2.1 Instrumentation - Langmuir Probe

In the mesosphere, the ion-neutral and electron-neutral collision frequencies are much higher than ion and electron gyro-frequencies, respectively and hence any perturbations produced in neutral density by any source such as turbulence will also be transmitted to ions and electrons as well (Thrane and Grandal, 1981). This aspect is utilized in rocket-borne electron and ion density measurements as well as in the MST radar technique to study turbulence and to derive neutral turbulence parameters in this region. In the present study Langmuir probes have been employed to measure the electron density fluctuations and thereby study the neutral turbulence in the equatorial and low latitude mesosphere and the lower thermosphere.

The Langmuir probe technique was employed for ionospheric studies for the first time in 1946 by Spencer and his colleagues in the USA. A modified version of the basic technique was developed at Physical Research Laboratory, Ahmedabad (Prakash and Subbaraya, 1967) and was flown on board different types of rockets.
from the Thumba Equatorial Rocket Launching Station (TERLS), Thumba (8.5°N, 76.9°E) and Satish Dhawan Space Centre (SDSC), Sriharikota (14°N, 80°E).

2.1.1 Theory of Langmuir Probe

The Langmuir probe experiment consists of exposing a metallic sensor to the medium under study and measuring the current collected as a function of the bias voltage applied to it, which is varied from a convenient negative value, through zero to a positive value. The resulting current - voltage (I-V) curve, shown in Fig. 2.1, called the probe characteristics is used to determine various plasma parameters such as ion density, electron density, electron temperatures, etc. A Langmuir probe system consists of a probing electrode, generally referred to as the ‘sensor’ and an electrically insulated reference electrode which in most cases is the rocket body (for rocket borne experiments). A guard electrode is used to give a definite geometry to the electric field of the sensor and reduce the leakage current from the sensor to the reference electrode thereby improving the performance of the probe.
The current collected by the sensor from the surrounding plasma is converted into a voltage suitable for telemetry by an amplifier and telemetered to the ground. With appropriate biasing of the LP with a fixed positive voltage, the collected current is used to determine fluctuations of electron density at different scales, which typically range between a few km to about a meter.

**Probe at Plasma Potential**

When a probe is at the same potential as the ambient plasma, it receives both electrons and positive ions that strike its surface during their random motion in the medium. The net current drawn by the probe is therefore,

$$J = J_e + J_i$$  \hspace{1cm} (2.1)

where $J_e$ is the random electron current reaching its surface and $J_i$ is the random positive ion current.

$$J_e = A j_e = \frac{A n_e e v_e}{4}$$  \hspace{1cm} (2.2)

where, $j_e$ is the random electron current per unit area, $A$ is the effective surface area of the probe, $n_e$ is the ambient electron number density, $e$ is the electron charge and $v_e$ is the electron thermal velocity

$$v_e = \sqrt{\frac{8k T_e}{\pi m_e}} = 6.21 \times 10^5 \sqrt{T_e}$$  \hspace{1cm} (2.3)

and $m_e$ is the electron mass, $T_e$ is the electron temperature, $k$ is the Boltzmann constant.

Similarly,

$$J_i = A j_i = \frac{A n_i e v_i}{4}$$  \hspace{1cm} (2.4)

where, $v_i$ is the ion thermal velocity

$$v_i = \sqrt{\frac{8k T_i}{\pi M_i}}$$  \hspace{1cm} (2.5)

$M_i$ is the ion mass and $T_i$ is the ion temperature.
And since \( M_i \gg m_e \), it results in \( v_i \ll v_e \) and hence \( j_i \ll j_e \). So the net current to the probe at plasma potential can be considered to be purely the electron current.

\[
J = J_e = A \cdot \frac{n_e e v_e}{4} = A \cdot n_e e v_e \frac{kT_e}{2\pi m_e}^{\frac{1}{2}} \tag{2.6}
\]

This property can be used to determine the electron density in the medium if the electron temperature is known and vice versa.

**The Plasma Sheath and Floating Potential**

As the typical thermal velocity of electrons in the E-region of the ionosphere is about 200 km/s and that of ions is 1 km/s, a metallic probe immersed in plasma is very soon surrounded by an excess electron cloud as \( v_e \gg v_i \). The accumulation of the electrons around the probe continues until the electron cloud presents to the surrounding medium a negative potential strong enough to retard the electrons and make the electron flux to the outer surface of this electron cloud equal to the random positive ion flux in the medium and thereby maintaining equilibrium. The intermediate medium separating the probe from the ambient plasma is called the plasma sheath. The thickness of plasma sheath varies with the probe potential, but its scale is usually expressed in terms of a plasma parameter known as Debye shielding length, \( \lambda_D \), which is given by

\[
\lambda_D = \sqrt{\frac{kT_e}{4\pi n_e e^2}} \tag{2.7}
\]

Typical value of Debye shielding length in the E-region ranges between a few cm to a few tens of cm. The probe thus acquires a negative potential due to the plasma sheath, *vis-à-vis* the space or plasma potential, and is called the floating or wall potential. And at the floating potential, the net current to the probe is zero. The floating potential \( V_f \) given by

\[
V_f = -\frac{kT_e}{e} \ln \frac{I_e}{I_i} \tag{2.8}
\]

where, \( T_e \) is the electron temperature, \( e \) is electronic charge, \( I_e (j_i) \) is the electron (positive ion) random current density. Although the exact floating potential is a
function of electron temperature, electron and ion mean velocities, around 100 km altitude its typical value is about $-0.5V$ relative to the plasma or space potential.

**Probe at a Negative Potential**

When the probe is at a large negative potential with respect to the plasma potential, it attracts positive ions and electrons are repelled and the total current collected by the probe is a net positive ion current. This is called the ion saturation region (Fig. 2.1). As the probe potential becomes less negative the ion current starts reducing and electron current starts increasing, due to energetic electrons that can overcome the negative potential of the probe. Thus for measurement of ion densities, the probe must be operated in the ion saturation region.

**Retarding Potential Analysis**

As the probe potential is decreased further, the electron current to the probe increases rapidly (Fig. 2.1). This is called the 'retarding potential regime' in which the positive ion current is very small in magnitude compared to the electron current and the electron current consists of those electrons which could overcome the negative (retarding) potential in the probe. For a Maxwellian distribution of electrons, the electron current, $J_e$, at a retarding potential $V$ is given by

$$J_e = J_0 \exp \left( \frac{eV}{kT_e} \right)$$

(2.9)

Thus, if measurements of probe current are made at a number of retarding potentials, a semilog plot of the probe current versus probe potential will be a straight line of slope $e/kT_e$. This property is used to determine the electron temperature of the medium.

$$\log J_e = \log J_0 + \frac{eV}{kT_e}$$

(2.10)
**Probe at a Positive Potential**

Even a small positive potential is sufficient to make the positive ion current negligible when compared to the electron component and the net current can be considered to be purely an electron current. The electron current to a positive probe depends in a complex manner both on size and shape of the probe. For a sufficiently small probe, whose dimensions are comparable to the Debye length and mean free path of the medium, Langmuir and Mott Smith (1926) have obtained the following expressions for spherical, planar and cylindrical geometries.

i. *A small sphere:* For a sphere of radius smaller than Debye length, the current is given by,

\[
J = J_e \left( 1 + \frac{eV}{kT_e} \right)
\]  
(2.11)

ii. *A large plane:* For a large plane whose thickness is comparable to the sheath thickness, the current is given by

\[
J = J_e
\]  
(2.12)

iii. *A long thin cylinder:* For a long cylinder whose radius is comparable to the Debye length, the current is given by

\[
J = J_e \left\{ \frac{2}{\sqrt{\pi}} \frac{eV}{kT_e} + \exp \frac{eV}{kT_e} P \left( \frac{eV}{kT_e} \right) \right\}
\]  
(2.13)

where \( P \) is the error function

\[
P(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^2} dy
\]  
(2.14)

### 2.1.2 Proportionality between Probe Current & Electron Density

Assuming that there are no large variations of electron temperature in regions of measurement, the probe current can be considered to be proportional to the electron density. Thus a conversion factor can be used to get the electron density from the probe current. The factor can be evaluated by comparison with the electron
density obtained by ionosonde \((\text{Smith, 1969})\) or some other experiments which can give electron density. If the Langmuir probe is flown along with a radio frequency resonance probe, the conversion factor can be evaluated by comparison with the densities obtained by these measurements, as the resonance probe gives absolute value of electron density. Based on the experimental observations, it is widely accepted that the proportionality factor remains fairly constant above 85 km up to approximately 180 km. In the region below 85 km also a constant factor is used to estimate the electron density, as no independent measurement of electron density was available. However, due to the above considerations, the measurements in this region are used to study the fluctuations of electron density and not the absolute electron densities. In the present study the conversion factor was obtained by normalizing the probe current at the E-region peak with the electron density obtained by an ionosonde located in the neighbourhood of rocket station. The conversion factor used in the present work is \(2.4 \times 10^4 \text{ cm}^{-3} \mu \text{ A}^{-1}\).

### 2.1.3 Limitations of LP onboard Rockets

**Effects of Negative Ions**

In the classical Langmuir theory, described in section 2.1.1, the system consists only of positive ions and electrons. In the ionosphere this assumption is true only above about 70 km as negative ions are also present below this altitude. Hence the current collected by the Langmuir probe below this altitude consists of electron as well as negative ion current. Hence in the strict sense the LP theory is applicable only for altitudes greater than 70 km.

**The Reference Potential**

The positive and negative potentials on the LP sensor, referred to in the LP theory, are with respect to the plasma or space potential. As it is not possible to realize plasma potential, the next best thing is to realize the floating potential, which as
mentioned in Sec. 2.1.1, is approximately $0.5\,\text{V}$ lower than the plasma potential in the E-region. As the rocket body also comes to the floating potential it is used as a reference potential to ensure that the LP sensor is biased well above the plasma potential. For working in the electron saturation regime, a voltage of $+4\,\text{V}$ is usually applied to the LP sensor with respect to the rocket body.

**Size of Sensor**

The current collected by a probe in space returns to the ionosphere through the surface of the vehicle, so that the impedance of this surface with respect to the plasma is actually an element in the probe circuit. The collected current therefore should not significantly change the reference potential. This implies that the ratio of the area of the rocket body to the area of the sensor should be greater than the ratio of the electron and ion saturation currents ($J_e/J_i$) that can be collected by the vehicle. In the ionosphere, this ratio of the electron and ion saturation currents is of the order of 150 to 200. For an RH-300 MK II rocket (height = 100 cm (after combustion), diameter = 30 cm) used in the present experiment (Ref: Sec. 2.1.4), the surface area of the rocket body is approximately 9000 cm$^2$. The diameter of the hemispherical sensor used is 4 cm, which results in the above ratio to be greater than 350. This ensures that the current collected by the probe does not disturb the rocket payload.

**Effects of Rocket Velocity**

Most probe theories are developed for a probe at rest but by its very nature, a rocket borne probe does not meet this criterion. However typical rocket velocities of the order of a kilometer per second to a few hundred meters per second are much smaller than electron thermal velocities in the ionosphere which are of the order of 100 km s$^{-1}$. Hence the probe can be considered to be at rest with respect to the electrons.
Wake Effects

When a rocket or a satellite is travelling with a velocity greater than the mean ion velocity in the medium, positive ions can not penetrate into the rear of the vehicle although electrons can. Hence the absence of the positive ions creates a negative potential in the wake of the vehicle. This potential affects the charge particle collection by the probe and therefore should be avoided. The best solution is to place the probe at the nose tip of the rocket or far into the medium using booms. However, a probe at the nose tip does enter the wake during descent of the rocket. Errors arising due to the wake effects have thus to be considered during descent.

Effects of Geomagnetic Field and Attitude Variations

The attitude of the rocket borne sensor changes during the rocket flight due to rocket spin, precession as well as changes in the inclination of the rocket axis during its trajectory. In the presence of the Earth's magnetic field this variation in attitude has an effect on the probe current collected since charged particles can travel more easily along the magnetic lines than across it due to much higher electrical conductivity along the field lines. The effective mean free path for a charged particle is the Larmor radius, $\rho = \frac{mv_{\perp}}{eB}$, where $m$ is the mass of the charged particle, $B$ is the magnetic flux density and $v_{\perp}$ is the velocity of the particle in a direction perpendicular to the magnetic field. In the lower ionosphere the Larmor radius for an electron is of the order of one cm and for an ion it is of the order of a meter. Therefore, any probe of practical dimensions will be larger than the mean free path of the electrons. Since motion along the field lines is uninhibited, the probe collects electrons from distant regions along the field lines and since motion across is inhibited, the probe collects electrons not far away across the field lines. The effect of rocket spin and precision can thus clearly be seen as a modulation in the electron probe current. The extent to which this perturbation occurs depends on various factors like shape of the sensor, position of the sensor with respect to
the rocket spin axis, asymmetries on the body of the rocket, since it is used as the reference electrode, as well as the potential at which the sensor is being operated.

Very large spin modulation was observed in Flights 1 and 2 (Sec. 2.1.4) when the Langmuir probe was placed away from the rocket spin axis. Fig: 2.2 shows two second data of July 2004 flight along with the magnetometer data. The effect of rocket spin on the electron current is clearly observed.

2.1.4 Langmuir Probe Flights

Three RH-300 MK II rockets, each carrying a Langmuir probe, were launched during daytime to study the equatorial and low latitude mesosphere and lower thermosphere. Two of these flights were launched from Sriharikota (14°N, 80°E, 13.8°Dip) on 23 July 2004 at 1142 hrs LT (UT+05:30 hrs) and on 08 April 2005 at 1125 hrs LT (hereafter called Flight 1 and Flight 2, respectively) and the third flight was
launched from Thumba (8.5°N, 76.9°E, 0.4°Dip) on 27 November 2005 at 1123 hrs LT (hereafter called Flight 3). The first two flights were conducted in coordination with the Indian MST radar at National Atmospheric Research Laboratory (NARL), Gadanki (13.5°N, 79.2°E, 6.4°Dip). Both these flights were launched when the MST radar observed strong echoes at mesospheric altitudes. A pre-campaign was conducted to understand the morphology of the mesospheric echoes over Gadanki during May to July 2003 (Harish Chandra, Personal Communication). For Flight 1 the month of July was chosen as the occurrence and the strength of echoes were found to be highest during June-July and for Flight 2 the month of April was chosen to study the relatively weaker echoes and understand the seasonal variation. Kamala et al. (2003) discussed this seasonal variation of the mesospheric echoes over Gadanki in detail. Fig. 2.3 gives the geographic location of the two rocket launching sites, Sriharikota and Thumba, and the location of the Indian MST radar at Gadanki.

Fixed bias Langmuir probes were used to measure electron density fluctuations
in all the three flights. The configurations explained below were almost similar in all three flights. For the Flights 1 and 2, the LP sensor was a split sphere of 50 mm diameter, whose upper hemisphere was biased at +4 V and was used to collect the electron current, and the lower hemisphere was used as a guard electrode. For Flight 3, the LP sensor was of ogive shape and was biased at +4 V. For Flights 1 and 2, the LP sensor was mounted on the top deck with the help of a 200 mm long boom while for Flight 3 the sensor was mounted of the rocket nose tip. The electrical connection to the signal conditioning electronics was provided by means of a coaxial cable. The electronics was accommodated in a package mounted on one of the instrument decks. To cover the large dynamical range arising from the change in current due to variation in the electron density an automatic gain amplifier was used to measure the current in the range of 1 nA to about 3μA. For studying the electron density fluctuations in different scale sizes the current collected by the LP sensor was processed on board in three channels with different gains having frequency response of 0-100 Hz, 30-150 Hz and 70-1000 Hz. These channels are named as LP Main, LP MF and LP HF and were sampled at 520 Hz, 1040 Hz and 5200 Hz, respectively.

2.1.5 Other Complementary Experiments

RH-200 Rockets with Chaff Payloads

The first RH-200 rocket with chaff payload was launched at 1215 hrs LT on 23 July 2004, *i.e.*, 33 minutes after the launch of Flight 1 and the second one was launched at 1158 hrs LT on 08 April 2005, *i.e.*, 33 minutes after the launch of Flight 2 to measure the neutral winds in the atmosphere. The metallic chaffs were tracked by radars, which provided zonal and meridional wind profiles in the height ranges of 20-76 km and 20-60 km, respectively.
Table 2.1: Specifications of the MST Radar Operating Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Gadanki (13.5°N, 79.2°E)</td>
</tr>
<tr>
<td>Frequency</td>
<td>53 MHz</td>
</tr>
<tr>
<td>Peak Power Aperture Product</td>
<td>$3 \times 10^{10}$ Wm$^2$</td>
</tr>
<tr>
<td>Peak Power</td>
<td>2.5 MW</td>
</tr>
<tr>
<td>Maximum Duty Ratio</td>
<td>2.5%</td>
</tr>
<tr>
<td>Number of Yagi Antennas</td>
<td>1024</td>
</tr>
<tr>
<td>Beam Width (3 dB full width)</td>
<td>3°</td>
</tr>
<tr>
<td>Beam Angle</td>
<td>Zenith and 10° off zenith</td>
</tr>
<tr>
<td>Number of Beams used</td>
<td>East, West, Zenith, North, South</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>3 μs uncoded (450 m)</td>
</tr>
<tr>
<td>Inter Pulse Period</td>
<td>0.9 ms</td>
</tr>
<tr>
<td>Maximum number of range bins</td>
<td>125</td>
</tr>
<tr>
<td>Number of Coherent integrations</td>
<td>40</td>
</tr>
<tr>
<td>Maximum number of FFT points</td>
<td>256</td>
</tr>
</tbody>
</table>

Indian MST Radar

The MST radar of the National Atmospheric Research Laboratory (NARL) is located at Gadanki and operates at 53 MHz (Rao et al., 1995). On 23 July 2004 and 08 April 2005 the MST radar was operated during 0830-1600 LT. Details of the experimental parameters used for the radar operation are presented in Table 2.1. Uncoded 3 μs pulses were used to provide a height resolution of 450 m during the rocket launch period. Five radar beam directions (North-10°, South-10°, East-10°, West-10° and Zenith) were used to get the return echoes from 60 km to 120 km. The angles refer to zenith angles used for the experiments.

2.2 Data

2.2.1 Langmuir Probe Data

The current collected by the Langmuir probe sensors in all the flights was processed on board through three separate channels, to accurately detect the large range of amplitudes associated with scale sizes ranging from a few kilometers
down to less than a meter. The main channel had a frequency response of DC-100 Hz and was sampled at 520 Hz. The rocket vertical velocity in the lower altitudes at ~70 km was ~850 m/s and hence the altitude resolution of electron density measurements was ~2 m. The smallest scale that could be studied using the main channel data was therefore ~4 m. At ~90 km, the velocity was ~600 m/s and the altitude resolution was higher and ~1 m. The smallest scale that could be studied was ~2 m.

The other two channels were AC channels of medium and high frequencies with frequency responses of 30-150 Hz and 70-1000 Hz and were sampled at 1040 and 5200 Hz, respectively. Using the data from these channels, the smallest scale sizes that could be studied were smaller by a factor two and ten, respectively. After combining the data from the three channels, information regarding various scale sizes ranging from a kilometer to less than a meter was obtained.

2.2.2 Photometer Data

One of the longest series of ground based photometric observations of the oxygen green line (OI 557.7 nm), from 1979 to 1994 at Kiso (35.8°N, 137.6°E), Japan was used in the present study for detecting the long term variations. Ground based measurements of integrated zenith intensities of the OI 557.7 nm emission taken at one minute interval for twelve years (1979-1990) and hourly averages for four years (1991-1994) were taken from the World Data Center (WDC) C2 for Airglow, Tokyo, Japan. One-minute values were not available at the WDC for the period 1991-1994. These measurements were made on moon-less clear nights.
2.3 Analysis

2.3.1 Basic Mathematics and Fourier Transform

Let $L^p(\mathbb{R})$ denote the collection of all measurable functions $f$ defined on $\mathbb{R}$ such that the integral, called the Lebesgue integral,

$$\int_{-\infty}^{\infty} |f(t)|^p \, dt$$

is finite for each $p$, where $1 \leq p < \infty$.

Let $x(t)$ be an integrable function (i.e., $x(t) \in L^1(\mathbb{R})$). The Fourier transform of such a function is given by,

$$\hat{x}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)e^{-i\omega t} \, dt$$

If a particular frequency exists in the given signal, the above integral will result in a large value at $\omega$ and if it does not exist then the integral will give a minimum value or zero, thus producing a frequency spectrum. $\hat{x}(\omega)$ satisfies the following:

i. $\hat{x} \in L^\infty(\mathbb{R})$ with $||\hat{x}||_\infty \leq ||x||_1$,

where $||x||_p$ is the norm of the function $x$ and is defined as

$$||x||_p = \begin{cases} \left\{ \int_{-\infty}^{\infty} |x(t)|^p \, dt \right\}^{\frac{1}{p}} & \text{for } 1 \leq p < \infty \\ \text{ess sup} |x(t)|; (\infty < t < \infty) & \text{for } p = \infty \end{cases}$$

ii. $\hat{x}$ is uniformly continuous on $\mathbb{R}$,

iii. if the derivative $x'$ of $x$ also exists and is in $L^1(\mathbb{R})$, then $\hat{x}'(\omega) = i\omega \hat{x}(\omega)$, and

iv. $\hat{x}(\omega) \to 0$, as $\omega \to \pm\infty$

Inspite of the last property of $\hat{x}$, for every $x \in L^1(\mathbb{R})$, it is not necessary that $\hat{x}$ is also in $L^1(\mathbb{R})$. Only when $\hat{x} \in L^1(\mathbb{R})$ can the signal $x(t)$ be recovered back by the
use of the Inverse Fourier Transform (IFT), which is defined as,

\[ x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{x}(\omega)e^{i\omega t} d\omega \]  

(2.18)

The Fourier transform is capable of finding the different frequencies present in the signal, but it does not show the time information. However, time information is not completely lost but is hidden in the phases. If it were lost, it would not be possible to reconstruct the signal by IFT (Eq. 2.18). Noise like features are also observed in the Fourier spectrum of non-stationary signals (signals that do not contain a set of frequencies at all times) that arise due to abrupt changes in the frequencies. Such noise is not observed in case of stationary signals (signals that contain a set of frequencies at all times). Hence Fourier transform best suits stationary signals and can also be used for non stationary signals when time information is not required.

To overcome the difficulty faced by the Fourier transform, the Short Time Fourier Transform (STFT), also called as the Window Fourier Transform (WFT), was developed. In this technique, the given signal is split up into number of segments in small intervals of time where it is assumed that in each interval the signal remains stationary. A non trivial function \( g(t) \in L^2(\mathbb{R}) \) can be used as a window function if it satisfies the condition,

\[ t \cdot g(t) \in L^2(\mathbb{R}) \]  

(2.19)

The center \( t^* \) and radius \( \Delta_g \) of a window function \( g(t) \) are defined as

\[ t^* = \frac{1}{\|g\|_2^2} \int_{-\infty}^{\infty} t |g(t)|^2 dt \]  

(2.20)

and

\[ \Delta_g = \frac{1}{\|g\|_2^2} \left\{ \int_{-\infty}^{\infty} (t - t^*) |g(t)|^2 dt \right\}^{1/2} , \]  

(2.21)

respectively; and the width of the window function \( g(t) \) is defined by \( 2\Delta_g \).

Such a window function, \( g(t) \), whose width is equal to the width of the segment in the signal is taken. It is first placed at the beginning of the signal and then
Figure 2.4: A few examples of window functions.

multiplied with the signal with an appropriate weightage of the window. This is taken as the new signal and the Fourier transform is used to find frequency spectrum for that part of the signal. The window is then translated along the signal and the process is repeated. Few examples of window functions are the Rectangular window, Gaussian window, Hamming window, Hanning Window, etc., (Fig. 2.4). By using a rectangular window function, the data is broken into parts and the data is not modified at all. The problem of leakage appears due to this as the phase at the beginning of the signal and the end of the signal might not be same. This causes a broadening of the frequency components that are picked up by the FT or the STFT. By using other window functions like the Hanning or the Hamming windows, which start at zero at \( t = 0 \), increase smoothly to 1 at \( t = T/2 \) and then decrease smoothly again to zero at \( T = 0 \), the leakage problem can be reduced. The Hanning window, \( H(t) \) is generated from a cosine function and for
0 \leq t \leq T, H(t) \text{ is given as } 

\begin{equation}
H(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi t}{T} \right) \right] \tag{2.22}
\end{equation}

The STFT of a function \( x(t) \) is given by,

\begin{equation}
\text{STFT}(\omega, t') = \int_t [x(t) * g(t - t')] * e^{-i\omega t} dt 
\end{equation}

The transformed signal is a function of two parameters, frequency and time. Thus information of various frequencies in the signal at different times can be obtained. The efficiency of this method depends to some extent upon the assumption that the signal segment is stationary. This method has a constant resolution in the time domain as well as in the frequency domain. This can be represented in the time frequency plane as a box whose width gives the time resolution and the height gives the frequency resolution (Fig. 2.5). If the signal \( w \) non-stationary in one of the segments considered, then the time information corresponding to that time interval would be lost. To have a higher resolution in time, the width of the window function can be further shortened and hence the length of the signal to be Fourier analyzed. But as the signal becomes shorter, the frequency resolution becomes
coarser because any number of frequencies can be expected in such a short signal. Thus, a narrow window will give a good time resolution but a poor frequency resolution while a wide window will give a poor time resolution but a good frequency resolution. In the time frequency plane, good frequency (time) resolution means long rectangles with the longer side along the time (frequency) axis. This also means a low time (frequency) resolution. There is thus, a trade off between the frequency resolution and the time resolution. This is the disadvantage of this technique. Hence STFT is not suitable for signals with both very high and very low frequencies. In addition, several window lengths must be employed to determine the appropriate choice. For a definite window, the time and frequency resolutions will be the same throughout, i.e., the boxes in the time frequency plane are of constant width and height. The resolution in both domains is limited by the Heisenberg's uncertainty principle to $1/4\pi$. This results in a finite area of the box in the time frequency plane (Fig. 2.5), i.e., it cannot be said that a particular frequency existed at a point of time. Rather, it can be said that a particular band of frequencies existed in an interval of time. A Gaussian window gives the best possible resolution in time and frequency simultaneously, with the smallest time frequency window, with an area of $1/4\pi$.

### 2.3.2 Continuous Wavelet Transform

The wavelet transform, relative to some basic wavelet (defined below), provides a flexible time frequency window which automatically narrows when observing high frequency components and widens when observing low frequency components. The technique of using wavelets to identify localized power events in time series is now being used and applied in various studies starting from stock market fluctuations to geophysical phenomena.

If the function $\psi \in L^2(\mathbb{R})$ satisfies the "admissibility" condition:

$$C_{\psi} := \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} - d\omega < \infty$$  \hspace{1cm} (2.24)
then $\psi$ is called a wavelet. The window function $\psi$ is called a *mother wavelet*. If, in addition, both $\psi$ and $\hat{\psi}$ satisfy Eq. 2.19, then the basic wavelet provides a time frequency window with finite area given by $4\Delta_\psi \cdot \Delta_\hat{\psi}$. Under this condition it follows that $\hat{\psi}$ is a continuous function, so that the finiteness of $C_\psi$ in Eq. 2.24 implies $\hat{\psi} = 0$, or equivalently,

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \quad (2.25)$$

A Gaussian window does not possess this property and hence cannot be used as a wavelet. In addition, the wavelet should also be well localized in physical and Fourier spaces. To study the $M^{th}$ derivative of a function $x(t)$, the wavelet should have cancellations up to order $M$, so that it does not react to lower-order variations of $x(t)$. Hence for all $m \leq M$,

$$\int_{-\infty}^{\infty} \psi(t)t^m dt = 0 \quad (2.26)$$

In other words, to be admissible as a wavelet, a function must have zero mean and be localized in both time and frequency space (Farge, 1992). This is the reason why $\psi$ is called a “wavelet”.

Relative to every basic wavelet $\psi$, the continuous wavelet transform (CWT), $W_x^\psi(s, \tau)$, of a function $x(t) \in L^2(\mathbb{R})$ is defined as,

$$W_x^\psi(s, \tau) = \frac{1}{\sqrt{|s|}} \int_t x(t) * \psi^*(\frac{t-\tau}{s})dt \quad (2.27)$$

The transformed signal, $W_x^\psi(s, \tau)$, is a function of two variables, $\tau$ and $s$, i.e., translation and scale. These two parameters correspond to time and frequency respectively. $\psi(\frac{t-\tau}{s})$ is called the mother wavelet and $\psi^*(\frac{t-\tau}{s})$ is its conjugate. It is so called because all the wavelets to be used in the analysis are derived from this function. The factor $\frac{1}{\sqrt{|s|}}$ is for energy normalisation.

Assume that both $\psi$ and $\hat{\psi}$ are window functions satisfying Eq. 2.19. The center and radius of the function $\psi(s, \tau)$ are given by $\tau + st^*$ and $s\Delta_\psi$, respectively and those of its Fourier transform, $\hat{\psi}(s\omega)$, are given by $\omega^*/s$ and $\Delta_\psi/s$, respectively.
So the wavelet transform $W_x^\psi(s, \tau)$ gives local information of an analog signal $f(t)$ with a time frequency window,

$$[\tau + st^* - s\Delta\psi, \tau + st^* - s\Delta\psi, \omega^*/s - \Delta\psi, \omega^*/s + \Delta\psi]$$

(2.28)

For small values of $s$, this window is thin and tall and for large values it is wider and short (Fig. 2.5). Scale is inversely proportional to frequency, i.e., high scale implies low frequency and vice versa. For a particular $s$, a different wavelet is derived from the mother wavelet. When scaling is done, the mother wavelet is dilated, i.e., it is either stretched or compressed. Larger scales corresponding to lower frequencies are stretched versions of the mother wavelet and smaller scales corresponding to high frequencies are compressed versions of the mother wavelet. As the transform is computed for every single frequency or scale, the width of the window changes and hence the resolution, as is evident from the Eq. 2.28. Hence, different resolutions are obtained at different frequencies. At lower frequencies there is a good resolution in frequency and poor resolution in time, and at high frequencies it is vice versa. Here also Uncertainty Principle comes into the picture and so the boxes always have some finite area. As the wavelet is translated through the signal, the frequency spectrum at different times is obtained. The process is same as shifting of the window function through the signal in STFT. For every value of $s$ and $\tau$, one point in the time frequency plane is computed.

Some basic properties of the continuous wavelet transform are as follows.

**Linearity:** The wavelet transform is linear because it is an inner product between the signal $x(t)$ and the wavelet $\psi$. Let $x(t) = x_1(t) + x_2(t)$, then

$$W_x^\psi(s, \tau) = W_{x_1}^\psi(s, \tau) + W_{x_2}^\psi(s, \tau)$$

(2.29)

Let $x(t) = k x_3(t)$, then

$$W_x^\psi(s, \tau) = k \cdot W_{x_3}^\psi(s, \tau)$$

(2.30)

**Covariance by Translation:** The continuous wavelet transform is covariant under any translation $t_0$. Let $x_0(t) = x(t - t_0)$, then

$$W_x^\psi(s, \tau) = W_{x_0}^\psi(s, \tau - t_0)$$

(2.31)
**Covariance by Dilation:** The continuous wavelet transform is covariant under dilation also. Let $x_d(t) = x(at)$, then

$$W^\psi_{x_d}(s, \tau) = \frac{1}{\sqrt{a}} W^\psi_x(as, a\tau)$$

(2.32)

**Continuous Wavelet Transform of Discrete Data**

For a discrete equally spaced time series $x_n$, where $n = 0, 1, \ldots, N - 1$ is the index in time, the convolution of $x_n$ with a scaled and translated version of $\psi$ gives the continuous wavelet transform (Torrence and Compo, 1998):

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \psi^* \left[ \frac{(n' - n)\delta t}{s} \right]$$

(2.33)

where the (*) indicates the complex conjugate and no subscript indicates that the wavelet function is normalized (Eq: 2.38). By varying the wavelet scale $s$ and translating along the localized time index $n$, the wavelet coefficients can be computed.

Although it is possible to calculate the wavelet transform using Eq. 2.33, it can be done considerably faster in the Fourier space. Using Eq. 2.33, the convolution should be done $N$ times for each scale. Whereas in the Fourier space the Convolution theorem allows us to do all $N$ convolutions simultaneously using a discrete Fourier transform. According to the theorem, the wavelet transform will be the inverse Fourier transform of the product of the Fourier transforms of the signal ($\hat{x}_k$), and the conjugate of the mother wavelet.

$$W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^*(s\omega_k)e^{i\omega_k n\delta t}$$

(2.34)

where the Fourier transform of the signal is given by,

$$\hat{x}_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-2\pi ink/N}$$

(2.35)

In the continuous limit, the Fourier transform of the function $\psi(t/s)$ is given by $\hat{\psi}^*(s\omega_k)$. The angular frequency $\omega_k$ is defined as,

$$\omega_k = \begin{cases} \frac{2\pi k}{N\delta t} : & k \leq \frac{N}{2} \\ -\frac{2\pi k}{N\delta t} : & k > \frac{N}{2} \end{cases}$$

(2.36)
The wavelet function at each scale $s$ is normalized to have unit energy such that the wavelet transforms are directly comparable to each other and to transforms of other series.

\[ \int_{-\infty}^{\infty} |\tilde{\psi}(\omega')|^2 d\omega' = 1 \]  
\hspace{1cm} (2.37)

\[ \tilde{\psi}(s\omega_k) = \left( \frac{2\pi s}{\delta t} \right)^{1/2} \tilde{\psi}_0(s\omega_k) \]  
\hspace{1cm} (2.38)

Using these normalisations, one has at each scale $s$

\[ \sum_{k=0}^{N-1} |\tilde{\psi}(s\omega_k)|^2 = N, \]  
\hspace{1cm} (2.39)

where $N$ is the number of points.

The wavelet transform $W_n(s)$ is real ($R\{W_n(s)\}$) or complex ($R\{W_n(s)\} + iI\{W_n(s)\}$) depending on whether the wavelet function $\psi(\eta)$ is real or complex, respectively. The wavelet power spectrum can thus be defined as $|W_n(s)|^2$. The analysis in the present thesis is done by the latter method to determine the wavelet power spectra (Torrence and Compo, 1998).\(^1\)

**Wavelet Functions**

A few examples of wavelet functions are the Morlet wave, Paul and Mexican Hat. In the present study a Morlet wave has been used, which consists of a plane wave modulated by Gaussian (Fig. 2.6).

\[ \psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2} \]  
\hspace{1cm} (2.40)

where $\eta$ is a dimensionless time parameter and $\omega_0$ is the frequency, here taken to be 6 to satisfy the admissibility condition.

For nonorthogonal wavelet analysis, one can use an arbitrary set of scales to build up a more complete picture. It is convenient to write the scales as fractional powers of two and so the set of scales to compute the wavelet transform in Eq. 2.33 are chosen as follows:

\[ s_j = s_0 2^{j\delta j}, j = 0, 1, ..., J \]  
\hspace{1cm} (2.41)

\(^1\)Software is available at URL: http://paos.colorado.edu/research/wavelets/
where $s_0$ is the smallest resolvable scale and $J$ determines the largest scale. $s_0$ is chosen such that the equivalent Fourier period is approximately $2\delta t$. The choice of a sufficiently small $\delta j$ depends on the width in spectral space of the wavelet function. For the Morlet wavelet, a $\delta j$ of about 0.5 is the largest value that still gives adequate sampling in scale, while for the other wavelet functions, a larger value can be used. Smaller values of $\delta j$ give finer resolution.

**Cone of Influence**

Since the data set is finite, errors are introduced into the spectrum at the beginning and end of the spectrum. This happens because the Fourier transform assumes the data to be cyclic. When this condition is not satisfied the data set is padded with zeroes to make the data length $N$ equal to the next highest power of two to speed up the Fourier transform. Errors due to such padding are called edge errors and the **Cone of Influence** (COI) is the region where these errors are important. It is defined as the $e$-folding time for the autocorrelation of wavelet power at each
scale. For a Morlet wavelet, the e-folding time, $\tau_s$ is,

$$\tau_s = \sqrt{2s} \quad (2.43)$$

Note that it is independent of the frequency of the wavelet ($\omega_0$). The Mexican hat wavelet has a much narrower COI. $\tau_s$ is large at larger scales (smaller frequencies) and forms the vertex of the cone and at smaller scales (larger frequencies), $\tau_s$ is small and forms the base of the cone. The size of the COI at each scale also gives a measure of the decorrelation time for a single spike in the time series. By comparing the width of a peak in the wavelet power spectrum with this decorrelation time, one can distinguish between a spike in the data (possibly due to random noise) and a harmonic component at the equivalent Fourier frequency.

**Wavelet scale and Fourier frequency**

The relationship between the equivalent Fourier period and the wavelet scale can be derived analytically for a particular wavelet function by substituting a cosine wave of a known frequency into Eq. 2.34 and computing the scale $s$ at which the wavelet power spectrum reaches its maximum. The wavelet scale ($s$) and the Fourier period ($T$, inverse of the Fourier frequency) for a Morlet wave with $\omega_0 = 6$, are related as $T = 1.03s$. They are almost equal. But this need not be the case for all wavelet functions (Torrence and Compo, 1998).

**Time-averaged Power Spectrum**

Slicing of the wavelet plot gives the local power spectrum and a time-averaged wavelet spectrum over a certain period is

$$W^2_n(s) = \frac{1}{n_a} \sum_{n=n_1}^{n_2} |W_n(s)|^2 \quad (2.44)$$

where the new index $n$ is arbitrarily assigned to the midpoint of $n_1$ and $n_2$, and $n_a = n_2 - n_1 + 1$ is the number of points averaged over. The extreme case is when
the average is over all the local wavelet spectra, which gives the *global wavelet spectrum* (Eq. 2.45).

\[ W^2(s) = \frac{1}{N} \sum_{n=0}^{N-1} |W_n(s)|^2 \]  

(2.45)

The global wavelet spectrum provides an unbiased and consistent estimation of the true power spectrum of a time series (Torrence and Compo, 1998). Further the global wavelet spectrum can be used as a background against which the peaks in the local wavelet spectrum can be identified.

**Confidence levels**

An appropriate background spectrum is chosen to determine the significance levels of the wavelet power spectrum. For many geophysical phenomena, an appropriate background spectrum is either white noise (with a flat Fourier spectrum) or red noise (increasing power with decreasing frequency) against which the actual wavelet power spectrum is compared. If a peak in the wavelet power spectrum is above this background, then it can be assumed to be a true feature with a certain percentage of confidence. And to determine for e.g., the 95% confidence level (significant at 5%), the background spectrum is multiplied by the 95th percentile value for \( \chi^2_v \), where \( \chi^2_v \) is the chi-square distribution with \( v \) degrees of freedom (Torrence and Compo, 1998, and references therein).

### 2.3.3 Analysis of Langmuir Probe Data

The current collected by the Langmuir Probe sensor was processed on board three channels, viz., the Main, MF and HF channels. The current obtained in the LP main channel was converted to electron density using a conversion factor of 2.4 \( \times 10^4 \) cm\(^{-3}\). µ A\(^{-1}\), as explained earlier in Sec. 2.1.2. The factor was obtained by normalizing the probe current at the E-region peak during the July 2004 flight with the electron density obtained by the ionosonde located at Sriharikota at 1142 hrs LT, on 23 July 2004, i.e., at the time of the July 2004 rocket flight.
The Wavelet Spectra

The continuous wavelet transform using the Morlet wavelet (Ref. Section 2.3.2) was used to find the power spectra in all the three channels. The smallest scale, \( s_0 \), taken was \( 2\delta t \). The \( \delta t \) was different for the three channels and was 0.00192, 0.00096 and 0.000192 sec for the Main, MF and the HF channels, respectively. It is the inverse of their sampling frequency. The \( \delta j \) was taken to be 0.125, to give sufficient scale resolution. The confidence levels were not determined in this study as the interest was in the shape of the power spectrum but not in the peaks of the spectrum.

The complete profile of the electron density was taken to compute the spectra of the Main channel. Time-averaged power spectra corresponding to an altitude region of 100 m were computed using Eq: 2.45. These spectra will be referred to as the altitude-averaged power spectra hereafter. The altitude-averaged power spectra calculated from the MF and the HF channel data in the same altitude regions were then normalized to the main channel altitude-averaged power spectra to obtain the final composite altitude-averaged power spectra ranging from DC to 1000 Hz, for every 100 m. A range of 100 m was chosen to make sure that the breakpoints observed in the spectra, are indeed present in the data set. Secondly, averaging the spectra over 100 m assures that consecutive spectra contain independent information on the relevant spatial scales between 10 and 100 m. Finally, the other reason for this choice is to ensure numerical stability of the fitting algorithm (Strelnikov et al., 2003). The percentage amplitudes were also calculated from all the spectra using the Eq. 1.23.

Heisenberg Model and Turbulence Parameters

All altitude-averaged power spectra were examined to find if the Heisenberg theoretical spectral model (Sec. 1.2.2) could be fitted. If the observed power spectrum contained the two characteristic slopes of \(-5/3\) and \(-7\), of the ISR and the VDR, respectively, then the model was fitted by varying the parameter \( k_0 = 2/l_0 \). The best
Instrumentation, Data and Analysis

fit identifies the break in the slope and hence the inner scale, \( l_0 \). For those spectra where the inner scale was identified unambiguously, other turbulence parameters were estimated as discussed in Sec. 1.2.3. The temperature values required for the calculations were taken from MSISE-90 model (Hedin, 1991) and the kinematic viscosity and Brunt-Väisälä frequency were also calculated using the temperatures and densities from the same model.

2.3.4 Analysis of Photometer Data

The hourly averages of integrated emission of the oxygen green line emission over Kiso were computed for the period 1979–1990 and concatenated with the hourly averages of 1991–1994. Monthly averages at each hour of the night from 1800 hrs Japan Standard Time (JST; UT+09:00 hrs) to 0500 hrs JST were then calculated to form twelve time series of 16 years each. In addition, a nightly averaged series was also computed by taking averages of emissions at each hour from 2000 hrs to 0300 hrs JST. In view of very few data points around 1800, 1900, 0400 and 0500 hrs JST, these were not used in the present analysis.

The continuous wavelet transform (Ref. Section 2.3.2) was applied to the eight data series of monthly averages at each hour of the night from 2000 to 0300 hrs JST, as well as to the nightly average for each month. The software written in IDL, provided by Torrence and Compo (1998), was used for the present analysis also. The Morlet wavelet was used as the mother wavelet. Since the code requires equally spaced data, about 20 points were interpolated in each of the series from 2000 to 0300 hrs JST, each of which is a series of 192 points. The least square quadratic fit was used for the interpolation. A number of other interpolation techniques were also tried but the results were very similar. The 90% and 95% confidence levels
were also computed using a white noise fourier spectrum to determine the statistically significant oscillations, which are the semi-annual, annual and the quasi-biennial oscillations, and also their period of occurrence. The percentage amplitudes of these components were computed using the 12-point, 24-point and 56-point running averages of the data, respectively.