The parametric excitation processes in plasmas have been receiving growing attention in recent years due to their numerous practical applications, e.g., plasma heating, gas discharge physics, plasma shocks and semiconductor physics. Furthermore, it is also helpful in understanding the theory of nonsteady processes. This chapter is divided into two parts: Part A contains parametric excitation of electron-acoustic waves by helicon waves in magnetised semiconductor-plasmas wherein a helicon wave is used as a pump wave. Part B contains parametric excitation of electron-acoustic waves in magnetised semiconductor-plasmas wherein the plane polarised extraordinary (X-wave) wave is used as a pump wave.
PART A

PARAMETRIC EXCITATION OF ELECTRON-AcouSTIC WAVES
BY HELICON WAVES IN MAGNETISED SEMICONDUCTOR-PLASMAS

2.1 Introduction

In recent years there has been increasing interest in the study of the nonlinear behaviour of helicon waves in solid-state plasmas. Aigrain 1960 has reported that the existence of propagating low-frequency electromagnetic waves in high conductivity media is due to the inhibiting effect of the magnetic field on the motion of the charges and since then numerous studies of waves in solids have been made. He gave the name 'helicon' referring to the helical configuration of the electric field. When one sign of charge carrier plays the dominant role in conduction processes, we have what is known as an uncompensated plasma. The helicon wave is one of a number of low-frequency electromagnetic excitations that is possible in an uncompensated plasma in a magnetic field. It has a frequency smaller than the plasma as well as the cyclotron frequency of electrons and could be excited to high amplitudes as high power sources in this frequency
regime, viz., radio and microwave frequencies are available in abundance. At high amplitudes, helicons should give rise to nonlinear effects. The use of helicons is well-established as the diagnostics of lightly and heavily doped semiconductors and metals for a long time (Buchsheilm and Platzman 1967).

The parametric excitation of helicon waves in a magnetooactive electron plasma in the presence of a weakly modulated high frequency electric field has been investigated by Poverman and Tskhakaya 1975. Guha et al. 1979 have studied the amplification of an acoustic wave and a helicon wave in a piezoelectric semiconductor-plasma in the presence of a strong oscillatory electric field and have obtained the threshold value of the electric field required for the onset of instability and the growth rate. The explosive instability of a helicon wave when it interacts with a transverse and longitudinal acoustic wave in a magnetised solid-state plasma has been studied by Fugakov 1976. It was observed that the wave energy of the helicon is transformed into a transverse acoustic wave when the pump helicon mode is a fast one. Guha and Namjoshi 1979 have shown the possibility of the parametric decay of a high power helicon into another helicon and an
acoustic mode in a piezoelectric semiconductor using coupled mode theory where the acoustic mode is assumed to propagate across the applied magnetic field.

A right-handed circularly polarised helicon wave decays resonantly into a low-frequency acoustic wave and a scattered helicon wave in a piezoelectric semiconductor where the low-frequency nonlinearity arises through the parallel ponderomotive force on electrons and the high-frequency nonlinearity arises through the equation of continuity (Sodha and Sharma 1979), Guha and Namjoshi 1980 have investigated the parametric excitation of a helicon and an acoustic wave in a piezoelectric semiconductor-plasma in the presence of a strong magnetic field using the coupled mode theory. They have observed that an acoustic wave of higher frequency and higher phase velocity than that of the pump wave cannot be excited. Experimental observation of nonlinear effects arising due to the propagation of high power helicon waves in a semiconductor has been reported by Laurinavichyus and Pozhela 1974 where microwave signal is a circularly polarized corresponding to a helicon wave in the semiconductor and a magnetic field is applied along the direction of propagation of the wave in the waveguide, Guha and Ghosh 1977.
have discussed analytically the propagation of high power helicon wave in n-InSb crystal at liquid nitrogen temperature (77°K) taking into account the heating of the carriers due to the amplitude of the wave. Salimullah and Ferdous 1984 have studied the modulational instability of a beam of high amplitude helicon wave in a magnetoactive piezoelectric semiconductor-plasma where the nonlinear response of electrons has been found by the fluid model of homogeneous plasma and the low-frequency nonlinearity has been taken through the nonlinear current density of electrons. For typical plasma parameters in n-InSb crystal and for a considerable power density of the incident helicon beam, the growth rate of the modulational instability is quite high (\( \sim 10^7 \text{ rad s}^{-1} \)).

In this chapter we have analytically studied the parametric excitation of an electron-acoustic wave by a helicon wave in a magnetised semiconductor-plasma where helicons produce nonlinear effects at large amplitudes. So far no attempt has been made to study such effects. In Section 2.2, we have derived the nonlinear dispersion relation for the low-frequency mode and the growth rate of the excited mode in the presence of a high power helicon pump in an n-type semiconductor. Here,
the low-frequency nonlinearity arises through the parallel ponderomotive force on electrons whereas the high-frequency nonlinearity comes through the nonlinear current density. The fluid equations are used to obtain the nonlinear response of electrons. In Section 2.3, we have given discussion of the results.

2.2 Theoretical Formulation

We consider the fluid model of an n-type homogeneous magnetised semiconductor-plasma which is subjected to an externally driven right-handed circularly polarised helicon wave propagating along the direction of the static magnetic field $\vec{B}_s$.

The basic equations used are as follows:

Equation of motion

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \vec{E} - \left[ \frac{e}{mc} (\vec{v} \times \vec{B}) + (\vec{v} \cdot \vec{B}) \right] - \frac{\vec{v}_{th}}{n_0} \vec{v}_n,$$

(2.1)

where $v_{th} \approx (k_B T_e/m)^{1/2}$ is the thermal speed of the electrons, $k_B$ is the Boltzmann constant and $T_e$ is the
temperature of the electrons, the quantity in the square bracket is the ponderomotive force (Sodha et al. 1974) on the electrons.

Continuity equation

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0
\]  

(2.2)

Poisson's equation

\[
\nabla \cdot \mathbf{E} = -4\pi n e
\]  

(2.3)

Maxwell's equations

\[
\nabla \times \mathbf{E} = \left( \frac{i\omega}{c} \right) \mathbf{B} \quad \text{and} \quad \nabla \times \mathbf{B} = \left( \frac{4\pi}{c} \right) \mathbf{J} - \left( \frac{i\omega}{c} \right) \mathbf{E}
\]  

(2.4)

where \( n \) is the total density of electrons and \( \varepsilon \) is the permittivity of the medium, \( \mathbf{\vec{E}}, \mathbf{\vec{v}} \) and \( \mathbf{\vec{B}} \) are the total electric field, velocity and total magnetic field respectively and \( \mathbf{\vec{J}} \) is the current density.

We consider the dependence of all the quantities as \( \exp \left[ i(kx - \omega t) \right] \) and choose the low-frequency mode \( (\omega, \mathbf{k}) \) to be purely electrostatic and express the various quantities as follows:
\[ \dot{E} = E_0 (\omega_0, k_0) + E_e (\omega, k) , \]
\[ \dot{B} = B_0 (\omega_0, k_0) + B_s, \]
\[ \dot{v} = v_0 (\omega_0, k_0) + v_e (\omega, k) \]
\[ n = n_0 + n_e (\omega, k) \]

(2.5)

where \( E_0, v_0, n_0 \) and \( B_0 \) are the zeroth order electric field, velocity, number density of the electrons and magnetic field of the pump respectively. \( \omega_0 \) and \( k_0 \) are the pump frequency and the wave number, respectively. \( E_e, v_e \) and \( n_e \) are the electric field of the electron-acoustic wave, the perturbed velocity of the electrons and the number density of the electrons, respectively and \( B_s \) is the uniform static magnetic field.

Equation (2.1) can be rewritten as

\[ \frac{\delta \dot{v}_e}{\delta t} = - \frac{1}{m} ( e \dot{E}_e + F_p ) - \frac{e}{mc} ( \dot{v}_e \times \dot{B}_s ) - \gamma_0 \gamma_v \dot{v}_e - \frac{\gamma_{th} \gamma_v}{n_0} n_e \]  

(2.6)

where \( F_p \) is the ponderomotive force on the electrons and it can be expressed as

\[ F_p = [ \frac{e}{c} ( \dot{v}_e \times \dot{B}_0 ) + m ( \dot{v}_e \cdot \nabla ) \dot{v}_e ] \]
From equations (2.1) and (2.5), we get in the 
Z - direction as

\[ v_{ez} = \frac{-e e}{m(\nu_0^{-1} - i\omega)} + \frac{ie k \varphi_p}{m(\nu_0^{-1} - i\omega)} - \frac{i(v_{th}^2) k n_e}{n_o(\nu_0^{-1} - i\omega)} \]  

(2.7)

and from equation (2.2),

\[ v_{ez} = \frac{n_e (\omega - v_{oz} k)}{n_o k} . \]  

(2.8)

The ponderomotive force \( \vec{F}_p = -e\vec{\nabla}\varphi_p \), \( \varphi_p \) is the 
ponderomotive potential for the electrons due to the incident wave which becomes for this case \( \approx (\varphi/2\omega)k v_{ox}^* \) 
(Tripathi and Liu 1982). The asterisk represents the 
complex conjugate of the quantity.

On equating the equations (2.7) and (2.8), we 
obtain \( n_e \) as

\[ n_e = \frac{k^2 x}{4 \pi e} (\varphi + \varphi_p) , \]  

(2.9)

where

\[ x = \frac{\omega_p^2}{k^2 v_{th}^2 - (1\nu_0^{-1} + \omega)(\omega - v_{oz} k)} . \]  

(2.1c)
The high-frequency nonlinear current density \( \vec{J}(\omega, \vec{k}) \) is given by

\[
\vec{J}(\omega, \vec{k}) = -n_0 \vec{v}_e - \frac{1}{2} n_0 e \vec{v}_o
\]

\[
= \frac{e}{m(\omega_0^2 - \omega^2)} \left[ e \phi \vec{v}_e - i e n_0 \vec{k} \phi_p \right]
\]

\[
+ i n_e m k v_{th}^2 - \frac{k^2 X}{\pi} (\phi + \phi_p) \vec{v}_o ,
\]

where \( \vec{v}_o \) is the unperturbed velocity of the electrons at \((\omega_0, \vec{k}_0)\) response (from equation 2.1) as

\[
\vec{v}_{ox} = \frac{-i e \omega_0 E_{ox}}{m(\omega_0^2 - \omega_{ce}^2)} ,
\]

\[
\vec{v}_{oy} = \frac{-e \omega_{ce} E_{ox}}{m(\omega_0^2 - \omega_{ce}^2)} ,
\]

and

\[
\vec{v}_{oz} = \frac{k v_{th}^2}{(i \chi \omega_0 + \omega)} .
\]

From equations (2.3), (2.4), (2.9) and (2.11), we obtain

\[
\vec{E} \cdot \phi = -X \phi_p ,
\]

and

\[
D \cdot \vec{E}_e = \beta \vec{v}_{ox}^* .
\]
where
\[ \beta = \frac{i \omega k^2 \chi^2}{2c^2} \phi, \] (2.15a)

\[ \epsilon \frac{\mu}{D} = (k^2 - \frac{\omega^2 \epsilon}{c^2}) \phi \] (2.15b)

and \( \phi \) is the unit dyadic. In equation (2.13), \( \epsilon \) is the linear dielectric function given by (Steele and Vural 1969) as

\[ \epsilon = \epsilon_L + \frac{\omega^2}{\omega^2 - \omega_0^2 - i \omega k^2 v^2_{th}/\omega} - \frac{2k^2 v^2_s}{\omega^2 - k^2 v^2_s} \] (2.16)

where \( \epsilon_L \) is the lattice dielectric constant. The last term of equation (2.16) is the piezoelectric contribution due to the lattice in which \( v_s \) is the speed of sound and \( k_c \) is the dimensionless electromechanical coupling coefficient. The numerical value of \( k_c^2 \) for most of the piezoelectric semiconductors is \( \approx 10^{-3} \) (Seeger 1973).

On eliminating \( \phi \) and \( \epsilon \) from equations (2.13) and (2.14), the nonlinear dispersion relation for the low-frequency electron-acoustic wave is obtained as

\[ \phi = \mu/D, \] (2.17)

where
\[ \mu = \frac{-k^2}{4c^2} \chi^2 v^2_{ox}. \] (2.18)
To simplify the expression for $X$, we assume the approximations $\nu_0 > \omega$, $\nu_0 \omega > k^2 v^2_{th}$ and $\omega > v_{oz} k$ so that
\[ X = \frac{1}{\nu_0 \omega} . \] (2.19)

Using equations (2.12), (2.18) and (2.19), we get
\[ \mu = \frac{e^2 k^2 \omega^2 \omega_p^2}{4c \nu_0^2 \omega^2_m} \left( \frac{e^2}{\nu_0 \omega p} \right) \left( \frac{\omega_o^2}{\omega_{ce}} \right)^{-1} \] (2.20)

Following Sharma and Tripathi 1979, the growth rate of the low-frequency electron-acoustic wave is obtained from the expression
\[ \gamma_0^2 = \mu \left\{ \frac{\delta \varepsilon}{\delta \omega_r} \frac{\delta \varphi}{\delta \omega_r} \right\} , \] (2.21)
where $\gamma_0$ is the growth rate in the absence of linear damping given by
\[ \gamma_0 = E_{ox} \left[ \frac{e^2 k^2 \omega^2 \omega_p^4}{8 \nu_0^2 m_e \omega^4} \left( \frac{\omega_0^2}{\omega_{ce}} \right)^{-1} \right]^{1/2} . \] (2.22)

It is seen from equation (2.22) that in the magnetoactive plasma, the growth rate of the excited mode well above
threshold is proportional to the pump amplitude; that is, in the absence of $E_0$, the growth rate disappears.

2.3 Results and Discussions

In order to calculate the growth rate of the parametrically excited electron-acoustic wave, we take a typical case of an n-InSb crystal at 77$^\circ$K with the physical constants as $m = 0.014 m_0$ ($m_0$ is the mass of a free electron), $\omega_L = 18$, $\gamma_0 = 3.5 \times 10^{11}$ sec$^{-1}$, $\omega_p = 2 \times 10^{13}$ sec$^{-1}$, $\omega_0 = 1.78 \times 10^{14}$ sec$^{-1}$ (corresponding to CO$_2$ laser), and $k = 5 \times 10^6$ Cm$^{-1}$. The crystal is irradiated with a pulsed 10.6 $\mu$m CO$_2$ laser beam. These values satisfy the condition for helicon propagation. The results are plotted. Fig. 2.1(A) shows the nature of the variation of ratio of the growth rate ($\gamma$) of the excited mode well above the threshold electric field ($E_0$)$_{th}$ to $E_{ox}$ with the magnetic field $B_0$ [equation (2.22)]. It is evident from the figure that the ratio increases with the increasing value of the magnetic field strength. The growth rate is affected directly by the collision frequency, that is, the role of the collision frequency is found to decrease the growth rate of the excited wave, as higher damping has to overcome in order to obtain
growth of the excited waves. Higher carrier concentrations in the semiconductor enhance the growth rate of the excited wave. The best results can be achieved by using moderately low values of the pump amplitude and by choosing large carrier concentration in the medium. The results of this investigation suggest that semiconductors could be effectively used to obtain a clearer understanding of the phenomenon of parametric excitation of an electron-acoustic wave by a large amplitude right-handed circularly polarised helicon wave in the presence of a dc magnetic field. These waves might be important to open a potential tool for semiconductor diagnostics at high-doping levels. Helicon propagation in solids is quite well understood within the framework of existing theories. The helicon wave should continue to be a useful probe in determining some of the electronic properties of metals.
Fig. 2.1(A): Variation of the growth rate ($\gamma$) of the excited mode well above threshold with the magnetic field strength ($B_0$).
References

2.1 Introduction

The excitation of ion-acoustic waves in hot collisionless isotropic plasmas by two Gaussian laser beams has been investigated by Sodha et al. 1979 wherein on account of the non-uniform intensity distribution along the wavefront of the beams, the ponderomotive force becomes finite. This leads to modification of the plasma density profile in the plane transverse to the beam axis and hence also to modification in the propagation characteristics of the waves propagating in the plasma. It is shown that self-focusing of the waves can occur and this considerably influences the beat wave power flux with the available high-power laser and microwave sources. Salimullah 1981 has analysed the parametric excitation of the ion-acoustic wave at the difference frequencies of two microwave beams in a semiconductor-plasma, viz., n-type InSb crystal where the ponderomotive force on electrons drives the
ion-acoustic wave at the difference frequency. He has found that the power density of the excited ion-acoustic wave is 1.76 kw Cm\(^{-2}\) in n-type InSb crystal by taking the microwave beams of power density 1 Mw Cm\(^{-2}\).

Tripathi and Liu 1982 have studied the parametric instabilities of electron-cyclotron waves at the second harmonic cyclotron layer in the EBT (Elmo bumpy torus) and at the cyclotron harmonic in the large tokamaks, e.g., PLT.

From the literature it seems that no such study as the excitation of the electron-acoustic wave in a magnetised semiconductor-plasma has been done. In this chapter, we have studied the parametric excitation of the electron-acoustic wave in a magnetised semiconductor-plasma, viz., n-type InSb. The pump wave is an extraordinary electromagnetic wave which excites the electron-acoustic wave.

When the extraordinary mode is launched as a pump wave on a homogeneous semiconductor, it excites an electron-acoustic wave, a lower-hybrid wave and an electron-cyclotron wave. However, among these the excitation of the electron-acoustic wave only is possible in
magnetized semiconductor-plasma wherein the electron-acoustic wave propagates almost transverse to the magnetic field with frequency $w_{ce} > \omega$. Other parametric effects can be ignored because they are possible in the low-density plasma.

We have used the fluid model of a plasma and have obtained the nonlinear dispersion relation of the excited electron-acoustic wave in Section 2.2. In Section 2.3, the dispersion relation has been solved to obtain the threshold electric field and the growth rate of the excited wave. The calculations of the threshold value of the amplitude of the electric field of the pump and the growth rate of the excited wave have been made for the case of n-type InSb crystal in Section 2.4.

2.2 Dispersion Relation for Electron Acoustic Wave

We use the hydrodynamic model of a homogeneous infinite semiconductor-plasma in which a linearly polarised extraordinary pump wave of the form

$$E_0 \exp \left[ -i (\omega t - k_0 x) \right]$$
propagates along the x-axis in the presence of a dc magnetic field $B_0$ applied in the z-direction. The linear response of the electrons to the pump wave is given by

\[
E_{ox} = -i \frac{\omega_p^2}{\omega_e} \frac{\omega_e E_{ov}}{u}, \quad (2.1a)
\]

\[
v_{ox} = \frac{e u E_{ox}}{i m \omega_p^2 \left( \omega_e^2 + (\gamma_0 - i\omega_0)^2 \right)} \frac{1}{u} \left\{ \frac{i(\omega_0 - \gamma_0) \omega_p^2}{u} + 1 \right\} \quad (2.1b)
\]

\[
u = \omega_0 \left[ \omega_e^2 + (\gamma_0 - i\omega_0)^2 \right] - i\omega_p^2 (\omega_0 - \gamma_0) \quad (2.1c)
\]

where $\omega_p$, $\omega_e$ and $\gamma_0$ are the electron-plasma frequency, electron-cyclotron frequency and electron-phonon collision frequency respectively, $\nu_0$ is the zeroth order velocity of an electron, $-e$ and $m$ are the electronic charge and mass respectively, $\omega$, $k$ and $E_0$ are the frequency, the wave number and the electric field of the pump respectively.

We use the following basic equations:
Equation of motion

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \vec{E} - \left[ \frac{e}{mc} (\vec{\nabla} \times \vec{B}) + (\vec{\nabla} \cdot \vec{v}) \vec{v} \right] - V_0 \vec{v} - \left( \frac{v_{th}^2}{n_0} \right) \vec{v}$$

(2.2)

where $v_{th} \approx (k_B T_e/m)^{1/2}$ is the thermal speed of the electrons, $k_B$ the Boltzmann's constant and $T_e$ the temperature of the electrons. $\vec{v}$ and $\vec{E}$ are the drift velocity and total electric field of the electrons respectively. $n_0$ and $\vec{B}$ are the zeroth order number density and the total magnetic field. The quantity in the square bracket in equation (2.2) is the ponderomotive force due to the incident pump wave (Sodha et al. 1974).

Continuity equation

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n \vec{v}) = 0$$

(2.3)

where $n$ is the total carrier concentration of the electrons.

Maxwell's equations

$$\vec{V} \times \vec{E} = \left( \frac{i\omega}{c} \right) \vec{B} \quad \text{and} \quad \vec{V} \times \vec{B} = \left( \frac{4\pi}{c} \right) \vec{J} - \left( \frac{i\omega}{c} \right) \vec{E},$$

(2.4)

where $\varepsilon$ is the permittivity of the medium, $\vec{J}$ is the current density.
We choose the low-frequency mode \((\omega, k)\) to be purely electrostatic and express the various quantities as follows:

\[
\begin{align*}
\vec{E} &= \vec{E}_0 (\omega_0, \vec{k}_0) + \vec{E}_e (\omega, \vec{k}), \\
\vec{B} &= \vec{B}_0 + \vec{B}_s, \\
\vec{v} &= \vec{v}_0 (\omega_0, \vec{k}_0) + \vec{v}_e (\omega, \vec{k}), \\
n &= n_0 + n_e (\omega, \vec{k}),
\end{align*}
\]

(2.5)

where \(\vec{E}_e\), \(\vec{v}_e\) and \(n_e\) are the electric field of the electron-acoustic wave, the perturbed velocity of the electrons and the number density of the electrons, respectively and \(\vec{B}_s\) is the static uniform magnetic field. \(\vec{B}_0\) is the pump magnetic field.

From equations (2.3) and (2.5) we get

\[
\vec{v}_{ez} = \frac{n_e (\omega - \nu_{ox} k)}{n_0 k},
\]

(2.6)

From equations (2.2) and (2.5) we get
On equating the equations (2.6) and (2.7), one obtains $n_e$ as

$$n_e = \frac{k^2 \beta}{4\pi e} (\phi + \phi_p) \quad (2.8)$$

where

$$\beta = -\frac{\omega_p^2}{\omega_e} \left\{ \frac{\omega_e}{(i\omega_0 + \omega_0)} - k^2 \frac{\omega_{ce}}{\omega_{th}} \right\} (\omega - \omega^*_{th}) = \frac{\omega_{ce}}{\omega_{th}} (\omega - \omega^*_{th}). \quad (2.9)$$

From Maxwell's equations (2.4) and Poisson's equation ($\nabla \cdot E = -4\pi n\epsilon$), one obtains

$$\left( \frac{\omega_p^2}{c^2} \right) k \phi = \left( \frac{4\pi \omega}{c^2} \right) J_x = \left( \frac{4\pi \omega}{c^2} \right) J_x. \quad (2.10)$$

The $x$-component of the high-frequency nonlinear current density $J_x$ is given by

$$J_x = -n_e v_{ex}^*. \quad (2.11)$$

Now substituting the values of $v_{ex}$ and $n_e$ (equations 2.6 and 2.8), we get
Using equations (2.10) and (2.11), the nonlinear dispersion relation for the low-frequency electron-acoustic wave is

\[ J_x = \left( \frac{-k_B}{4\pi} (\omega - v_{ox}^\ast k) \right) (\phi + \phi_p) . \] (2.11)

Using equations (2.10) and (2.11), the nonlinear dispersion relation for the low-frequency electron-acoustic wave is

\[ \mu = \frac{\varepsilon_c}{\varepsilon_p} = \left( 1 - \frac{v_{ox}^\ast k}{\omega} \right) \left( 1 + \frac{v_{ox}^\ast k}{2\omega} \right) . \] (2.12)

When the ponderomotive potential is neglected, the dispersion relation reduces to that of the electron-acoustic wave.

2.3 Growth Rate and Threshold

To study the instability of the electron-acoustic wave, the dispersion relation (2.12) is reduced to the form as

\[ \omega^2 + \frac{i\gamma_0 \omega^2}{\varepsilon (\gamma_0^2 + \omega_{ce}^2)} \omega + \frac{i\gamma_0 \omega^2 v_{ox} k}{\varepsilon (\gamma_0^2 + \omega_{ce}^2)} = 0 \]

which is a quadratic equation in \( \omega \) under the assumption of collision-dominated plasma, i.e., \( \gamma_0 > \omega \) and
$\omega_0 > k^2 \nu_{th}^2$. The value of $\beta$ is substituted in the above quadratic equation and we get two roots which are

$$\omega_1 = \frac{i\nu_0 \omega_p^2}{2(\omega_0^2 + \omega_{ce}^2) \varepsilon} - 2kv_{ox}^* \quad (2.13)$$

and

$$\omega_2 = \frac{-3i\nu_0 \omega_p^2}{2(\omega_0^2 + \omega_{ce}^2) \varepsilon} + 2kv_{ox}^* \quad (2.14)$$

if when $4k \varepsilon \nu_{ox}^* (\omega_0^2 / \omega_{ce}^2) / \omega_p^2 \ll 1$.

Writing $\omega_1 = \omega_{1r} + i\omega_{1i}$, equation (2.13) yields after some algebraic simplifications as

$$\omega_{1r} = \frac{\text{Re}P_{E_{ox}}}{m(C^2 + D^2)\omega_p^2} \left[ M (\omega_o D+\nu_0 \omega_p^2) + N (C+\omega_p^2) \omega_o \right] \quad (2.15)$$

and

$$\omega_{1i} = \frac{\nu_0 \omega_p^2}{2\varepsilon(\omega_0^2 + \omega_{ce}^2)} + \left[ N (\omega_o D+\nu_0 \omega_p^2) - N \omega_o (C+\omega_p^2) \right]$$

$$+ \frac{\varepsilon P_{E_{ox}}^R}{m(C^2 + D^2)\omega_p^2}, \quad (2.16)$$

where

$$M = \frac{\omega_o \omega_p^2 (CA - DB) - \nu_0 \omega_p^2 (CB + DA)}{(A^2 + B^2)} - D, \quad (2.17a)$$
\[
N = \frac{\omega_o \omega_p^2 (DA + CB) + \gamma_o \omega_p^2 (CA - DB)}{(A^2 + B^2)}, \quad (2.17b)
\]

\[
R = \frac{\gamma_o \omega_p^2}{\varepsilon (\gamma_o^2 + \omega_{ce}^2)}, \quad (2.17c)
\]

\[
P = \frac{2 \varepsilon k (\gamma_o^2 + \omega_{ce}^2)}{\gamma_o \omega_p^2}, \quad (2.17d)
\]

\[
A = \omega_o (\omega_{ce}^2 + \gamma_0^2 - \omega_o^2 + \omega_p^2), \quad (2.17e)
\]

\[
B = \gamma_o (2 \omega_o^2 - \omega_p^2), \quad (2.17f)
\]

\[
C = (\omega_{ce}^2 + \gamma_0^2 - \omega_o^2) \quad (2.17g)
\]

and

\[
D = 2 \gamma_o \omega_o. \quad (2.17h)
\]

Equation (2.14) does not yield an instability. In the presence of the magnetic field (i.e. \( B_o \neq 0 \)), the threshold value of the pump electric field is obtained from equations (2.1) and (2.16) as

\[
(E_{ox})_{\text{th at } B_o \neq 0} = \frac{m(C^2 + D^2)\omega_p^2}{2eP[N(\omega_o D + \gamma_0 \omega_p^2) - M\omega_o(C + \omega_p^2)]}. \quad (2.18)
\]
Equation (2.18) gives a finite value of the threshold electric field when \( N(\omega_0 D_0 + \omega_0^2) \gg M \omega_0 (\omega_0 + \omega_p^2) \). Under the condition \( \omega_0 > \omega_p > \omega_0 \left( \propto \omega_{ce} \right) \), the threshold electric field can be obtained from equations (2.17) and (2.18) as

\[
(E_{ox})_{th} \text{ at } B_0 \neq 0 = \frac{m \gamma_0^4 \omega_p^4}{4e \omega_0 k \epsilon (\gamma_0^2 + \omega_{ce}^2)} . \tag{2.19}
\]

Equation (2.19) can be employed suitably to various magnetoactive n-type semiconductors for the case of parametric excitation of the electron-acoustic wave.

In an isotropic plasma with \( B_0 = 0 \), the threshold electric field (2.19) becomes as

\[
(E_{ox})_{th} \text{ at } B_0 = 0 = \frac{m \omega_p^4}{4e k \epsilon \omega_0 \gamma_0} . \tag{2.20}
\]

Thus, it can be seen from equation (2.20) that the effect of the magnetic field is to decrease the threshold value of the pump electric field.

We obtain the growth rate of the excited mode well above the threshold in a magnetoactive semiconductor-
plasma for the case when \( \omega_o > \omega_p > \nu_o \left( \approx \omega_{ce} \right) \) from equations (2.1), (2.16) and (2.17) as

\[
(\omega_{11})_{B_o \neq 0} = \frac{2ek\nu_o E_{ox}}{m\omega_p^2} + \frac{\nu_o^2 \omega_p^2}{2\epsilon(\nu_o^2 + \omega_{ce}^2)}.
\]  

(2.21)

It is seen from equation (2.21) that in the magneto-active plasma the growth rate of the excited mode depends linearly on the pump amplitude. Equation (2.21) can be used for the determination of the growth rate of the excited mode.

In an isotropic plasma with \( B_o = 0 \), the growth rate of the excited mode well above the threshold reduces to

\[
(\omega_{11})_{B_o = 0} = \frac{2ek\nu_o E_{ox}}{m\omega_p^2} + \frac{\omega_p^2}{2\epsilon \nu_o^2}.
\]  

(2.22)

Thus, it can be seen from equation (2.22) that the effect of the magnetic field is also to decrease the growth rate of the excited wave.
2.4 Results and Discussions

From equations (2.19) and (2.20), one obtains

\[
\begin{align*}
\frac{(E_{ox})_{th} \text{ at } B_0 = 0}{(E_{ox})_{th} \text{ at } B_0 \neq 0} &= (1 + \frac{\omega_0^2}{\gamma_0^2}) .
\end{align*}
\]

(2.23)

In order to calculate the parametric excitation of the electron-acoustic wave, we take a typical case of an n-InSb crystal at 77°k with the physical constants 
\[ m = 0.014 \, m_0 \, (m_0 \text{ is the mass of a free electron}), \quad \epsilon = 18, \]
\[ \gamma_0 = 3.5 \times 10^{11} \, \text{s}^{-1}, \quad \omega_p = 2 \times 10^{13} \, \text{s}^{-1}, \quad \omega_0 = 1.78 \times 10^{14} \, \text{s}^{-1} \]
and obtain from equations (2.19) and (2.20) that

\[ (E_{ox})_{th} \approx 0.68 \times 10^3 \, \text{V cm}^{-1} \text{ when } B_0 = 2T, \quad k = 5 \times 10^6 \, \text{cm}^{-1} \]
and in the absence of the magnetic field, \( (E_{ox})_{th} \approx 1.35 \times 10^3 \, \text{V cm}^{-1} \). Thus the magnetic field \( B_0 = 2T \) decreases the threshold electric field to about 0.5 times that in a nonmagnetic situation. Such an electric field can be obtained by irradiating the crystal with a pulsed 10.6 μm CO₂ laser beam. It may be noted that for the parametric excitation of an acoustic wave in an n-InSb crystal high values of the threshold electric field have been proposed by Guha and Sen 1979a, Guha and Sen 1979b.
Our result is in general agreement with their result i.e. the threshold electric field decreases with the magnetic field.

Using the above material parameters for the crystal, one can obtain from equation (2.21) and (2.22),
\[ \omega_{li} \approx 2.3 \times 10^{13} \text{ s}^{-1} \text{ at } E_{ox} \approx 1 \times 10^3 \text{ V cm}^{-1} \text{ and } B_0 = 2T, \] while for \( B_0 = 0 \), one finds, \( \omega_{li} = 4.7 \times 10^{13} \text{ s}^{-1} \) at \( E_{ox} \approx 2 \times 10^3 \text{ V cm}^{-1} \).

The role of the collision frequency found to decrease the threshold electric field as well as the growth rate of the excited wave.
References


