10R_{22} + 7R_{23} + 5R_{31} + 9R_{32}
+ 8R_{33} + 16D_{11} + 12D_{12} + 9D_{13} + 8D_{21} + 10D_{22} + 12D_{23} + 9D_{31} + 14D_{32}
+ 6D_{33}
\]

Total Cost + \(n_c - p_c = 40,000\)

**Table 2.4:**

<table>
<thead>
<tr>
<th>Penalize</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>(1/40000 = 0.00003)</td>
</tr>
</tbody>
</table>

**GOAL 2**: Keep each refinery operating at least 75% of its capacity

For \(R_1\), 75% of 1000 = 750

For \(R_2\), 75% of 1200 = 900

For \(R_3\), 75% of 800 = 600

Therefore,

\[S_{11} + S_{21} + S_{31} + nr_1 - pr_1 = 750\]
\[S_{12} + S_{22} + S_{32} + nr_2 - pr_2 = 900\]
\[S_{13} + S_{23} + S_{33} + nr_3 - pr_3 = 600\]
Also,

\[ S_{11} + S_{21} + S_{31} \]
\[ S_{12} + S_{22} + S_{32} \leq 1200 \]
\[ S_{13} + S_{23} + S_{33} \leq 800 \]

Refinery capacity is taken as 75%. The quantity of fuel transported from refinery to depot should be restricted to the amount of fuel transported from three sites to each refinery.

Therefore,

\[ R_{11} + R_{12} + R_{13} + S_{11} + S_{31} \]
\[ R_{21} + R_{22} + R_{23} \leq S_{12} + S_{22} + S_{32} \]
\[ R_{31} + R_{32} + R_{33} \leq S_{13} + S_{23} + S_{33} \]

Table 2.5:

<table>
<thead>
<tr>
<th>Penalize</th>
<th>1 (R₁)</th>
<th>1(R₂)</th>
<th>1(R₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75% of refinery capacity</td>
<td>750</td>
<td>900</td>
<td>600</td>
</tr>
<tr>
<td>weight</td>
<td>1/750 = 0.001</td>
<td>1/900 = 0.00111</td>
<td>1/600 = 0.002</td>
</tr>
</tbody>
</table>

Similarly, the quantity of fuel transported from depot to customer should be restricted to the amount of fuel transported from three refineries to each depot.

Therefore,

\[ D_{11} + D_{12} + D_{13} + R_{21} + R_{31} \]
\[ D_{21} + D_{22} + D_{23} \leq R_{12} + R_{22} + R_{32} \]
D_{31} + D_{32} + D_{33} \leq R_{13} + R_{22} + R_{32}

**GOAL 3:** Keep each depot at a level of between 50% and 90% of its capacity

For D_1,

50\% \text{ of } 1400 = 700

90\% \text{ of } 1400 = 1260

For D_2,

50\% \text{ of } 900 = 450

90\% \text{ of } 900 = 810

For D_3,

50\% \text{ of } 900 = 450

90\% \text{ of } 900 = 810

Therefore,

R_{11} + R_{21} + R_{31} + n_1 p_1 = 700

R_{12} + R_{22} + R_{32} + n_2 p_2 = 450

R_{13} + R_{22} + R_{32} + n_3 p_3 = 450

Similarly,

R_{11} + R_{21} + R_{31} + n_1 p_1 = 1260

R_{12} + R_{22} + R_{32} + n_2 p_2 = 810

R_{13} + R_{22} + R_{32} + n_3 p_3 = 810

Also,

R_{11} + R_{21} + R_{31} \leq 1400

R_{12} + R_{22} + R_{32} \leq 900
\[ R_{13} + R_{22} + R_{32} \leq 900 \]

**Table 2.6:**

<table>
<thead>
<tr>
<th>Penalize</th>
<th>1 ((D_1))</th>
<th>1((D_2))</th>
<th>1((D_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% of depot capacity</td>
<td>700</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>weight</td>
<td>1/700 = 0.0014</td>
<td>1/450 = 0.002</td>
<td>1/450 = 0.002</td>
</tr>
<tr>
<td>90% of depot capacity</td>
<td>1260</td>
<td>810</td>
<td>810</td>
</tr>
<tr>
<td>weight</td>
<td>1/1260 = 0.0008</td>
<td>1/810 = 0.0012</td>
<td>1/810 = 0.0012</td>
</tr>
</tbody>
</table>

**Priority 1:** Goal 2 and 3

\[ P_1 \left( 0.001 \text{nr}_1 + 0.001 \text{nr}_2 + 0.002 \text{nr}_3 + 0.0014 n_1 + 0.0022 n_2 + 0.002 n_3 + 0.0008 pp_1 + 0.0012 pp_2 + 0.001 pp_3 \right) \]

**Priority 2:** Goal 1

0.00003 \(P_2 p_c\)

**Priority 3:** Goal 4 (Assign any unused capacity to site three)

**Table 2.7:**

<table>
<thead>
<tr>
<th>From →</th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>To S₁</td>
<td>0</td>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>S₂</td>
<td>0</td>
<td>0</td>
<td>500</td>
</tr>
</tbody>
</table>
\[ n_{ri} = 0 \]
\[ p_{ri} = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_1 )</td>
<td>0</td>
<td>800</td>
<td>200</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>450</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( D_3 )</td>
<td>50</td>
<td>0</td>
<td>700</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0</td>
<td>0</td>
<td>750</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>900</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>100</td>
<td>450</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ n_i = 0, \ p_1 = 300, \ p_2 = 0, \ p_3 = 300; \]
\[ n_{ni} = 260, \ n_{n2} = 360, \ n_{n3} = 60, \ p_{pi} = 0; \]
\[ n_c = 0, \ p_c = 26550. \]

**Priority 1:** 0

**Priority 2:** 0.664
Priority 3: (Total deviation): 0.664.

2.4 CONCLUSION

In this chapter, we have been able to demonstrate that LGP approach is a better technique than the single objective criterion when multiple conflicting objectives are involved. There are several practical applications of the technique proposed in this chapter for petroleum industry. Other constraints may be included in the model based on the situation surrounding the decision processes on the business. The model is general enough to incorporate many of the incommensurable and incompatible economic and operational goals of industries. Some of the practical aspects of the case study have not been studied thoroughly.