5.1 Putter's method

Putter (1955) pointed out that in the absence of random velocities (that is when the pattern is fixed in shape), all the time shifts obtained would be related to one another by a simple straight line—called Putter's line.

This can be seen from the equation (1) by eliminating \( \gamma \) that:

\[
\frac{t_x}{\frac{\gamma}{\gamma'}} \cos \phi + \frac{t_y}{\gamma'} \sin \phi = 1/V \quad \ldots \ldots (2)
\]

Thus for a fixed pattern, the plot of \( t_x \) and \( t_y \) against each other would be a straight line given by equation (2), which has a slope angle \( \lambda \), given by

\[
\tan \lambda = \frac{\frac{t_x}{\gamma'}}{\frac{t_y}{\gamma'}} \tan \phi \quad \ldots \ldots (3)
\]

Solving the equations (2) and (3), one can determine the values of \( V \) and \( \phi \) unaffected by the curvature of the line of maximum amplitude. Geometrically, a line from origin and perpendicular to Putter's line would give the velocity of the drift of the pattern both in magnitude and direction.

The above treatment does not in any way depend on the statistical distribution of the points corresponding to different maxima and it is the relative orientation of
the points rather than their actual positions which is important. Putter's method therefore applies even to an anisotropic diffraction pattern, which does not change its shape as it moves. Another advantage of this method lies in the fact that far fewer number of maxima are needed and hence it is quicker to apply.

BANERJI (1960) has modified this technique to the case of a turbulent anisotropic pattern. This is discussed separately later in Sec. 5.3.

5.2 Method of Briggs - Page and Spencer (Probability distribution of time lags)

BRIGGS and SPENCER (1955) showed how to deduce the properties of the changing diffraction pattern from the statistical observations of time lags alone. If the pattern is anisotropic the problem is shown to be insoluble if only two sets of time lag $t_x$ and $t_y$ are available. The availability of a time shift $t_z$ in a third direction only enables the determination of all the involved parameters.

The distribution $P(t_x)$ and $P(t_y)$ for $t_x$ and $t_y$ is shown by BRIGGS et al. (1955) to be:

$$P(t_x) = \frac{t_x}{\left(\frac{t_x}{\lambda^2 + \nu^2 - \nu_0^2}\right)}$$

Similarly for $P(t_y)$
When the above theoretical distribution is compared with the experimental distribution of $t_x$ and $t_y$, it is found that the two curves give a best fit only for a particular value of $V_e$.

5.3 Banerji's statistical method for anisometric and turbulent patterns

BANERJI (1958) demonstrated, that even in the presence of random variations of the pattern as it moves, the value of $\phi$ can be obtained equally easily by drawing a mean straight line through the plot of $t_x$ against $t_y$, if the ground pattern is isotropic. The magnitude of $V$ obtained by the mean straight line has, however, to be corrected by using the correlation coefficient $\rho$ between $t_x$ and $t_y$ values.

(a) Isometric Turbulent Pattern

In the presence of turbulence for an isotropic pattern, the plot of $t_x$ against $t_y$ will have some spread instead of a straight line. The lines of equal probability on this diagram would be ellipses, the major axis of which will be perpendicular to the direction $\lambda$, given by equation (3). Hence the major axis lies in the same direction as the Futter's line in the absence of turbulence.
and hence \( \phi \) is same. However, the intersection of this line with \( t_y/t_x = \gamma / \gamma_o \), \( \tan \phi \) does not yield the magnitude of the velocity but leads to a new parameter \( V(1 + Vc^2/v^2) \). The spread of the points about the major axis is an independent indication of the magnitude of \( V_c \). It can be shown that \( V \) and \( V_c \) are related by the following equation:

\[
\frac{2}{V^2 + Vc^2} = \left( \frac{\tau_{xx} + \tau_{yy}}{\sigma_x^2 + \sigma_y^2} \right)^2 + \left( \frac{\tau_{yx} + \tau_{xy}}{\sigma_x \sigma_y} \right)^2 + 4 \left( \frac{\tau_{xy}}{\sigma_x \sigma_y} \right)^2 \ldots (5)
\]

where \( \rho \) is the correlation coefficient between \( t_x \) and \( t_y \) values. Hence \( V \) and \( V_c \) can be determined unambiguously.

(b) Anisometric Turbulent pattern

When the pattern is anisotropic as well as turbulent, two additional unknowns appear in the equations, namely the axial ratio \( r \) and the orientation \( \psi \) of the auto-correlation ellipse. This necessitates the evaluation of another statistical parameter. BANESLI (1960) used the results of BRIGGS and SPENCER (1955) for an anisotropic pattern by a suitable transformation of the coordinate system, in which the pattern appears to be isotropic and the results of BRIGGS and SPENCER could be used. Standard deviation of \( t_x \) and \( t_y \) was the additional parameter.
Following scheme of calculations is given by Banerji to determine various parameters of drift and anisotropy by this method:

(i) Calculate $\bar{x}_x, \bar{x}_y, \sigma_{x_x}^2, \sigma_{x_y}^2$ and $\bar{x}^2$

(ii) Calculate $A = (1/\sigma_{x_x^2})$, $B = (\bar{x}_x/\sigma_{x_x^2})$, $C = (2\sigma_{x_x^2}/\sigma_{x_x^2})^2$

(iii) Calculate $\psi = \frac{1}{2} \left( \frac{\rho \sqrt{C - D} + \sqrt{C}}{\sqrt{C - D}} \right)$ ....(6)

(iv) Calculate $P_x = \cos \psi \left[ \frac{x + y}{x - y} \right]$ ....(7)

(v) Calculate $r = \left( \frac{P + 1}{P - 1} \right)^{1/2}$ ....(7)

(vi) Calculate $\beta = \psi + \tan^{-1} \left[ \frac{1}{\sqrt{C}} \left( \frac{\cos \psi - (A/C) \sin \psi}{\sin \psi - (A/C) \cos \psi} \right) \right]$ ....(8)

(vii) Calculate $\nu = \frac{1}{\sqrt{C}} \cos \psi \left( \frac{1}{\sqrt{C}} - \eta \right)$ - $\sin \psi \eta$ ....(9)

$M = \frac{1}{\sqrt{C}} \sin \psi - \eta$ + $\cos \psi \eta$

$N = \frac{1}{\sqrt{C}} \cos \psi \left( \frac{1}{\sqrt{C}} - \eta \right)$ + $\sin \psi \left( \frac{1}{\sqrt{C}} - \eta \right)$

(viii) Calculate $V_x = (A/C - M/L)$

$V_y = (B/D - M/L)$

$V = V_x V_y / (V_x^2 + V_y^2)$ ....(9)

(ix) Calculate $V_x = (V/L - N/V^2)^{1/3}$ ....(10)

5.4 Method of Lyon, Morris and Bamgboye using Standard deviation of time lags

As shown earlier, the method of BRIGGS and SPENCER (1955) requires a large number of time lags.
t_x and t_y in order to get a good fit of the actual distribution (BRIGGS and PAGE, 1955). Briggs and Spencer used about an hour long record. It is however doubtful that the conditions remain stationary over such a long period. Since the experience shows that the distribution of time lags is approximately Gaussian in majority of the cases, it was proposed by LYON, MORRIS and BAMBOYE (1969) that the standard deviation of a series of time shifts i.e. t_x and t_y, should be used as an adequate means of fitting the distribution and it is then possible, to use Briggs-Page- Spencer's method for a relatively short drift records of a few minute duration, which are normally used in the drift measurements.

If t_x has the distribution given by equation (4) of Briggs and Spencer, then the variance of the quantity\
\( \frac{t_x - \bar{t}_x}{(x - \bar{t}_x)} \) where \( X = \frac{\bar{t}_x^2}{(V^2 + V_c^2)} \) will be 0.37 (for a normal distribution).

As \( t_x \) is mean and \( X \) is a constant,
\[ \frac{t_x^2}{(x - \bar{t}_x)} = 0.37 \]
or \( X = \frac{t_x^2}{tx} + 2.70 \frac{t_x^2}{tx} \) \( ....(11) \)

Similarly for Y and Z

If a better value of variance of the function \( \tilde{t} \) is \( \tilde{t}^2 \) instead of 0.37,
then \( X = \frac{\bar{t}_x^2}{tx} + \frac{t_x^2}{\tilde{t}^2} \) \( ....(12) \)
This modified time delay method yields unambiguous values for the parameters of the characteristic ellipse, exactly in the same way as the correlation technique. The two methods are related by the equation:

\[ X = \left( \frac{2}{v_{0}'} \right)^2 = \tau_1^2 \]
\[ Y = \left( \frac{\rho'}{v_{0y}'} \right)^2 = \tau_2^2 \]  
\[ Z = \left( \frac{\rho_{0x}}{v_{0z}'} \right)^2 = \tau_3^2 \]  

Having obtained \( V'_o \) and \( \tau_1, \tau_2, \) and \( \tau_3 \) from this, all the relations of PHILLIPS and SPENCER (1955) can be applied to obtain \( r \) and \( \psi \) and also \( V_c \) and \( V_{co} \) (\( V_c \) along the minor axis of ellipse).

The mean distance \( L_0 \) between the irregularities can be deduced from the mean time \( 'To' \) between the maxima of the pattern.

\[ L_0 = V_{co}' To \]

where \( V_{co}' \) is the value of fading velocity \( V_c \) along the minor axis of the ellipse.

Following relations are given LYON et al. (1969) for the drift speed and direction as well as the anisotropy parameters of the pattern for an equatorial station where the irregularities are field aligned (\( \psi = 0 \)) by substituting the values of \( \tau_1, \tau_2, \) and \( \tau_3 \) and \( V_c \)
from equation (13), in the Phillips-Spenccer's relations for full-correlation analysis, as given in the Chapter III. These equations hold for an isosceles right angled triangle with the diagonal 'd' and two sides along the north-south and east-west directions.

\[ V_E = d \sqrt{2} x \]
\[ V_N = d \sqrt{2} y \]  
\[ r = \frac{x}{y} \]  
\[ V_{oo} = \frac{d}{(2x)^{\frac{3}{2}}} \]  
\[ V_{oo} = \frac{d}{(2x)^{\frac{1}{2}} (1 - \sqrt{2} x - \sqrt{2} y)} \]  
\[ L_0 = T_0 \frac{d}{2x} \]

**Comparison of Lyon-Morris method with the Phillips-Spenccer's method**

1. The shift of the maxima of cross-correlograms corresponds to the mean time lag \( t_x \) or \( t_y \).
2. The time lag \( \tau_1 \), \( \tau_2 \) and \( \tau_3 \) correspond to the quantities \( X^\frac{1}{2} \), \( Y^\frac{1}{2} \) and \( Z^\frac{1}{2} \).
3. Comparing the equation (16) with the one derived by BRIGGS, PHILLIPS and SHINN (1950), according to which:

\[ \tau_c^2 = \tau_{xx}^2 + \tau_{mm}^2 \]

or \( \tau_{mx} = 1.64 \tau_{tx} \)  

Similarly for \( \tau_{my} \) and \( \tau_{mz} \).
Here $c$ stands for $T_1$, $T_2$ and $T_3$ and for the shift of maxima of cross-correlogram $T_m$ is the time lag on auto-correlogram corresponding to the maximum cross-correlation coefficient. Lyon et al. (1969) have also discussed the condition for the real values of $r$ and $V_{co}$ and the possible causes of errors.

5.5 Sprenger and Schminder's method (Variant-2)

Sprenger and Schminder (1969) have shown that there are at least two different ways of making similar fades analysis, which may possibly yield considerably different drift velocity.

Variant-1 is the conventional Mitra's method described in Chapter II and consists of measuring the individual time delays between similar fading maximum or minimum and averaging them separately for any two pairs of records. The mean drift vector $V_{1}$ is deduced from these two mean time delays $t_x$ and $t_y$. Variant-2 (Sprenger and Schminder, 1969) consists in measuring the individual pair of time lags for each fading maximum or minimum which occurs similarly at all the three traces and determining the drift vector for individual pairs of $t_x$ and $t_y$. The mean drift vector $V_{2}$ is found out by averaging all these individual drift vectors. In short, Variant-1 is the
method of averaging the time delays and variant-2 is the method of averaging drift vectors. Instead of using the means, median values of this may be used with advantages as recommended by the URSI-C19 working Committee on Ionospheric Drift analysis (Wright and Kent, 1968).

Variant-1 gives the apparent drift velocity, where the effect of the curvature of line of maxima is eliminated by averaging a large number of \( t_x \) and \( t_y \) but the Variant-2 takes into account of the different values of drift velocity at different set of similar fades within the same record and the vector sum of these will result into a velocity slightly lower than \( V_2' \). Fade to fade variation of the drift vector can also be due to genuine variation of drift during the course of an observation.

5.7 Six point correlation method of Yerg and its modification by Keneheea and by Fedor for 3-D case

(a) Yerg (1955) developed a slightly different method of analysis for determining \( V, V_c \) and \( V_c' \), which requires only six values of correlation coefficients rather than the whole curve (as in Phillips and Spencer's method) and therefore offers much saving of time. These six correlation coefficient are: 

\[
\rho(\tilde{z}, 0, t), \rho(\gamma, \eta, \tau) / \rho(\gamma, \eta, 0), \rho(\tilde{z}, 0, \tau), \rho(0, \eta, \tau) \ldots \rho(0, 0, \tau)
\]
The first three are cross-correlation coefficients between three pairs of aerials at zero time. Next three at an arbitrary time.

The correlation coefficient \( \rho \) is a function of space and time. It is assumed that:

1. This function may be represented as a Taylor's series in three variables (i.e., it has a parabolic form) and

2. Near the origin, only first and second order terms are retained.

Thus we have:

\[
f(\frac{3}{\lambda}, \gamma, \tau) = 1 + A \cdot \frac{\gamma^2}{\lambda^2} + B \cdot \frac{\tau^2}{\lambda^2} + C \cdot \frac{\gamma^2}{\lambda^2} + 2M \cdot \frac{\gamma \cdot \tau}{\lambda^2} + 2N \cdot \frac{\gamma^2}{\lambda^2} \]

The six coefficients \( A, B, C, H, M \) and \( N \) are evaluated of the pattern, then it can be shown that:

\[
V_x = \left( \frac{MT}{AC} - H/A \right) / \left( 1 - \frac{N^2}{AC} \right)
\]

\[
V_y = \left( \frac{HN}{AC} - K/C \right) / \left( 1 - \frac{N^2}{AC} \right)
\]

Hence \( V = (V_x^2 + V_y^2) \)

and \( \theta = \tan^{-1} \left( \frac{V_x}{V_y} \right) \)

and \( V_c = B \left( 1 + \tan^2 \theta \right) / \left( A + C \tan^2 \theta + 2N \tan \theta \right) \)

and so, \( V_c = \left( V_c^2 - V^2 \right) ^{\frac{1}{2}} \)

This method is simple as it needs only six values of . Also it includes the effect of random motion as well as anisotropy of the pattern.
YIGALI (1962) modified Yerg's six-point method by using a triangular shape of the correlation function rather than a parabolic shape. According to him:

\[ \beta = 1 - \left( A \gamma^2 + B \eta^2 + C \tau^2 + 2H \eta \gamma + 2M \eta \tau + 2 N \gamma \tau \right) \]

The main defects of Yerg's method are:
1. The arbitrary choice of time lag \( \gamma \) and
2. The assumed parabolic shape of the correlation function near the peak of the curve.

(b) Best fit correlation method.

The basic assumption of the BRIGGS et al. (1950) in their full correlation analysis of the drift records was that the contours of the constant correlation are similar concentric ellipsoids of the same orientation in the ground plane and time coordinate system. Since the actual data deviates more or less from this assumption, KENESHEA et al. (1965) proposed a new method in which first the correlation ellipsoids are computed which give the best overall fit to the measurements.

The generalized equation of the correlation ellipsoids is:

\[ f(\xi, \eta, \tau) = 1 - \left( A \xi^2 + B \eta^2 + C \tau^2 + 2H \eta \xi + 2M \eta \tau + 2 N \xi \tau \right) \]

\[ \ldots \ldots (24) \]
Using six points on the correlogram, one can find out the values for the constants $A, B, C, H, M$ and $N$ as in case of Yerg's method as well as the time lags $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$, and $\tau_6$. The slopes $(\tau_1^2/\tau_2)$, $(\tau_3^2/\tau_4)$ and $(\tau_5^2/\tau_6)$ of these three BPS lines are made to satisfy the following equation for an ellipsoid by rotating the straight lines about their best-determined points.

$$\frac{\tau_1^2}{\tau_2} + \frac{\tau_3^2}{\tau_4} - \frac{\tau_5^2}{\tau_6} = 0 \quad \ldots \ldots (25)$$

The six resulting intercepts are then used to determine the parameters $V, \dot{\varphi}, V_c, r$ and $\varphi$ as the Yerg's method described earlier in Sec 6. This method uses all available values of cross-correlation down to some arbitrary minimum to determine the best straight lines, but treats each receiver pair separately; even the 'best fitting' straight lines so achieved are then abandoned to satisfy the relation (25) amongst their slopes. PEDEOR (1967) extended this method to 3-dimensional case and a larger number of aerials.

(c) Statistical determination of the three dimensional Ionsopheric drifts

A general method of analysis, based upon the concepts introduced by BRIGGS et al. (1950) and
KENESHEA et al. (1965) has been developed by EIDOR (1967).

From any adequate combination of the cross-correlation functions among spaced measurements of the random fading of a pulsed radio echo from the ionosphere, the method extracts the best-fitting value of average drift, velocity of random changes and parameters describing the average size, life-time, spatial orientation and characteristic shape of the irregularities in the echo amplitude. The analysis was developed in three spatial dimensions plus time and is useful when the fadings at two closely spaced frequencies are also available. The method offers three distinct advantages over previous treatments of this problem:

(i) it provides a best fit to the available data in accord with the principle of least squares,

(ii) it places no upper limit on the quantity of the data that may be so fitted and in particular permits the mathematical over determination of the problem, which is desirable from its inherently statistical nature,

(iii) the method permits a complete description of the average three dimensional space and time structure of the measured quantity.

It is shown that the horizontal velocity components are not given correctly by the two dimensional
methods if vertical movements of anisometric irregularities occur. The main results of this method are as follows:

Expending 3-dimensional correlation function upto first order even terms of a taylor's series, we have

\[ \rho^2(x, y, z, t) = \exp(-\frac{1}{2} f(x, y, z, t)) \]

where

\[ f(x, y, z, t) = (a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{44}t^2 + a_{12}xy + a_{13}xz + a_{14}xt + a_{23}yz + a_{24}yt + a_{34}zt) \]

The coefficients \( a_{11}, a_{12} \) et. can be determined by using ten known values of \( \rho^2(x, y, z, t) \).

Thus one gets

- \( V_x \) (eastward) = \( (a_{12}a_{24} - 2a_{22}a_{44}) / \Delta_1 \)
- \( V_y \) (northward) = \( (a_{12}a_{14} - 2a_{11}a_{24}) / \Delta_1 \)
- \( V_z \) (vertical) = \( (\Delta_2 - \Delta_1 \Delta_3 - \Delta_1 \Delta_4) / (\Delta_1 \Delta_5 - \Delta_2 \Delta_4) \)

where

- \( \Delta_1 = (4a_{11}a_{22} - a_{12}^2) \)
- \( \Delta_2 = (2a_{11}a_{23} - a_{12}a_{13}) \)
- \( \Delta_3 = (2a_{11}a_{24} - a_{12}a_{14}) \)
- \( \Delta_4 = (2a_{11}a_{34} - a_{14}a_{13}) \)
- \( \Delta_5 = (4a_{11}a_{33} - a_{13}^2) \)

Also

- \( a_{11} = (\frac{1}{11}/d_1)^2 + (\frac{1}{12}/d_2)^2 + (\frac{1}{13}/d_3)^2 \)
- \( a_{22} = (\frac{1}{21}/d_1)^2 + (\frac{1}{22}/d_2)^2 + (\frac{1}{23}/d_3)^2 \)
- \( a_{33} = (\frac{1}{31}/d_1)^2 + (\frac{1}{32}/d_2)^2 + (\frac{1}{33}/d_3)^2 \)
- \( a_{12} = 2(\frac{1}{11}\frac{1}{13}/d_1^2 + \frac{1}{12}\frac{1}{22}/d_2^2 + \frac{1}{13}\frac{1}{23}/d_3^2) \)
where $d_1$, $d_2$, and $d_3$ are the semi-axes of the spatial characteristic ellipsoid and $l_{ij}$ are their direction cosines.

Solving these equations, one gets the size in $d_1$, $d_2$, $d_3$ and orientation $l_{ij}$ for the characteristic ellipsoid.

Horizontal drift velocity, $V_\text{H} = \left( V_x^2 + V_y^2 \right)^{\frac{1}{2}}$ (27) 

$\phi_\text{H} = \tan^{-1} \left( \frac{V_y}{V_x} \right)$ 

and the drift velocity $V = \left( V_\text{H}^2 + V_z^2 \right)$ \ldots (28) 

$V_c = \frac{d'^2}{b^2}$ \ldots (29)

where $d'$ is the spatial separation at which the correlation drops to $1/e$ and $b$ is the lifetime of the irregularities.

This technique of analysis has been applied to a few spaced antenna closed frequency simultaneous fading records taken at Thumba to determine 3-dimensional drift. Results are discussed sec. 5.12. Following ten correlation coefficients are used to determine the values of the ten coefficients $a_{11}$, $a_{12}$ etc.

\[
\begin{align*}
\rho(0,0,0,0), & \rho(0,0,0,0), \rho(0,0,0,0), \\
\rho(0,0,0,0), & \rho(0,0,0,0), \rho(0,0,0,0). \\
\rho(0,0,0,0), & \rho(0,0,0,0), \rho(0,0,0,0). \\
\rho(0,0,0,0), & \rho(0,0,0,0). \\
\end{align*}
\]
Briggs spatial correlation method of analysis

The analysis of a moving random diffraction pattern where the whole pattern can be recorded, was given by Briggs (1968). The pattern is assumed to have a certain mean velocity and also to change randomly in shape, as it moves. It is shown how the parameters used in the standard correlation analysis, namely $V, V_c, r, b$ and $\gamma$ can be determined from two records of a two-dimensional moving pattern obtained at a known time interval. The velocity of motion can be obtained without making the assumption that the spatial and temporal correlation functions have the same shape. If any successive records of the pattern are available separated by different time intervals, then the form of the temporal correlation function of the random changes alone can be determined.

The details of this method are not discussed here as it was not applicable to Thumba drift records, due to limited number of receiving aerials. Using the Buckland Park array of 89 receiving aerials at Adelaide, Golley and Rossiter (1970), have used this technique to calculate each spatial Fourier component of drift velocity. This is a useful method of analysis when it is suspected that the dispersive wave motions play a part in the movements of the pattern and the effect of the variability of drift
speed during the course of a record is to be eliminated (see 7.5c).

5.8 Validity of correlation analysis

WRIGHT (1968) and WRIGHT and FEDOR (1969) compared the results of the radio spaced antenna experiment with those of the rocket and gun launched visible trial measurements of neutral winds and found a good agreement between the two for the total reflection (while the standard theory requires the echo pattern from a point source transmitter to move at twice the ionospheric speed) and a poor agreement when the partial reflections were involved. As it does not seem likely that the ionospheric irregularities themselves move at half the neutral wind speed throughout the height range (90 - 140 km) of observations (KATO et al. 1970), one can doubt the simple scattering models from which the doubled velocities are deduced for a pattern from a point source transmitter.

A few workers have however pointed out that such a disagreement can result from the obscure defects in the much involved correlation analysis. BRIGGS (1969) found an increasing true drift speed with the increasing aerial separation and doubted this to be the results of the assumption of the similarity of space and time variations of the pattern. HINES (1964) has questioned the
validity of correlation analysis in the presence of the wavelike motions in the ionosphere.

It has however been recently shown (PITTMAY et al. 1971) using a computer simulated fading records, that the correlation analysis introduced by BRIGGS et al. (1950) and PHILLIPS and SPENCER (1955) and developed in an optimal form by FEDOR (1967) detects the mean direction and speed of the diffraction pattern and estimates random velocity fluctuations directly related to the random changes in the model.

5.9 Results and comparison of various methods of analysis for Thumba (0.6°S dip)

Fig. 5.1 gives the plot of $t_x$ and $t_y$ values for a few records at Thumba, in an attempt to apply the Putter's method of analysis. It is found that owing to extreme anisotropy of the ground diffraction pattern, the values are invariably low and the plot of $t_x$ and $t_y$ does not produce a straight line as shown theoretically by PÜTTER (1955). The points are clustered at one place and it is not possible to estimate the drift velocity.

Fig. 2a compares the results of similar fades method of Mitra (Variant-1) with those of the similar fades methods of Sprenger and Schminder (Variant-2) for F-region.
PUTTERS METHOD

PLOT OF T_x - T_y FOR THUMBA DRIFT

NIGHT TIME
30^{th} NOV. 67
2300 HR
4.7 F

31^{st} DEC. 67
0000 HR
4.7 F

3^{rd} NOV. 67
2300 HR
4.7 F

DAY TIME
30^{th} NOV. 67
0700 HR
4.7 F

31^{st} NOV. 67
0800 HR
2.2 E

30^{th} NOV. 67
1700 HR
4.7 F

30^{th} NOV. 67
1200 HR
4.7 F

THUMBA F-REGION 1968-69

APPEARENT DRIFT

SPEED IN K/S

DIRECTION EAST OF NORTH

BY VARIANT 2 METHOD

BY VARIANT 1 METHOD

BY FULL CORRELATION METHOD
The dotted lines represent the equal values of the two parameters. It is however found that \( V_2 \) is always lower than \( V_1 \). As the drift direction is predominantly east-west at Thun, the points in \( \theta_2 \) \( \theta_1 \) plot are clustered either around 90° or 270° east of north. Comparison of variant-2 results with the true drift velocity of full correlation analysis is shown in Fig.3b. This shows a close correspondence in the values of speed and the direction. This is in accordance with the results of SPRENGER and SCHEMINDAR (1969) and shows that Variant-2 leads to the values close to the true velocity.

SPRENGER and SCHEMINDAR (1969) analysed drift records for Central Europe by the two variants of similar fades method and also full-correlation method and compared the velocities and found a one to one correspondence between the velocities by similar fades and the full-correlogram method. The correlation was 0.91. The slight deviation of values between \( V_1 \) and \( V' \) (by full correlation taking anisotropy into account) was accounted for the anisotropy which is not taken into account by the similar fades method.

As already mentioned \( V_2 \) is always smaller than \( V_1 \) and the difference increases with increasing spread of the individual time delays. Just the same tendency can be
found from the equation \( V' = V \left( 1 + \left( \frac{V_c}{V} \right)^2 \right)^{\frac{3}{2}} \) of BRIGGS et al. (1950) between \( V' \) and \( V \), where \( V \) is always smaller than \( V' \) and the difference increases with increasing magnitude of \( V_c \). Thus \( V_c' \) may be comparable to the true drift velocity \( V \), assuming that there is some relationship between the spread of the individual time shifts and the magnitude of \( V_c \).

SPRINGER and SCHMINDAER (1969) also compared \( V_c' \) and \( V \) and found a good correlation. The close correspondence of \( V_c' \) with \( V \) on one hand and \( V_1' \) and \( V' \) on the other hand, leads us to substitute \( V_1' \) and \( V_c' \) instead of \( V' \) and \( V \) respectively into the equation of BRIGGS et al. (1950) for the random drift parameter \( V_c' \).

Hence \( V_{cs} = V_2' \left( \frac{V_1'/V_2'}{2} - 1 \right)^{\frac{3}{2}} \) ...........................................(30)

SPRINGER and SCHMINDAER (1969) also compared the \( V_c' \) with \( V_c \) (by full-correlation method) and found a close correspondence in the two values.

Drift speed and directions were also calculated for each fade of a number of drift records for Thumba P-region and the results are given in the table I and II. It is noticed that the drift speed is rarely consistent and this variability of the drift of the pattern from one fade to another can arise either from the actual changes in the ionospheric drift or from the continuously changing
shape of the pattern. The direction $\bar{\varphi}$ however does not vary much. The variant-2 method of determining average drift during the course of a record is therefore more accurate as it takes care of the changing drift directions also. KASHIN et al. (1970) have given the modified similar fades method when drift changes during the period of observation.

Fig. 5.2b shows the comparison of the apparent drift calculated from the Vatra's method (Var-1) and the correlation peak method. The results are very smaller excepting some scatter of the points which can be accounted for by the presence of dispersion in the drift as discussed in Chapter VII (see 7).

Fig. 5.3a compares the results for true drift speed $V$ as determined by the full-correlation method and that by Yerg's six point correlation method. The correspondence is fairly good for the direction of drift while for the speed there is slight deviation for the higher values of the drift speed. The main defects of Yerg's six point method lie in the choice of an arbitrary time lag $\tau$. The method yields correct results (equal to those obtained by full-correlation analysis) for smaller values of $\tau$, which corresponds to the near peak of the correlogram and hence is subject to less statistical variation.
Yorg's method also assumes a parabolic shape of the correlation function whereas the full correlation analysis of BRIGGS et al. (1950) assumes the contours of constant correlation to be similar concentric ellipsoids. This could result in a slight difference in the values estimated by two methods.

Fig. 5.4 compares the $V$ and $\theta$ results of Banerji's method and the method of Lyon et al. to those of the full-correlation method. Though the statistical method of Lyon et al. yields fairly close values, the Banerji's method does not seem to be fully applicable for the equator as the values of both speed and direction are widely distributed around the $45^\circ$ line. There is a tendency of getting slightly lower values for $V$. Lyon et al. have modified their method for the case of equator by assuming that the characteristic ellipse is oriented along the magnetic lines of force, and then determined $V$ and $\theta$. Banerji's method on the other hand first estimates the value of the tilt angle and then the value of $V$ and $\theta$ with the suitable corrections. A slight error in the estimation of $\alpha$ can result in large change in the value of $\theta$ and hence $V$. For Thumba, the $ty$ values (along magnetic north-south) are negligibly small compared to $tx$ and the value of $B$ as well as $C$ worked out to be nearly zero and the estimates of various parameters are not accurate.
Fig. 5.5 compares the random drift parameter $V_c$ as evaluated by four different methods, namely variant-2, Banerji's, Lyon-Morris and Yerg's with that from the full-correlation analysis. The variant-2 yields nearly the same values as full-correlation method and this has been discussed earlier in this Chapter along with the results of SPRENGER and SCHMINDER (1969). The values of $V_c$ by Yerg's six point method deviate slightly for the high magnitudes and the limitations applicable to $V$ also hold for $V_c$. The results of Banerji's method for $V_c$ like those for $V$, are not in very good correlation with full-correlation analysis results. The effect of anisotropy does not seem to be removed completely from these values for a station like Thumba, where the pattern is highly anisotropic. The method of Lyon et al gives values of $V_c$ closely agreeing with those of full-correlation values.

Fig. 5.6 compares the anisotropy parameters of the ground diffraction pattern as derived by the Banerji's method and Lyon-Morris method, with those from the full-correlation method. Banerji's method yields mostly a lower value of axial ratio $'r'$ than the complete correlation analysis. The tilt angle $\psi$ of the correlation ellipse is also fairly large in case of Banerji's method compared to full-correlation method. The Lyon-Morris method gives the $'r'$ values distributed equally on both sides of 45° line.
THUMBA F-REGION 1968-69
ANISOTROPY PARAMETERS

RANDOM DRIFT SPEED IN M/S

AXIAL RATIO 'γ'
ORIENTATION EAST OF NORTH

BY VARIANCE METHOD

BY FULL-CORRELATION METHOD

BY NEUMANN'S METHOD

BY FULL-CORRELATION

Y IS ASSUMED TO BE ZERO FOR EQUATOR BY LYON-MORRIS
The tilt angle $\gamma$ is assumed to be zero for the equator in this method, which is more or less the case for Thumba. The results of the three dimensional statistical correlation method suggested by PEDOR (1967) for a few cases at Thumba are summarized in the table No. III.

Conclusions:

The modified time delay method of LYON et al. (1969) offers a good substitute for full-correlation analysis for an equatorial station like Thumba when a complete knowledge of the ground diffraction pattern is to be obtained from the time shift values alone. The methods proposed by PUTTER (1955) and BANERJI (1958) fail completely for the equator due to extreme elongation of irregularities in magnetic N-S direction. Variant-2 offers a good substitute for full-correlation method when only the true drift parameters and the random drift is to be determined but not the anisotropy parameters. Both these methods, namely the Variant-2 and the Lyon-Morris method result in great saving of time and can be applied even in the absence of electronic computer. The method of Yerg, though simpler as far as the computations are concerned, does not result in any special reduction of labour as the fading records have still to be read completely for their amplitudes, as in
the full-correlation method. The values obtained by Yerg's method are however closest to those of the full-correlation technique. On the whole, full-correlation technique of drift analysis is the most accurate and should be used for regular reduction of drift data if the electronic computer is available.

Fedor's three-dimensional correlation method is the most ideal and accurate method of analysis provided close frequency spaced antenna simultaneous records are available, as it takes into account the effect of the vertical drift component also.
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Table - I
### Fade to fade variation of the drift direction at Thumba during 1968-69

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Table - II
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Where:
- $V_H$ = horizontal drift in m/s
- $\varphi_H$ = horizontal drift direction measured east of north.
- $V_z$ = vertical drift in m/s
- $\varphi_z$ = vertical drift direction measured clockwise from vertical.
- $V$ = total drift speed in m/s
- $V_0$ = random drift speed
- $d_1$ = semi-minor axis of characteristic ellipsoid in metres
- $d_2$ = semi-major axis of characteristic ellipsoid in metres
- $d_3$ = semi-vertical axis of characteristic ellipsoid in metres
- $\alpha$ = tilt angle of third axis to the vertical
- $\gamma$ = tilt angle of major axis to the magnetic N-S

**Table - III**