5.1 BASICS OF ANTENNA

Field region in antenna: Antenna fields region near far fields. These are produce electromagnetic or EM fields near and far from antenna. All electromagnetic fields could not developed radiations in the space. An antenna field region of simple dipole antenna is show in figure 5.1.

Figure 5.1: Antenna field region [57]

Antenna region: This is physical commutative antenna area defined as \( R \leq L/2 \) and called as antenna region.

Near field region: This is seen imaginary part of antenna resistance which not radiated and simply stored near antenna called as near field region.

Fresnel region: It is radiating field behaves the fraunhofer and near field region. Radiation pattern is exists in this region but changes with respect to a phase center distance at which radiation reduces by different rates.
Radiation intensity: Normalized power density is represents radiation intensity. Power density is vanished by moving from antenna and intensity proportional to squared of distance. In radiation intensity independent far-field pattern plot is made by cancelling the dependency of inversely proportional square distance. Normally radiation pattern in 3D indicates radiation intensity of antenna.

Basics antenna nomenclature: Antenna performance is understanding by knowing the under defined terms.

1) Antenna pattern: The plot which describes direction characteristics of antenna is called an antenna pattern. This pattern shows magnetic or electrical function. It is called a field pattern. Radiation intensity defines the antenna pattern and referred as power pattern describe by antenna measurements results not by functional and is referred as power or field pattern.

Figure 5.2 (a) and 5.2 (b) are two and three dimensional field pattern. First figure shows field plot in rectangular and polar co-ordinate.

![Field Pattern in a) Plot in rectangular b) Plot in polar co-ordinates](image)

Figure 5.2: Field Pattern in a) Plot in rectangular b) Plot in polar co-ordinates [57]

2) Antenna boresight: Maximum gain direction or physically aimed antenna direction is the antenna boresight. It is a antenna main-lobe central axis is also called as antenna bore sight Figure 5.2 show a boresight or main lobe central axis at Z-axis, $\theta = 0$
3) Principal plane patterns: Considering slices of power or field pattern in the two three dimension in different ways. One of the ways is to plot E and H-plane at linearly polarised radiation. In which E-plane indicates E vector of the field in maximum radiation direction. In other H-plane indicates H vector of the field in maximum radiation direction. $\theta$ constant plane in spherical coordinate is referred as azimuth and $\phi$ constant plane in spherical coordinate is referred as elevation plane. This principal plane pattern is shown in Figure 5.3. In this figure planes as azimuth and elevations are parallel to polarized fields of the system expressed in spherical coordinate.

![Figure 5.3: Principle Planes](image)

4) Beamwidth: In the radiation pattern three db points measured the beamwidth. Figure 5.4 is a two dimensional slice indicates beamwidth Figure shows two half power point at three db in a power pattern.
5) Directivity: It is a relational radiation of isotropic antenna with individual antenna measure how directive with same total power is called directivity. It is definite as the ratio of an isotropic antenna power density to radiating the same power by isotropic antenna. When directivity is only isotropic source radiates energy equally in all direction.

6) Beam solid angle: Solid angle $\Omega$ of a beam is the angle by which antenna power radiates with its radiation intensity equal to maximum radiation intensity. It is represented by steradians. One steradian is solid angle which is containing area on surface equal to square of radian.

7) Gain: Directionality of antenna is directivity. It directs the energy of an antenna in a specific direction. It is indicates that there is no antenna losses. Modification of directivity is a gain of antenna including its in inefficiencies. So that pattern generated by directivity and gain are equal with exception of scale factor in directivity.

8) Effective aperture: Reciprocity is principal of received power and radiation power in the same specific direction in this case received antenna can be create as an effective aperture which causes area of the present power density. This aperture is related to directivity of antenna. Transmitting power and effective power density $W$ is shown in Figure 5.5. In Figure incident power density is delivering to load by intercepting power density by receiving antenna. Now the received antenna expressed as effective aperture of area.
5.2 BASICS OF ARRAY ANTENNAS

Adaptive antenna is the set of more than one antenna works in such way product a specific radiation pattern. These address or antenna elements are working in synchronization by phasing used in antenna elements. This is achieved by hardware which is digital signal processing. Antenna arrays can use several geometric forms. These are planner, circular linear and comfort array.

Linear array: Linear array has maximum length of two element array. Two element arrays is the fundamental array used to explain the behaviour of larger array. This helps to study the phase relation of nearby array elements the electric field of two elements. The electric field of two elements is the fundamental array consists of two parts that is element factor is the one dipole for field equal and array factor is related to the array geometry pattern function. Pattern multiplication is the multiplication of array factor and antenna pattern. Means knowing an array factor, pattern multiplication is derived from antenna pattern. From the discussion it is clear that A.F. depends or electrical phase between each element, spacing between elements and geometric arrangement of the element.

The phase between the uniformly placed linear array elements is directly proportional to the wavelengths. This wavelength is the spacing distance between the elements that means entire behaviour of assortment is understand through the phase relation of each array constituent. The term used in the basic linear array, array factor is the sum of array vector or array steering vector.
Array weighting: Array factor plot shows number of side lobe. Out of these largest sidelobes has 24% of the peak value. Sidelobes in the array factor indicates that array radiates the energy in the interference or unwanted undesired direction. Along with this array received energy from undesired direction. In multipath communication by multiple angles each side lobe received the same signal as that of the main lobe or this is main principal of the fadding in wireless communication. By knowing transmission angle, which is possible to guide the ray in the route of desired user and suppress the sidelobes to minimize un-wanted signals. The supersession of sidelobes is possible by windowing, shading or weighting the array components. Array weighting has very important role in the digital signal processing. Sidelobes cancellers are choose the array weight in such a way that will provide deep nulls in the route of unrequired interference signal in contrast at the similar instance main lobe maximum is towards the desired direction.

The overall radiation pattern of adaptive antenna may be different from pattern of the other antenna which is a combination of individual elements pattern in array antenna. The antenna array can achieve the same performance, resolving mechanical problems by tuning the antenna and electrical problems by tracing combined output of several antennas. This performance is achieved by varying phase in the array antenna as well as amplitude in each outputs of element. In general antenna pattern of the array is focused in the direction of the user without mechanically tuning each element.

Orientation and position of each element gives radiation pattern which determine the overall radiation of the array. Amplitudes and phase of the relative feeding currents also determines the overall of the array radiation pattern. The array radiation pattern i.e. adaptive R(W, a, Øe ) is given by product of the radiation patterns of the entity constituents and array factor in the adaptive array r(W, a, Øe) and a(W,a, Øe) respectively is as given in an equation follows:

\[ R(W,a, Øe)=r(W, a, Øe) *a (W, a, Øe) \] (5.1)

Feeding currents and the phase factor in space related to the amplitude level of the array factor is determined by the relevant elements. The array factor is related to
the array amplitude level and the overall pattern scanned or adopted by updating current at each element [58].

Figure 5.6 shows the simplest geometry of linear array allied middles of antenna components in a straight line. The planar arrangement is one in which all elements of the midpoints are in the same plane. Examples of these are linear array, circular array and arbitrary array. For a uniformly spaced linear array, based on principal axis generates the current with a angle of arrival. A travel distance $s \cos \alpha$ between the elements of the signal from the signal at the second instance by the propagation phase factor $B = \frac{2\pi}{\lambda}$, a phase factor lags by a $B \cos \alpha$. In the case of a narrowband, distance travel by the signal without the effect of modulation on it.

Observing signal in a complex envelope which is transmitted in narrowband given by an equation written as follows:

$$T(t) = A(t) \exp \left[ -j[w_0 t + v(t)] \right]$$  \hspace{1cm} (5.2)

Where,

$$A(t): \text{The signal amplitude and}$$
\( w_0 \): The center frequency.

An isotropic array element, assuming first element as a reference, signals \( s(t) \) can be received and characterize at the condition array must be linear and distance linking them is uniform, the matrix form given by an equation written as follows:

\[
\begin{bmatrix}
1 \\
\exp[-jB_1scos(a)] \\
\exp[-jB_2scos(a)] \\
\vdots \\
\exp[-jB_(N-1)scos(a)]
\end{bmatrix}
T(t) = v(a) \ T(t)
\tag{5.3}
\]

Where,

\( N \): The quantity of components in array,

\( a \): The angle of arrival, and

\( v(a) \): The vector wheel.

For easily understanding, assumed the \( \Phi_e \) elevation angle equal to zero and dropped the frequency dependent \( v(w_0, a, \Phi_e) \) in narrowband case relative to the elements bore sight. To store multiple vectors \( w_0, (a, \Phi_e) \) is known as steering angles of array vectors. In the direction finding experiments to calibrate the array, the array manifold must be carefully evaluated.

Adaptive beamforming in narrowband multiplying each element output \( O(t) \) with a \( w_i^* \), complex weight which changes amplitude and phase related between them given by an equation as follows:

\[
O(t) = [w_1^*, w_2^*, w_5^*, \ldots, w_N^*]
\begin{bmatrix}
1 \\
\exp[-jB_1scos(a)] \\
\exp[-jB_2scos(a)] \\
\vdots \\
\exp[-jB_(N-1)scos(a)]
\end{bmatrix}
T(t) = w^Hs(t) \tag{5.4}
\]
Branch of the amplitude and phase of the array antenna pattern, the amplitude and phase factor is based on the weight vector, continuing the overall pattern by adjusting the weight vector arrays can be improved.

In general pattern of the array is given by

\[ R(w, a, \varnothing_e) = |w^H a(w, a, \varnothing_e)| \] \hspace{1cm} (5.5)

Linear uniform narrowband isotropic element array has the pattern given by an equation written as follows:

\[
R(a) = \begin{bmatrix}
w_1^* & w_2^* & w_3^* & \cdots & w_N^*
\end{bmatrix} \begin{bmatrix}
1 \\
\exp[-jB_1\cos(a)] \\
\exp[-jB_2\cos(a)] \\
\vdots \\
\exp[-jB_{(N-1)}\cos(a)]
\end{bmatrix} = r(a) \] \hspace{1cm} (5.6)

Reminder that given the isotropic array element has \( r(w_o) = 1 \) create the pattern which becomes the overall pattern. Based on the pattern reproduction principle and the pattern of each array element give the overall beam pattern given by an equation as follows:

\[ R(a, \varnothing_e) = a(a, \varnothing_e) r(a) \] \hspace{1cm} (5.7)

Each array element has the dependence on \( \varnothing_e \) into its antenna pattern so that it is included into overall antenna pattern. The \( \varnothing_e \) is absent in the array factor so does not have dependence due to the chosen linear array geometry.

5.3 BEAMFORMING AND SPATIAL FILTERING

Traditionally, time domain filtering criteria presented in adaptive algorithm updates tap weights to reduce error. In narrowband antenna array complex weight vectors are adapted. The array samples incoming waveforms contrasting to a temporal
dimension in space. Convenient array patterns and weight updating reduce error using spatial filtering. Users occupy same frequency in a band separated using adaptive beamforming and spatially separated at a time same time.

The input vector in discrete form can be expressed by an equation as follows:

$$S_M = [s_1, s_2, \ldots, s_M]^T$$  \hspace{1cm} (5.8)

![Figure 5.7: Beamformer for Narrowband [58]](image)

The weight vector is given by an equation written as follows:

$$W_M = [w_1, w_2, \ldots, w_M]^T$$  \hspace{1cm} (5.9)

Attenuators and digitized phase shifters in an analog or digital domain receiver using the weight vector can filter and digitized the signal.

The scalar output in narrowband array is the product of complex weight and linear inputs is given by an equation given as follows:

$$O = S^H W_M$$  \hspace{1cm} (5.10)
Where Hermitian transpose is denoted by \((\cdot)^H\), which is the combination of conjugation operations and transposition.

Tapped delay line replace each complex weight in wideband array provides spatial and frequency response adaptation. By changing frequency in signal pass band changes phase relationship from each element. Wideband arrays gives confront so signal must digitized at high rate to utilized signal complete bandwidth. Lastly each element is responsive to the signal to cross in enormously wideband array.

The error is calculating on basic knowledge of transmitted original narrowband signal given by an equation as follows:

\[
e = d - 0 = d - S^H w_M
\]  

(5.11)

Where desired signal is \(d\) at the receiver and which is the same the signal of communication. The mean square error \((\varepsilon)\) can be given by an equation as follows:

\[
\varepsilon = E[e^2] = E[d^2] + w^H C_{xx} w_k - 2C_{xx}^H w_k
\]  

(5.12)

Where,

\(C_{xx}\): Covariance or correlation matrix of the input and

\(C_{xx}^H\): Cross correlation matrix between received signal and desired.

An adaptive algorithm using MSE improves signal to interference and noise ratio, reduce errors which express the weight vector in quadratic form. One real optimal and minimum unique global solution exists given by the error surface.

Weiner - Hopf solution gives optimum solution which minimizes the MSE. Derivative of mean square error with respect to Weight vector gives this solution and settles to null. Gradient or a derivative in vector space can be given by an equation:
\[ \Delta = 2C_{xx}w - 2C_{xx}^H = 0 \]  \hspace{1cm} (5.13)

Rearranging,

\[ C_{xx}w = C_{xx}^H \]  \hspace{1cm} (5.14)

Assuming \( C_{xx} \) is non-singular, the optimum solution is found as

\[ w = C_{xx}^{-1}C_{xx}^H \]  \hspace{1cm} (5.15)

The process required sufficient enough time for input data to develop or estimate \( C_{xx}^H \) and \( C_{xx} \). The optimum value of a weight vector is achieved by calculating \( C_{xx}^{-1} \) and it is non-trivial to implement on DSP. Iterative techniques are used in the process for the optimum solution. Generally using iterative methods, equation is updated and expressed by an equation given as follows:

\[ w_1 = w_0 + \zeta (-\Delta) \]  \hspace{1cm} (5.16)

The optimum solution is find by stochastic gradient algorithms which is constantly increasing in the unconstructive route of the gradient. The weight vector is adjusted to shift in minimum of MSE surface on which optimum solution present. The parameter \( \zeta \) controls step size and several algorithms use differing gradients. One of popular method LMS algorithm is an immediate gradient to calculated accurate gradient.

A significant transaction exits in the selection of \( \mu \) for iterative methods. Algorithm to improve the convergence uses large step size can gives significant optimum best solution. To get the best solution for small step size, long time needs to get optimum solution using the same algorithm. To access the array, the correct choice of step size depends on the propagation environment. The lager \( \zeta \) value is required for high mobility. Additional steps size is required to for the stability. Step size in LMS algorithm that guarantee the best solution is in the range of
\[ 0 < \zeta < \frac{1}{\lambda_{max}} \]  
(5.17)

Where, \( \lambda_{max} \) is the greatest eigenvalue of the input covariance matrix \( C_{xx} \) [44].

5.4 ADAPTIVE BEAMFORMING ALGORITHMS

5.4.1 LEAST MEAN SQUARES (LMS)

Least mean square algorithm used for digital beamforming in adaptive array antenna. The functions of this algorithm is to sum all processed signals of each antenna elements and tunes particular signal in the direction of desired user. It is done by spatial filtering. The output response is given by an equation as follows:

\[ O = w^H S \]  
(5.18)

Where,

\( w^H \): Multifaceted weight vector and

\( S \): The received signal vector.

The beamforming algorithm in adaptive array gives multifaceted weight vector \( w \). Gradient approach in least mean square algorithm gives the error vector by an equation as follows:

\[ e = d - w^H S \]  
(5.19)

Where,

\( d \): The sequence of reference or training symbols
Mean square error is minimized using an error signal $\varepsilon$ updates the weight vector $W$. MSE minimize with the weights which radiates the reference signal in the preferred route and nulls in the intrusion track.

The steepest descent based LMS algorithm updates and calculated array weight vector. Best value of the weight vector update should be reasonable to minimize the MSE using the negative gradient vector method. In the normal LMS algorithm, the weight vector $w$ is arbitrary and is given by an equation as follows [20]:

$$w(m + 1) = w(m) + \zeta S e^*$$  \hspace{1cm} (5.20)

Where,

$w (m + 1)$: The weight vector to be calculated at process $m + 1$ and

$\zeta$: The LMS tread mass which is connected to the rate of convergence.

The adaptive tread mass is chosen such way, it gives the constancy and junction of the algorithm, and its range is specified as

$$0 \leq \zeta \leq \frac{1}{Y_{max}}$$  \hspace{1cm} (5.21)

Where, $Y_{max}$ is the utmost Eigenvalue of the enter correlation matrix.

5.4.2 NORMALIZED LEAST MEAN SQUARE (NLMS)

The algorithm avoids the need for the calculation of the eigenvalue of autocorrelation matrix and it uses data based step size for iteration. The equations described in the LMS algorithm remain the same and only the weight function changes in this algorithm [45][57][59].

The NLMS algorithm update weight equation for the analysis is given by an equation as follows:
\[ W_{(m+1)} = W_{(m)} + \zeta e_{(m)} \frac{s_{(m)}^H}{s_{(m)}^H x_{(m)}} \] (5.22)

Where,

\((H)\): The Hermitian transpose, used for multifaceted conjugate of the input signal \( S_{(m)} \).

The optimum solution is obtained in adaptive array antenna by convergence with \( N \) number of elements spaced distance \( d \) using step size \( \zeta \). This gives minimum mean square error.

5.5 DIRECTION FINDING

In 1979, Schmidt proposed a time-delay evaluation and conditional fewer signals to noise ratio of sinusoids used in MUSIC algorithm for multipath channels [54]. The direction of arrival and several incident signal parameters are evaluating by this algorithm. The input covariance matrix in this algorithm is obtained by eigen structure method [25].

MUSIC signal classification algorithm takes the form of the geometry of the problem. Input data vector with \( D \) signal incident and an \( N \) number of array elements is defined as a combination of linear signal along with noise is given by an equation as follows:

\[
\begin{bmatrix}
S_1 \\
S_2 \\
\vdots \\
S_N
\end{bmatrix} = \begin{bmatrix}
a(a_1) & a(a_2) & \cdots & a(a_D)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_D
\end{bmatrix} + \begin{bmatrix}
n_{i1} \\
n_{i2} \\
\vdots \\
n_{iD}
\end{bmatrix}
\] (5.23)

\[ s = Ax + n_i \] (5.24)

Where

\( x \): The matrix of incident signals,
$n_i$: The noise matrix and

$a(a_1)$: The direction-finding vector w. r. t. the DOA of the $j^{th}$ signal.

Observing received and guiding vector in space of $N$ dimension indicates $x$ is combination of guiding vector of linear arrays and that gives $x_1$ to $x_D$ coefficients. Model given above gives the input signal covariance matrix $C_{xx}$ by an equation as follows:

$$C_{xx} = E[ss^H] = AE[xx^H]A^H + E[n_in_i^H]$$  \hspace{1cm} (5.25)

$$C_{xx} = AC_{ss}A^H + \sigma^2I$$  \hspace{1cm} (5.26)

Where,

$C_{ss}$: signal correlation matrix.

A matrix has column full rank for linearly independent individual guiding vector. At partially correlated incident signal correlation $C_{ss}$ is non singular matrix.

$AC_{ss}A^H$ matrix with rank $D$ is positive semi-definite at the $D$ number of incident signal. By linear algebra $N$-$D$ gives zero eigenvalues. $N$-$D$ small eigenvalues are equal to $\sigma^2$ gives $E\lambda_1$, $E\lambda_2$, .... $E\lambda_M$ eigenvectors and $E\lambda_1$, $E\lambda_2$, ...., $E\lambda_M$ eigenvalues of matrix $AC_{ss}A^H$. Data sample of $C_{xx}$ is closely spaced cluster of eigenvalue decreasing its variance with increasing number of samples in $C_{xx}$. Smallest eigenvalue multiplicity is determine number of signal given by an equation as follows:

$$D = N - K$$  \hspace{1cm} (5.27)

Where,

$N$: is the quantity of aerial element

$K$: is the quantity of eigenvalues related with a noise subspace.
Depends upon the description of eigenvectors and eigenvalues, the eigenvectors related to a minimum eigenvalues proved by an equation as follows:

\[ C_{xx} E v_i = \sigma^2 E v_i \quad ; \quad i = D+1, \ldots, N. \quad (5.28) \]

Examining (5.26), an equation becomes as follows:

\[ A C_{ss} A^H E v_i = 0 \quad ; \quad i = D+1, \ldots, N \quad (5.29) \]

At the moment \( C_{ss} \) is non-singular and \( A \) is full column rank matrix, given by an equation as follows:

\[ A^H E v_i = 0 \quad ; \quad i = D+1, \ldots, N \quad (5.30) \]

These involve that the eigenvectors \( v_{D+1}, \ldots, v_N \) related to the noise are perpendicular to the column vectors of \( A \).

Covariance matrix eigenvectors are related to orthogonal subspaces. The direction of arrival of signal related to guiding vectors orthogonal to span signal and noise subspace then array manifold search eigenvector. Such that this DOA estimates MUSIC spectrum which is given by an equation as follows:

\[ P_{MUSIC}(a) = \frac{1}{a_H(a) P_N P_N^H a(a)} \quad (5.31) \]

Where,

\[ P_N = [E v_{D+1}, \ldots, E v_N] \quad (5.32) \]

Eigenvectors is a assortment of noise. Matrix is the projection on the subspace \( P_N P_N^H \) noise. Steering vectors orthogonal to the noise subspace gives very small value
at the denominator of equation (5.31). Because of this the peaks results in Music spectrum $P_{MUSIC}(a)$ with respect to the signal of angle of arrival (AOA). The peaks assume true AOA of the signal fall upon array and the perfect calibration of array is done. Therefore, the direction-finding vectors with a full array of calibration are so popular. MUSIC arbitrary algorithm can resolve closely spaced signals. The accuracy and precision at multiple independent storage array manifolds is limited to change during the calibration and the data can be measured by the array.