CHAPTER 6

ELASTIC-PLASTIC ANALYSIS OF HOMOGENEOUS THICK-WALLED CIRCULAR CYLINDER UNDER INTERNAL AND EXTERNAL PRESSURE WITH STEADY STATE TEMPERATURE

6.1 INTRODUCTION

Circular cylinders play an important role in machine design. The problem of uniformly long thick-walled circular cylinders arises in the design of turbine rotors. For an ideal plastic material without strain hardening, the stress distribution in solid rotating cylinders has been described by Nadai [77]. The addition of a central hole and the consideration of a rigid plastic material with linear strain-hardening studied by Davis and Connelly [18]. The elastic-plastic deformation of a solid cylinder in the presence of a distributed heat source, subjected to a lateral pressure and axial force is investigated using Tresca’s yield condition, its associated flow rule, and a linear work hardening law by Kammash [60]. The simplest and most general theoretical treatment of the partially plastic thick-walled cylinder using the Tresca’s yield criterion was given by Davidson [17]. In the report by Chen [16], a new theoretical model for high strength steel is proposed and a closed-form solution for determining the residual stresses in autofrettaged tubes has been obtained. A complete analytical procedure has been presented which encompasses representation of elastic-plastic uniaxial loading by Parker [89, 90]. Perry and Aboudi [94] used finite difference method for calculating stresses in thick-walled cylinders. Leu [66] studied the analytical solution for rotating hollow cylinders with nonlinear strain-hardening, visco-plastic materials. Analytical solutions of plastic limit angular velocities are compared with numerical results and it was found that the analytical results were in agreement with numerical results. Liew et. al. [67] presented an analysis of the thermo mechanical behavior of hollow circular cylinders made up of functionally graded material. The solutions are obtained by a limiting process that employs the solutions of homogeneous
hollow circular cylinders, with no recourse to the basic theory or the equations of non-homogeneous thermo elasticity. Nie et. al. [81] presented a technique for functionally graded linear elastic hollow cylinders and spheres to investigate stresses. The volume fractions of two phases of a functionally graded material are assumed to vary only with the radius and the effective material properties are estimated by using either the rule of mixtures or the Mori–Tanaka scheme. A unified numerical method was developed by You and Zhang [144] for the analysis of stresses in elastic–plastic rotating disks with arbitrary cross-sections of continuously variable thickness and arbitrarily variable density made of non-linear strain-hardening materials. The finite element model of the moderate rotation theory has been applied on composite plates and shells by Palmerio et. al. [86]. These authors assumed incompressibility of the material, yield conditions and power law relationship between stresses and strains for the analysis. In fact, in most of the cases, it is not possible to find a solution in closed form without these assumptions.

Transition theory [100, 111] does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It has been successfully applied to several problems, i.e. Sharma et.al. [118] evaluated plastic stresses in thick-walled circular cylinder made of transversely isotropic material while Sharma and Sahni [117] evaluated creep stresses in thick-walled rotating cylinder under internal pressure made of transversely isotropic materials. Elastic-plastic transition in a thin rotating disc with edge load has been studied by Gupta et. al. [42]. Elastic-plastic stresses in thin rotating disc with edge loading have been studied by Gupta et. al. [43]. It is concluded that a rotating disc with edge load requires high increase in angular speed to become fully plastic. Sharma [115] evaluated plastic stresses in thick-walled circular cylinder made up of non-homogeneous materials while Gupta and Sharma [37] investigated the elastic-plastic stresses in non-homogeneous circular cylinder under internal pressure with temperature.

In this chapter, elastic-plastic stresses for a isotropic thick-walled circular cylinder with internal and external pressure under steady state temperature have been obtained by using Seth’s transition theory [100, 111]. The stresses as safety factors in cylinders with internal and external pressure are in very high demand and are used in nuclear industry. Results obtained have been discussed numerically and depicted graphically also.
6.2 MATHEMATICAL FORMULATION

Consider a thick-walled circular cylinder of internal and external radii ‘a’ and ‘b’ respectively, subjected to internal and external pressure \( p_1 \) and \( p_2 \) respectively with temperature \( \theta_0 \) applied at the internal surface.

![Geometrical representation of the problem.](image)

The displacement components in cylindrical polar co-ordinates [8-11, 111] are given by

\[
u = r(1 - \beta), \quad \omega = 0, \quad \alpha = dz,
\]

where \( \beta \) is a function of \( r = \sqrt{x^2 + y^2} \) and \( d \) is a constant.

Using Almansi strain components from equation (1.36), the generalized components of strain [108] are

\[
e_{rr} = \frac{1}{n}[1 - (r\beta' + \beta)^n], \quad e_{\theta\theta} = \frac{1}{n}[1 - \beta^n], \quad e_{zz} = \frac{1}{n}[1 - (1 - d)^n], \quad e_{r\theta} = e_{\theta z} = e_{r z} = 0.
\]  (6.2)

The thermal stress-strain relationships for an isotropic material is given as,

\[
T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij} - \xi \theta \delta_{ij}, \quad (i, j = 1, 2, 3),
\]  (6.3)

where \( T_{ij}, e_{ij} \) are stress and strain tensors respectively, \( I_1 = e_{kk} \) are strain invariants, \( \lambda, \mu \) are Lame’s constant, \( \delta_{ij} \) is Kronecker’s delta, \( \xi = \alpha(3\lambda + 2\mu) \), \( \alpha \) being coefficient of thermal expansion and \( \theta \) is temperature.

Equation (6.3) can also be written as

\[
T_{rr} = \left(\frac{\lambda + 2\mu}{n}\right)(1 - (r\beta' + \beta)^n) + \frac{\lambda}{n}(1 - \beta^n) + \lambda k - \xi \theta,
\]
\[ T_{\theta\theta} = \left( \frac{\lambda}{n} \right) \left( 1 - (r \beta' + \beta) \right) + \left( \frac{\lambda + 2 \mu}{n} \right) \left( 1 - \beta \right) + \lambda k - \xi \theta, \]

\[ T_{\theta z} = \left( \frac{\lambda}{n} \right) \left( 1 - (r \beta' + \beta) \right) + \left( \frac{\lambda}{n} \right) \left( 1 - \beta \right) + (\lambda + 2 \mu) k - \xi \theta, \]

\[ T_{rz} = T_{\theta z} = T_{r \theta} = 0, \quad (6.4) \]

where \( k = \frac{1}{n} \left( 1 - (1 - d) \right). \)

Equations of equilibrium are all satisfied except,

\[ \frac{d}{dr} \left( T_{rr} \right) + \left( \frac{T_{rr} - T_{\theta \theta}}{r} \right) = 0. \quad (6.5) \]

The temperature field satisfying Laplace equation \( \nabla^2 \theta = 0 \), with boundary conditions \( \theta = \theta_0 \) at \( r = a \) and \( \theta = 0 \) at \( r = b \),

where \( \theta_0 \) is a constant, given by \( \theta = \frac{\theta_0}{\log(a/b)} \log(r/b) \) \( (6.6) \)

### 6.3 IDENTIFICATION OF THE TRANSITION STATE

We know that as the point in the material has yielded, the material at the neighbouring points is on their way to yield rather than they remain in their complete elastic state or fully plastic state. Thus we can assume that there exists some state in between elastic and plastic state which is called as transition state. So, at transition the differential system defining the elastic state should attain some criticality.

The differential equation which comes out to be non–linear at transition state is obtained by substituting equations (6.4) and (6.6) in equation (6.5), we get

\[ \beta P (1 + P)^{n-1} \frac{dP}{d\beta} + \left( P + \frac{C}{n} \right) (1 + P)^n + P (1 - C) - \frac{C}{n} + \frac{C \xi \bar{\theta}_0}{2 \mu \beta^n} = 0, \quad (6.7) \]

where \( r \beta' = \beta P \) and \( \bar{\theta}_0 = \frac{\theta_0}{\log(a/b)}. \)

The transitional or critical points of \( \beta \) in equation (6.7) are \( P \to -1 \) and \( P \to \pm \infty \).

The boundary conditions are,

\[ T_{rr} = -p_1 \quad \text{at} \quad r = a, \quad \text{and} \quad T_{rr} = -p_2 \quad \text{at} \quad r = b. \quad (6.8) \]
The resultant force normal to the plane \( z = \text{constant} \) is
\[
2\pi \int_{a}^{b} r T_{z} \, dr = p_{2} - p_{1} .
\] (6.9)

### 6.4 SOLUTION THROUGH PRINCIPAL STRESS

The asymptotic solution at the transition points gives the solution for the transition state of a particular configuration of the problem. The solution for the fully plastic state may be obtained when the Poisson’s ratio is made to approach half. The material from elastic state can go over into (i) plastic state, (ii) creep state or to any other state under external forces. All these final states are reached through a transition state. As there are only principal stresses so the transition can take place only through the principal stresses or through the principal stress difference. It has been shown [8-11, 32, 44, 111-112, 115, 118, 128, 137] that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point \( P \to \pm \infty \) and thus, we define the transition function \( R \) as,
\[
R = T_{\eta} - \frac{\lambda}{n} k + \alpha (3 - 2C) \theta
\]
\[
= \frac{2\mu}{nC} \left[ (2 - C) - \beta^{n} \left( (P + 1)^{n} + (1 - C) \right) \right] + \lambda k \left( 1 - \frac{1}{n} \right) - \alpha (3 - 2C) \left( \frac{\lambda}{1 - C} - 1 \right) \theta_{0} \log \left( \frac{r}{b} \right) .
\] (6.10)

Substituting the value of \( \frac{dP}{d\beta} \) from equation (6.7) and taking the asymptotic value \( P \to \pm \infty \) after taking the logarithmic differentiation of equation (6.10) w.r.t. \( r \) which on integrating yields
\[
R = Ar^{-c} .
\] (6.11)

where \( A \) is a constant of integration and \( C = \frac{2\mu}{\lambda + 2\mu} \).

Using equation (6.11) in equation (6.10), we get
\[
T_{\eta} = Ar^{-c} + B - \alpha (3 - 2C) \theta ,
\] (6.12)

where \( B = \frac{\lambda}{n} k \).

Using boundary conditions (6.8) in equation (6.12), we get
\[
A = \frac{\alpha \theta_{0} (3 - 2C) - (p_{1} - p_{2})}{a^{-c} - b^{-c}} , \quad B = -p_{2} - Ab^{-c} .
\] (6.13)

Substituting the value of \( A \) and \( B \) in equation (6.12), we get
\[ T_{rr} = \left\{ \frac{\alpha \theta (3-2C) - (p_1 - p_2)}{a^{-c} - b^{-c}} \right\} \left( r^{-c} - b^{-c} \right) - p_2 - \alpha \theta (3-2C) \]  \hspace{1cm} (6.14)

Using equation of equilibrium (6.5), we get
\[ T_{\theta\theta} = \left\{ \frac{\alpha \theta (3-2C) - (p_1 - p_2)}{a^{-c} - b^{-c}} \right\} \left\{ (1-C)r^{-c} - b^{-c} \right\} - \alpha \bar{\theta} (3-2C) \left( 1 + \log \left( \frac{r}{b} \right) \right) - p_2 \]  \hspace{1cm} (6.15)

The axial stress is obtained from equation (6.4) as
\[ T_{zz} = \left( \frac{1-C}{2-C} \right) (T_{rr} + T_{\theta\theta}) + 2\mu \left( \frac{3-2C}{2-C} \right) e_{zz} - 2\mu \left( \frac{3-2C}{2-C} \right) \alpha \theta , \]  \hspace{1cm} (6.16)

\[ e_{zz} = \frac{\left( p_2 - p_1 \right)}{2\pi} - \int_a^b \frac{r(1-C)}{(2-C)} [T_{rr} + T_{\theta\theta}] dr + \mu \alpha \theta \left( b^2 - a^2 \right) \left( \frac{3-2C}{2-C} \right) \]  \hspace{1cm} (6.17)

From equations (6.14) and (6.15), we get Tresca’s yield criterion as
\[ T_{\theta\theta} - T_{rr} = -ACr^{-c} - \alpha \bar{\theta} (3-2C) \]  \hspace{1cm} (6.18)

**Initial Yielding**

It is found that the value of \( |T_{\theta\theta} - T_{rr}| \) is maximum at \( r = a \), which means yielding of the cylinder will take place at the internal surface. Therefore, we have
\[ |T_{\theta\theta} - T_{rr}|_{r=a} = -AC a^{-c} - \alpha \bar{\theta}_0 (3-2C)| = Y \text{ (say)}, \]  \hspace{1cm} (6.19)

where \( Y \) is yield stress.

The necessary effective pressure required for initial yielding is given by
\[ |P_i| = \frac{p_1 - p_2}{Y} = \frac{1}{A_2} \pm \theta_i \frac{A_3}{A_2}, \]  \hspace{1cm} (6.20)

where \( A_2 = \frac{Ca^{-c}}{a^{-c} - b^{-c}} \), \( A_3 = \left\{ \frac{(3-2C)Ca^{-c}}{a^{-c} - b^{-c}} + \frac{3-2C}{\log \left( \frac{a}{b} \right)} \right\} \) and \( \theta_i = \frac{\alpha \theta_0}{Y} \).

In the fully plastic state \([8-11, 33, 42, 110-111, 115, 118, 121, 131]\) i.e. \( C \to 0 \), equation (6.18) become
\[ |T_{\theta \theta} - T_{rr}|_{r=b} = \left| \frac{(p_1 - p_2) - 3\alpha \theta_0 - 3\alpha \theta_0}{\log \left( \frac{b}{a} \right)} \right| = Y_1 \text{ (say)}, \]  
\[ (6.21) \]

where \( Y_1 \) is yield stress in fully plastic state.

The effective pressure required for fully plastic state is given from equation (6.21) as

\[ |P_f| = \left| \frac{p_1 - p_2}{Y_1} \right| = \left| \log \left( \frac{b}{a} \right) \right|. \]  
\[ (6.22) \]

The radial, circumferential and axial stresses for fully plastic state are obtained by taking \( C \to 0 \) in equation (6.14) – (6.16) as

\[ T_{rr} = \left( 3\alpha \theta_0 - p \right) \frac{\log \left( \frac{r}{b} \right)}{\log \left( \frac{b}{a} \right)} - p_2 - 3\alpha \theta, \quad (6.23) \]

\[ T_{\theta \theta} = \left( 3\alpha \theta_0 - p \right) \frac{\left( \log \left( \frac{r}{b} \right) - 1 \right)}{\log \left( \frac{b}{a} \right)} - p_2 - 3\alpha \theta - 3\alpha \theta_0, \quad (6.24) \]

\[ T_{zz} = \frac{1}{2} \left( T_{rr} + T_{\theta \theta} \right). \quad (6.25) \]

Now we introduce the following non-dimensional components as:

\[ R_0 = \frac{a}{b}, \quad R = \frac{r}{b}, \quad \sigma_r = \frac{T_{rr}}{Y}, \quad \sigma_\theta = \frac{T_{\theta \theta}}{Y}, \quad \sigma_z = \frac{T_{zz}}{Y}, \quad P_i = \frac{p_{1i} - p_{2i}}{Y} = p_{1i} - p_{2i} \]

\[ P_f = \frac{P_{1f} - P_{2f}}{Y_1} = P_{1f} - P_{2f}. \]

The initial yielding in non-dimensional form from equation (6.20) is given as

\[ |P_i| = \left| \frac{1}{A_5} \pm \theta_1 \frac{A_6}{A_5} \right|, \]  
\[ (6.26) \]

where \( A_5 = \frac{C}{1 - R_0^C} \) and \( A_6 = \left( \frac{(3 - 2C)C}{1 - R_0^C} + 3 - 2C \right) \log(R_0) \).

The transitional radial, circumferential and axial stresses in non-dimensional form from equation (6.14) – (6.16) becomes,
\[
\sigma_r = \frac{T_r}{Y} = \frac{\theta_1 (3 - 2C) - P_i (R^{-c} - 1) - P_{2i} - \frac{\theta_1 (3 - 2C) \log (gR)}{\log (R_0)}}, \tag{6.27}
\]
\[
\sigma_\theta = \frac{T_\theta}{Y} = \frac{\theta_1 (3 - 2C) - P_i (1 - C) R^{-c} - 1) - \frac{\theta_1 (3 - 2C)(1 + \log(R))}{\log(R_0)} - P_{2i}, \tag{6.28}
\]
\[
\sigma_z = \left(\frac{1 - C}{2 - C}\right)(\sigma_r + \sigma_\theta) + e^* \sigma_{zz} - \frac{\theta_1 \log R}{\log R_0}, \tag{6.29}
\]
\[
-\frac{P_i}{2\pi} - \int_{R_0}^{1} b R 2^{-\frac{1 - C}{2 - C}}(\sigma_\theta + \sigma_r) dR + \frac{\theta_1 (\log R / \log R_0)}{2} (1 - R_0^2)
\]

where \( e^* = \frac{1}{2} (1 - R_0^2) \).

The pressure required for fully plastic state \( C \to 0 \) in non-dimensional form from equation (6.22) is given as
\[
|P_f| = \left| \log \left( \frac{1}{R_0} \right) \right|, \tag{6.30}
\]

The radial, circumferential and axial stresses for fully plastic state in non-dimensional form from equation (6.23) – (6.25) are given as
\[
\sigma_r = \frac{T_r}{Y_1} = \left[ \frac{(-P_f) \log (R)}{\log (R_0)} \right] - P_{2f}, \tag{6.31}
\]
\[
\sigma_\theta = \frac{T_\theta}{Y_1} = \left[ \frac{(-P_f)(1 + \log(R))}{\log (R_0)} \right] - P_{2f}, \tag{6.32}
\]
\[
\sigma_z = \frac{T_z}{Y_1} = \frac{1}{2} (\sigma_r + \sigma_\theta). \tag{6.33}
\]

### 6.5 CYLINDER UNDER EXTERNAL PRESSURE ONLY

The pressure required for fully plastic state \( C \to 0 \) in non-dimensional from equation (6.30) is given as
\[
|P_f| = \left| \frac{P_2}{Y_1} \right| = \left| \log \left( \frac{1}{R_0} \right) \right|, \text{ where } P_f \text{ is external pressure only.} \tag{6.34}
\]

The radial, circumferential and axial stresses for fully plastic state in non-dimensional form from equation (6.31) – (6.33) are given as

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\[ \sigma_r = \frac{T_{rr}}{Y_1} = \frac{(P_I) \log (R)}{\log (R_0)}, \]  
\[ \sigma_\theta = \frac{T_{\theta\theta}}{Y_1} = \frac{(P_I)(1 + \log (R))}{\log (R_0)}, \]  
\[ \sigma_z = \frac{T_{zz}}{Y_1} = \frac{1}{2} (\sigma_r + \sigma_\theta). \]  

These equations are same as obtained by Sharma and Sahni [128].

### 6.6 NUMERICAL DISCUSSION AND CONCLUSION

As a numerical illustration, the values of pressure required for initial yielding and for fully plastic state at different temperatures are taken as: \( p = 0, 25, 50, 75, \theta_I = 0.1, 0.3, 0.5 \). The values of the compressibility factor is taken as \( C = 0.15, 0.35, 0.5 \).

From table 6.1, it is observed that percentage increase in effective pressure required for initial yielding to become fully plastic increases with the increase in temperature significantly. Also, it is clear from table 6.1 that percentage increase in effective pressure required for initial yielding to become fully plastic is high for circular cylinder with radii ratio 0.2 as compared to cylinder with other radii ratios.

It is observed from figure 6.2 that without temperature, effective pressure required for initial yielding maximum at internal surface. It is also noticed that pressure required for initial yielding is less for circular cylinder with \( C = 0.5 \) as compared to homogeneous circular cylinder \( C = 0.15 \). With the introduction of temperature, effective pressure required for initial yielding decreases significantly.

From figure 6.3, it is observed that cylinder whose internal pressure is fixed (say, 25) transitional stresses are maximum at internal surface. When internal pressure is more than that of external pressure, these stresses are compressible, otherwise tensile. It is clear from figure 6.4 that with the introduction of temperature, circumferential stresses increases significantly which further increases with the increase in temperature, as can be seen from figures 6.5 and 6.6. Also, it is observed that circular cylinder with \( C = 0.5 \) has less circumferential stress as compared to circular cylinder \( C = 0.15 \).
Transitional stresses increases significantly with the increase in external pressure (when external pressure is greater than that of internal pressure), otherwise transitional stresses decreases (figure 6.7). Also it is observed from figure 6.8 and 6.9 that circumferential stresses decrease with the increase in temperature. From figure 6.10, it is noticed that fully plastic stresses are maximum at internal surface. Theses stresses are tensile when internal pressure is more than that of external pressure otherwise compressive. When internal pressure increases (internal pressure is greater than that of external pressure) fully plastic stresses increase, otherwise decreases as can be seen from figure 6.11. With the introduction of temperature, fully plastic stresses increases significantly (figure 6.12). From figures 6.13, it is observed that with the increase in internal pressure (internal pressure is greater than that of external pressure) a circumferential stresses are maximum at internal surface otherwise these are maximum at external surface.

**Conclusion:** From the analysis, it is observed that circular cylinder with $C = 0.15$ and steady state temperature for radii ratio $(R_0 = 0.2)$ is on the safer side of the design as compared to other parameters of cylinder because pressure required for initial yielding is less for circular cylinder $C = 0.5$ as compared to homogeneous circular cylinder with $C = 0.15$. This means that introduction of temperature gradient reduces the possibility of fracture of cylinders under internal and external pressure.

**Table 6.1:** Pressure required for initial yielding and full plasticity for different values of temperature.

<table>
<thead>
<tr>
<th>Temp</th>
<th>Rₐ = 0.2</th>
<th>Rₐ = 0.3</th>
<th>Rₐ = 0.4</th>
<th>Rₐ = 0.5</th>
<th>R₀ = 0.2</th>
<th>R₀ = 0.3</th>
<th>R₀ = 0.4</th>
<th>R₀ = 0.5</th>
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<td>0.904555</td>
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<td>19.7756</td>
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<td>Pₐ  1.60944</td>
<td>1.20397</td>
<td>0.916291</td>
<td>0.693147</td>
<td>31.307163</td>
<td>24.86898</td>
<td>19.7756</td>
<td>15.48892</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>Pₐ  1.60944</td>
<td>1.20397</td>
<td>0.916291</td>
<td>0.693147</td>
<td>35.197336</td>
<td>29.00014</td>
<td>24.09202</td>
<td>19.95796</td>
<td></td>
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<td>32.72497</td>
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<td>45.52497</td>
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Figure 6.2: Effective pressure required for initial yielding (at different temperature) of a homogeneous cylinder.
Figure 6.3: Transitional stresses for a cylinder under different external pressures 0, 50, 75 respectively when internal pressure is 25.

Figure 6.4: Transitional stresses for a cylinder under different external pressures 0, 50, 75 respectively when internal pressure is 25 and temperature $\theta_i = 0.1$. 
**Figure 6.5:** Transitional stresses for a cylinder under different external pressures 0, 50, 75 respectively when internal pressure is 25 and temperature $\theta_i = 0.3$.

**Figure 6.6:** Transitional stresses for a cylinder under different external pressures 0, 50, 75 respectively when internal pressure is 25 and temperature $\theta_i = 0.5$. 
Figure 6.7: Transitional stresses for a cylinder under different external pressures 0, 25, 75 respectively when internal pressure is 50.

Figure 6.8: Transitional stresses for a cylinder under different external pressures 0, 25, 75 respectively when internal pressure is 50 and temperature $\theta_i = 0.1$. 
Figure 6.9: Transitional stresses for a cylinder under different external pressures 0, 25, 75 respectively when internal pressure is 50 and temperature $\theta_i = 0.3$.

Figure 6.10: Transitional stresses for a cylinder under different external pressures 0, 25, 75 respectively when internal pressure is 50 and temperature $\theta_i = 0.3$. 
Figure 6.11: Fully plastic stresses for a cylinder under different external pressures 0, 50, 75 respectively when internal pressure is 25.

Figure 6.12: Fully plastic stresses for a cylinder under different external pressures 0, 25, 75 respectively when internal pressure is 50.