We have seen from the previous chapters that baryon number violation, CP violation and non-equilibrium processes are necessary conditions for BAU. In the aftermath of the establishment of the inflationary cosmology, it seemed necessary that baryon number violation may have to be imposed as an initial state condition after inflation, as all matter is diluted away in the inflationary process. Aesthetically and practically this was unsatisfactory, for, even if there is a baryon number violation at high temperatures, when thermal equilibrium is established, the asymmetry is wiped out. Thus, models were proposed to account for the separation of matter and antimatter on a scale large enough to account for the local baryon asymmetry of the universe. In the context of particle production in expanding universe, the problem
was first addressed by B. Touissant et al [57] and refined by N. J. Papastamatiou and L. Parker [25] who showed that baryon asymmetry can arise if there are B, C and T violating processes which result from the interaction between matter and gravitational fields. These interactions result from the minimal coupling of gravity and matter and can violate B and CP symmetry. In the context of Hawking radiation from black holes, which would have a net baryonic flux, the C and CP violating Lagrangian introduced by B. Touissant et al [57] was of the form

$$L = \sqrt{-g}ig[ g^{\mu \nu} \partial _\mu \phi ^*_i \partial _\nu \phi _i + -m_i^2 \phi _i^* \phi _i - \phi _i^* V_{ij} \phi _j R_{\alpha \beta \gamma \delta} R_{\alpha \beta \gamma \delta} \big], \quad (4.1)$$

where, $R_{\alpha \beta \gamma \delta}$ is the Riemann curvature tensor, $\phi _i$ are the scalar fields and $V_{ij}$ is a matrix which contains the baryon number, C and CP violating interactions. In the strong gravitational limit the $V_{ij}(x)$ is a function of space. The particles interact with the gravitational field via $V_{ij}(x)$ and antiparticles interact via $V_{ji}(x) = V_{ij}^*(x)$. With this Lagrangian [57] calculated the asymmetry in a strong gravitational field in the vicinity of black holes.

N. J. Papastamatiou and L. Parker [25] modified the above model for the generation of the BAU in the context of an isotropically expanding universe. They considered the following Lagrangian with two complex scalar fields, $\phi$ which has a non zero baryon number and $\psi$ which has a zero baryon number given by

$$L = \sqrt{-g}ig[ g^{\mu \nu} \partial _\mu \phi ^* \partial _\nu \phi + g^{\mu \nu} \partial _\mu \psi ^* \partial _\nu \psi - (m_1^2 + \xi_1 R)\phi ^* \phi - (m_2^2 + \xi_2 R)\psi ^* \psi - \lambda R(\phi ^* \Lambda \psi + \psi ^* \Lambda ^* \phi) \big], \quad (4.2)$$

where $m_{1,2}$ are masses of fields $\phi$ and $\psi$ respectively. In the presence of expansion, the spacetime dependent potential $V_{ij}(x)$ becomes a function of time $\Lambda(t)$ and the gravitational field $R_{\alpha \beta \gamma \delta} R_{\alpha \beta \gamma \delta}$ is replaced with the Ricci scalar $R$ and is treated as a classical background. The interaction term $\lambda R(\phi ^* \Lambda \psi + \psi ^* \Lambda ^* \phi)$ may be regarded as representing an effective two particle interaction with the graviton that violates baryon number conservation as well as CP invariance (since $\Lambda$ is complex), thus satisfying the first and second Sakharov’s conditions. The fields participating in the interaction are quantized and their time development is found
by perturbation theory. The BAU is calculated for the cosmological model in which universe contracts to minimum and then expands to present time. In such a model \(a(t)\) (expansion parameter) is treated as constant at early and late times. All the particles present at the late times are generated by the gravitational field and by the asymmetric interaction. They found the created baryon-number density has no explicit dependence on the parameters which characterize the purely gravitational pair production. BAU in this model is calculated perturbatively and first appears in the order \(\lambda^2\) as

\[
\Delta N^{(\text{out})}_{\phi K} = \langle N^{(\text{out})}_{\phi K} \rangle - \langle \tilde{N}^{(\text{out})}_{\phi K} \rangle = \lambda^2 (|I_2|^2 - |I_3|^2), \tag{4.3}
\]

where \(N^{(\text{out})}_{\phi K}\) are number of particles and \(\tilde{N}^{(\text{out})}_{\phi K}\) are number of anti-particles produced at late times,

\[
I_2 = \int_{-\infty}^{\infty} dt R(t) \Lambda(t) \chi_{\phi k}^*(t) \chi_{\psi k}^*(t) \tag{4.4}
\]

and

\[
I_3 = \int_{-\infty}^{\infty} dt R(t) \Lambda(t) \chi_{\phi k}(t) \chi_{\psi k}(t) \tag{4.5}
\]

where \(\chi_{\phi k}(t)\) and \(\chi_{\psi k}(t)\) are the c-number solutions for equation of motion for the fields \(\phi\) and \(\psi\) in Fourier space. They found that \(\Delta N^{(\text{out})}_{\phi K} \neq 0\) even when \(m = 0\). This implies that gravity acts an external field which induces asymmetric pair production. To be consistent with Einstein equations with zero cosmological constant at \(t \geq G^{-\frac{1}{2}}\), one requires \(\lambda \sim .8\). With such a high value of \(\lambda\), the perturbation results are inaccurate. For small values of \(\lambda\) one cannot generate sufficient asymmetry in this model. This model also has the ad-hoc approximation of taking \(R\) to replace the graviton field (requiring justification). This model was proposed prior to the inflationary scenario. The time scales in this model correspond to the time scales for inflation, hence it was natural to consider this model in the context of reheating after inflation.

R. Rangarajan and D. V. Nanopoulos [26] studied the possibility of creating BAU by particle production during the reheating phase by the decay of complex inflaton field, with
a suitable modification of the above model. The modified Lagrangian is given by

\[
L = \sqrt{-g}[g^{\mu\nu}\partial_{\mu}\phi^{*}\partial_{\nu}\phi + g^{\mu\nu}\partial_{\mu}\psi^{*}\partial_{\nu}\psi - (m_{\phi}^{2} + \xi_{\phi}R)\phi^{*}\phi - (m_{\psi}^{2} + \xi_{\psi}R)\psi^{*}\psi + g^{\mu\nu}\partial_{\mu}\eta^{*}\partial_{\nu}\eta - (m_{\eta}^{2} + \xi_{\eta}R)\eta^{*}\eta - V(\eta) - \lambda(\eta^{2}\phi^{*}\psi + \eta^{*2}\psi^{*}\phi)].
\] (4.6)

A minimally coupled complex inflaton field \( \eta^{2} \) replaces the term \( R\Lambda \) in the interaction in (4.2). The initial velocity of the \( \eta \) field and the shape of the inflaton potential ensures that its phase varies as the inflaton rolls down its potential. This gives rise to dynamic CP violation.

To calculate the BAU they followed the same perturbative methods of N. J. Papastamatiou and L. Parker [25], with a slight modification. In their calculation, they use the fact that the annihilation and creation operators are not same during the phase immediately after inflation and late times after reheating. By using the Bogoliubov transformations, which relates the annihilation and creation operators of early times to that of late times, BAU in this model is found to be

\[
\Delta N^{(out)}_{\phi k} = \langle N^{(out)}_{\phi k}\rangle - \langle \bar{N}^{(out)}_{\phi k}\rangle = \lambda^{2}(|I_{2}|^{2} - |I_{3}|^{2}),
\] (4.7)

where \( N^{(out)}_{\phi k} \) are number of particles and \( \bar{N}^{(out)}_{\phi k} \) are number of anti-particles produced at late times.

\[
I_{2} = \int_{-\infty}^{\infty} dt\eta^{2}\chi_{k}^{\phi^{*}}(t)\chi_{k}^{\psi}(t)
\] (4.8)

and

\[
I_{3} = \int_{-\infty}^{\infty} dt\eta^{2}\chi_{k}^{\phi}(t)\chi_{k}^{\psi}(t)
\] (4.9)

where \( \chi_{k}^{\phi}(t) \) and \( \chi_{k}^{\psi}(t) \) are the complex functions that solve the equation of motion for the \( \phi \) and \( \psi \), with \( \lambda = 0 \) in Fourier space. They considered chaotic and natural inflationary reheating scenarios. In the former case, the complex decaying field is the inflaton itself and, in the latter case, the phase of the complex field is the inflaton.

Even though the resultant BAU in all models is insufficient, the value of this work is that one should not ignore the inflationary and reheating era while calculating the asymmetry.
4.1 The Formalism.

We now consider the Lagrangian of R. Rangarajan and D. V. Nanopoulos [26] and calculate particle production, via parametric resonance during reheating, using general formalism of squeezed rotated states. This method allows us to calculate the BAU without resorting to perturbation theory, which may be the reason why insufficient BAU is generated in the above models. We give the details of the method in the following sections.

Considering the Lagrangian of (4.6), we will treat the inflaton field $\eta$ as a background classical field. Its temporal evolution is determined by a Euler Lagrange equation of motion, with the inflaton potential $V(\eta)$ in the Friedman-Robertson-Walker (FRW) metric $ds^2 = dt^2 - a^2(t) dx^2$,

$$\ddot{\eta} + 3 \frac{\dot{a}}{a} \dot{\eta} + \frac{\partial V}{\partial \eta} = 0. \quad (4.10)$$

In our analysis, the background gravitational field is coupled minimally i.e. ($\xi_\phi = \xi_\psi = \xi_\eta = 0$) and the action is

$$S = \int d^3 x dt a^3(t) \left( \frac{1}{2} (\dot{\phi}^*)(\dot{\phi}) - \frac{1}{2 a^2(t)} (\nabla \phi^*)(\nabla \phi) + \frac{1}{2} (\dot{\psi}^*)(\dot{\psi}) - \frac{1}{2 a^2(t)} (\nabla \psi^*)(\nabla \psi) - m_\phi \phi^* \phi - m_\psi \psi^* \psi - \lambda (\eta^2 \phi^* \psi + \eta^* 2 \psi^* \phi) \right). \quad (4.11)$$

Applying a Legendre transformation

$$H = \sum \pi_i \dot{\phi}_i - L, \quad (4.12)$$

where, $\pi_\phi$ and $\pi_\psi$ are the canonical momenta of the fields $\phi$ and $\psi$, given by

$$\pi_i = \frac{\partial L}{\partial \dot{\phi}_i}, \quad (4.13)$$

the Hamiltonian is

$$H = \int d^3 x dt a^3 \left[ \frac{1}{a^3} \pi_\phi^* \pi_\phi + \frac{1}{a^2} \nabla \phi^* \nabla \phi + \frac{1}{a^3} \pi_\psi^* \pi_\psi + \frac{1}{a^2} \nabla \psi^* \nabla \psi + m_\phi^2 \phi^* \phi + m_\psi^2 \psi^* \psi + \lambda (\eta^2 \phi^* \psi + \eta^* 2 \psi^* \phi) \right]. \quad (4.14)$$
We quantize the fields $\phi$ and $\psi$, using the mode expansion

\[ \psi(x^\mu) = \int d\tilde{k} \left[ a_\psi^\dagger k e^{-ik \cdot x} + b_\psi^\dagger k e^{ik \cdot x} \right], \quad (4.15) \]

\[ \psi^*(x^\mu) = \int d\tilde{k} \left[ a_\psi^\dagger k e^{ik \cdot x} + b_\psi^\dagger k e^{-ik \cdot x} \right], \quad (4.16) \]

\[ \phi(x^\mu) = \int d\tilde{k} \left[ a_\phi^\dagger k e^{-ik \cdot x} + b_\phi^\dagger k e^{ik \cdot x} \right], \quad (4.17) \]

\[ \phi^*(x^\mu) = \int d\tilde{k} \left[ a_\phi^\dagger k e^{ik \cdot x} + b_\phi^\dagger k e^{-ik \cdot x} \right], \quad (4.18) \]

with

\[ k \cdot x = k_\mu x^\mu = \omega t - k_i x_i, \quad (4.19) \]

\[ d\tilde{k} = \frac{d^3k dt}{[2\pi]^3 \omega}, \quad (4.20) \]

The quantized Hamiltonian is then

\[
H = \int d^3k \left[ \frac{\omega_\phi}{a^6} a_\phi^\dagger k a_\phi k + \frac{\omega_\psi}{a^6} a_\psi^\dagger k a_\psi k + \frac{\lambda \eta^2 a^3}{2\sqrt{a} \omega \psi} a_\psi^\dagger k a_\phi k + \frac{\lambda \eta^2 a^3}{2\sqrt{a} \omega \phi} a_\phi^\dagger k a_\phi k \right] \\
+ \frac{\omega_\phi}{a^6} b_\phi^\dagger k b_\phi k + \frac{\omega_\psi}{a^6} b_\psi^\dagger k b_\psi k + \frac{\lambda \eta^2 a^3}{2\sqrt{a} \omega \psi} b_\psi^\dagger k b_\psi k + \frac{\lambda \eta^2 a^3}{2\sqrt{a} \omega \phi} b_\phi^\dagger k b_\phi k \\
+ \frac{\lambda \eta^2 a^3}{2\sqrt{a} \omega \phi} [a_\psi^\dagger k b_\phi k + a_\phi^\dagger k b_\psi k], \quad (4.21) \]

where

\[ \frac{\omega_\phi^2}{a^6} = \frac{k^2}{a^2} + m_\phi^2, \quad (4.22) \]

\[ \frac{\omega_\psi^2}{a^6} = \frac{k^2}{a^2} + m_\psi^2. \quad (4.23) \]

Here, $k = \frac{k}{a}$ is the physical wave (co-wave) number of the mode $k$. By defining the following

\[ \frac{\Omega_\psi^2(t)}{a^6} = \frac{\omega_\psi^2}{a^6} + \sqrt{\frac{\omega_\phi}{\omega_\psi}} \lambda \eta^2(t), \quad (4.24) \]

\[ \frac{\Omega_\phi^2(t)}{a^6} = \frac{\omega_\phi^2}{a^6} + \sqrt{\frac{\omega_\phi}{\omega_\psi}} \lambda \eta^2(t), \quad (4.25) \]
the Hamiltonian is written as follows

\[ H = \int d^3k \left[ \frac{\omega}{a^3} a_k^\dagger a_k + \frac{\omega}{a} a_k^\dagger a_k^\dagger \right] + \frac{\omega}{a^3} \left[ \frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1 \right] a_k^\dagger a_k + \frac{\omega}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] b_k^\dagger b_k \]

This Hamiltonian has two symmetries \( su(2) \) and \( su(1,1) \). To illustrate these symmetries, the following number operators are defined

\[ N_1 = a_k^\dagger a_k, \quad N_2 = a_k^\dagger a_k^\dagger, \quad N_3 = b_k^\dagger b_k, \quad N_4 = b_k^\dagger b_k^\dagger. \]  

(4.27)

Then, the operators

\[ J_+ = a_k^\dagger a_k, \quad J_- = a_k^\dagger a_k^\dagger, \quad J_0 = \frac{1}{2} (N_1 - N_2), \]  

\[ M_+ = b_k^\dagger b_k, \quad M_- = b_k^\dagger b_k^\dagger, \quad M_0 = \frac{1}{2} (N_3 - N_4), \]

satisfy the \( su(2) \) algebras

\[ [J_+, J_-] = -2J_0, \quad [J_+, J_0] = -J_+, \quad [J_-, J_0] = J_. \]  

(4.29)

\[ [M_+, M_-] = -2M_0, \quad [M_+, M_0] = -M_+, \quad [M_-, M_0] = M_. \]  

(4.30)

Another set of operators are:

\[ K_+ = a_k^\dagger b_k^\dagger, \quad K_- = b_k^\dagger a_k, \quad K_0 = \frac{1}{2} (N_1 + N_3 + 1), \]  

\[ L_+ = a_k^\dagger b_k^\dagger, \quad L_- = b_k^\dagger a_k, \quad L_0 = \frac{1}{2} (N_2 + N_4 + 1). \]  

(4.31)

These satisfy the \( su(1,1) \) algebras

\[ [K_+, K_-] = 2K_0, \quad [K_+, K_0] = K_+, \quad [K_-, K_0] = -K_. \]  

(4.32)
\[ [L_+, L_-] = 2L_0, \quad [L_+, L_0] = L_+, \quad [L_-, L_0] = -L_- \]  

(4.33)

We can write the Hamiltonian as

\[
H = \int d^3k \left[ \frac{2\omega_\psi}{a^3} K_0 + \frac{2\omega_\phi}{a^3} L_0 + \frac{\omega_\psi(t)}{a^3} \left( \frac{\Omega^2_\psi(t)}{\omega_\psi^2} - 1 \right) (J_+ + M_+) \right. \\
+ \frac{\omega_\phi}{a^3} \left( \frac{\Omega^2_\phi(t)}{\omega_\phi^2} - 1 \right) (J_- + M_-) + \frac{\omega_\psi}{a^3} \left( \frac{\Omega^2_\psi(t)}{\omega_\psi^2} - 1 \right) (K_+ + L_-) \right. \\
+ \frac{\omega_\phi}{a^3} \left( \frac{\Omega^2_\phi(t)}{\omega_\phi^2} - 1 \right) (L_+ + K_-)].
\]

(4.34)

The \( \text{su}(1,1) \) and \( \text{su}(2) \) symmetries of the Hamiltonian are now manifestly evident.

These symmetries of the Hamiltonian will aid us to diagonalise it, so that one can go over to the parametric amplification picture. This Hamiltonian has resemblance to quantum optical Hamiltonians in beam splitting and parametric amplification processes [58]. Thus, we will use quantum optical methods to diagonalize it. The Hamiltonian can be written in terms of new creation and annihilation operators using the following unitary transformation

\[
H' = U^\dagger(R_2)U^\dagger(R_1)HU(R_1)U(R_2),
\]

(4.35)

where

\[
U(R_1)U(R_2) = \exp[\theta(J_+e^{2i\xi} + J_-e^{-2i\xi})]\exp[\theta(M_+e^{2i\xi} + M_-e^{-2i\xi})],
\]

(4.36)

and operator \( U(R_1) \) provides the well known (beam splitting) transformation relations:

\[
U^\dagger(R_1) \begin{pmatrix} a_k^\psi \\ a_k^\phi \end{pmatrix} U(R_1) = \begin{pmatrix} \cos(\theta) & e^{2i\xi}\sin(\theta) \\ -e^{-2i\xi}\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} a_k^\psi \\ a_k^\phi \end{pmatrix} = \begin{pmatrix} A_k \\ B_k \end{pmatrix},
\]

(4.37)

while \( U(R_2) \) provides the relation

\[
U^\dagger(R_2) \begin{pmatrix} b_k^- \phi \\ b_k^- \psi \end{pmatrix} U(R_2) = \begin{pmatrix} \cos(\theta) & e^{2i\xi}\sin(\theta) \\ -e^{-2i\xi}\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} b_k^- \phi \\ b_k^- \psi \end{pmatrix} = \begin{pmatrix} C_k \\ D_k \end{pmatrix},
\]

(4.38)

The angle \( \theta \) is determined from the relation \( \tan(\theta) = \sqrt{\omega_\phi/\omega_\psi} \), so that \( \cos(\theta) = \sqrt{\omega_\psi/(\omega_\phi + \omega_\psi)} \) and \( \sin(\theta) = \sqrt{\omega_\phi/(\omega_\phi + \omega_\psi)} \). Then the Hamiltonian assumes the form

\[
H = H_1 + H_2,
\]

(4.39)
where \( H_1 \) and \( H_2 \) are given by

\[
H_1 = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3} \left[ \beta [A_k^\dagger A_k + C_k C_k^\dagger] + \frac{\lambda \eta^2}{2 \omega_\phi \omega_\psi} A_k^\dagger C_{-k} + \frac{\lambda \eta^2}{2 \omega_\phi \omega_\psi} C_{-k} A_k \right],
\]

\[
H_2 = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3} \left[ \alpha [B_k^\dagger B_k + D_k D_k^\dagger] + \frac{\lambda \eta^2}{2 \omega_\phi \omega_\psi} D_{-k} B_k + \frac{\lambda \eta^2}{2 \omega_\phi \omega_\psi} B_k^\dagger D_{-k}^\dagger \right].
\]

Here

\[
2\alpha = 1 - \frac{\lambda |\eta|^2 a^6}{2 \omega_\phi \omega_\psi},
\]

\[
2\beta = 1 + \frac{\lambda |\eta|^2 a^6}{2 \omega_\phi \omega_\psi}.
\]

The complexity of the \( \eta \) field gives rise to the CP violation. After the \( su(2) \) rotations we get

\[
H' = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3} \left[ \beta(t) [A_k^\dagger A_k + C_{-k} C_{-k}^\dagger] \right.
\]

\[
+ \alpha(t) [B_{-k}^\dagger B_{-k} + D_k D_k^\dagger] \]

\[
+ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \left[ A_k^\dagger C_{-k} + D_k B_{-k} \right]
\]

\[
+ \frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1 \left[ B_{-k}^\dagger D_k^\dagger + C_{-k} A_k \right].
\]

Now define operators:

\[
D_{1+} = A_{k}^\dagger C_{k}^\dagger, \quad D_{1-} = C_{-k} A_k, \quad D_{10} = \frac{1}{2} \left( A_{k}^\dagger A_k + C_{-k} C_{-k}^\dagger + 1 \right),
\]

\[
D_{2+} = B_{-k}^\dagger D_k^\dagger, \quad D_{2-} = D_k B_{-k}, \quad D_{20} = \frac{1}{2} \left( B_{-k}^\dagger B_{-k} + D_k^\dagger D_k + 1 \right),
\]

satisfying the \( su(1,1) \) algebras

\[
[D_{1+}, D_{1-}] = 2D_{10}, \quad [D_{1+}, D_{10}] = D_{1+}, \quad [D_{1-}, D_{10}] = -D_{1-},
\]

\[
[D_{2+}, D_{2-}] = 2D_{20}, \quad [D_{2+}, D_{20}] = D_{2+}, \quad [D_{2-}, D_{20}] = -D_{2-}.
\]

Then in terms of the \( su(1,1) \) generators

\[
H' = \int d^3k \frac{2(\omega_\phi + \omega_\psi)}{a^3} \left[ \beta(t) D_{10} + \alpha(t) D_{20} \right]
\]

\[
+ \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] [D_{1+} + D_{2-}] + \frac{\omega_\psi}{a^3} \left[ \frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1 \right] (D_{2+} + D_{1-}).
\]
This $\text{su}(1,1)$ symmetry implies diagonalizability through a squeezing (Bogoliubov) transformation from the quantum optical analogies that we are applying, we see here that a product of two squeezing transformations will provide us with a diagonal Hamiltonian. The squeezing transformations we use are given by

\[
S(\zeta_1)S(\zeta_2) = \exp[\zeta_1 D_{1+} - \zeta_1^* D_{1-}] \exp[\zeta_2 D_{2+} - \zeta_2^* D_{2-}],
\]

where, $\zeta_1 = r_1 \exp[i\gamma_1]$ and $\zeta_2 = r_2 \exp[i\gamma_2]$ are the squeezing parameters. The operators $S(\zeta_1)$ and $S(\zeta_2)$ provide the relevant Bogoliubov transformations for the Hamiltonian. The creation and annihilation operators $A_k$, $B_k$, $C_k$ and $D_k$ in terms of new creation and annihilation operators are given by

\[
A_s(k,t) = \mu_1 A_k + \nu_1 C_{-k}^\dagger, \tag{4.51}
\]
\[
A_s^\dagger(k,t) = \mu_1^* A_k^\dagger + \nu_1^* C_{-k}, \tag{4.52}
\]
\[
B_s(k,t) = \mu_2 B_{-k} + \nu_2 D_k^\dagger, \tag{4.53}
\]
\[
B_s^\dagger(k,t) = \mu_2^* B_{-k}^\dagger + \nu_2^* D_k, \tag{4.54}
\]

The Bogoliubov coefficients can be read off as follows

\[
\mu_1 = \cosh(r) = \frac{2\omega_\phi \omega_\psi - \lambda \eta a^6}{2 \sqrt{\omega_\phi \omega_\psi (\omega_\phi \omega_\psi - \lambda \eta a^6)^2}} = \frac{2\alpha(t)}{(4\alpha(t) - 1)^{1/2}}, \tag{4.55}
\]
\[
\nu_1 = \sinh(r) = \frac{\lambda r^2 a^6}{2 \sqrt{\omega_\phi \omega_\psi (\omega_\phi \omega_\psi - \lambda \eta a^6)^2}} = e^{-i\gamma_1} \frac{2\alpha(t) - 1}{(4\alpha(t) - 1)^{1/2}}, \tag{4.56}
\]
\[
\mu_2 = \cosh(r) = \frac{2\omega_\phi \omega_\psi + \lambda \eta a^6}{2 \sqrt{\omega_\phi \omega_\psi (\omega_\phi \omega_\psi + \lambda \eta a^6)^2}} = \frac{2\beta(t)}{(4\beta(t) - 1)^{1/2}}, \tag{4.57}
\]
\[
\nu_2 = \sinh(r) = \frac{\lambda r^2 a^6}{2 \sqrt{\omega_\phi \omega_\psi (\omega_\phi \omega_\psi + \lambda \eta a^6)^2}} = e^{-i\gamma_2} \frac{2\beta(t) - 1}{(4\beta(t) - 1)^{1/2}}, \tag{4.58}
\]

where,

\[
|\mu_1|^2 - |\nu_1|^2 = 1, \quad |\mu_2|^2 - |\nu_2|^2 = 1. \tag{4.59}
\]
Thus the final diagonalized Hamiltonian is

\[
H_f = S(\zeta_2)\{S(\zeta_1)U^\dagger(R_2)U(R_1)H|U(R_1)U(R_2)S(\zeta_1)S(\zeta_2) \nonumber \\
= \int d^3k \frac{(\omega_\phi + \omega_{\phi})}{2a^3} (4\alpha(t)-1)^{\frac{3}{2}} [A_s^\dagger(k,t)A_s(k,t) + 1] \nonumber \\
+ \frac{(\omega_\phi + \omega_{\phi})}{2a^3} (4\beta(t)-1)^{\frac{3}{2}} [B_s^\dagger(k,t)B_s(k,t) + 1]. \tag{4.60} \]

We use the methods presented at the end of chapter 3 of the importance of the Bogoliubov transformations and the consequent population of a vacuum at a later time with particles of another vacuum at an earlier time to relate the squeezed rotated vacuum state \(|0(k,t),0(k,t)\rangle > H_f(t)|0(k),0(k)\rangle > H'\) the rotated vacuum state of \(H'\) and the original vacuum for \(H\). \(|0(k),0(k)\rangle > H\) is defined as \(a_k^\dagger|0(k),0(k)\rangle > H = 0\), \(a_k|0(k),0(k)\rangle > H = 0\), \(b_k^\dagger|0(k),0(k)\rangle > H = 0\), and \(b_k|0(k),0(k)\rangle > H = 0\). The vacuum of \(H_f\) is related to Vacuum of \(H'\) by

\[
|0(k,t),0(k,t)\rangle := e^{\int \frac{d^3k}{(2\pi)^3} \zeta_1(D_{1+}-D_{1-}) + \zeta_2(D_{2+}-D_{2-})} |0(k),0(k)\rangle > H'. \tag{4.61} \]

In turn the rotated vacuum of \(H'\) is related to the original vacuum of \(H\) by

\[
|0(k),0(k)\rangle > H' := e^{\int \frac{d^3k}{(2\pi)^3} \theta e^{2i\xi + J_+e^{2i\xi}} + [\theta e^{2i\xi + J_-e^{2i\xi}}]} |0(k),0(k)\rangle > H. \tag{4.62} \]

Thus, the vacuum of \(H_f\) is related to \(H\) by

\[
|0(t),0(t)\rangle := e^{\int \frac{d^3k}{(2\pi)^3} \zeta_1(D_{1+}-D_{1-}) + \zeta_2(D_{2+}-D_{2-})} e^{\int \frac{d^3k}{(2\pi)^3} \theta e^{2i\xi + J_+e^{2i\xi}} + [\theta e^{2i\xi + J_-e^{2i\xi}}]} |0(k),0(k)\rangle > H, \tag{4.63} \]

and is a squeezed rotated vacuum. The number of particles and anti-particles can be calculated by the relationship between the creation and annihilation operators of the initial freequanta \(a_k^\dagger, b_k^\dagger, a_k^\phi, b_k^\phi\) to the final creation and annihilation operators \(A_s\) and \(B_s\) by

\[
A_s(k,t) = (\mu_1\cos(\theta))a_k^\dagger + (\nu_1\sin\theta e^{2i\xi})b_k^\dagger + (\mu_1\sin\theta e^{2i\xi})a_k^\phi + (\nu_1\cos(\theta))b_k^\phi, \tag{4.64} \\
B_s(k,t) = (\mu_2\cos(\theta))a_k^\dagger + (\nu_2\sin\theta e^{-2i\xi})b_k^\dagger + (\mu_2\sin\theta e^{-2i\xi})a_k^\phi + (\nu_2\cos(\theta))b_k^\phi. \tag{4.65} \]
Note that it is the $\phi$ field which carries the baryon number. This gives us the number of baryons generated at time $t$ as

$$N_B(t) = \sum_k \langle A_s^\dagger(k,t) A_s(k,t) \rangle = \sum_k |\nu_{k1}|^2,$$  

(4.66)

while the number of anti-baryons is

$$N_{\overline{B}}(t) = \sum_k \langle B_s^\dagger(k,t) B_s(k,t) \rangle = \sum_k |\nu_{k2}|^2.$$  

(4.67)

This shows clearly that the vacuum $|0(k,t), 0(k,t)\rangle$ is populated with particles and anti-particles with respect to vacuum $|0(k), 0(k)\rangle$. So we have a resultant baryon asymmetry given by

$$N_B(t) - N_{\overline{B}}(t) = \sum_k (|\nu_{k1}|^2 - |\nu_{k2}|^2),$$

$$= \sum_k \left[ \frac{(\lambda a^6\eta^2)^2}{(\omega_\phi\omega_\psi - \lambda |\eta|^2 a^6)} \right] - \left[ \frac{(\lambda a^6\eta^2)^2}{(\omega_\phi\omega_\psi + \lambda |\eta|^2 a^6)} \right].$$  

(4.68)

where the k dependence is through $\omega_\phi, \omega_\psi$. The time dependence of the asymmetry parameter comes from the inflaton field $\eta(t)$ and the expansion parameter $a(t)$. To get a proper dynamical estimation of the parameter in an expanding universe, we will study the time evolution the number of particles and anti-particles produced.

### 4.2 Evolution Of the Asymmetry Parameter:

To obtain the time evolution equations for the wave functions of the particles (baryon number 1) and the anti-particles (baryon number -1), in a realistic expanding universe, we
go over to the co-ordinate representation by defining the following operators

\[
A_s(k, t) = e^{i \frac{\Omega_+(t)dt}{a^6}} \left( \frac{\Omega_+(t)}{a^3} \Pi_A(k, t) + iP_{\Pi_A}(k, t) \right),
\]
\[
A_s^\dagger(k, t) = e^{-i \frac{\Omega_+(t)dt}{a^6}} \left( \frac{\Omega_+(t)}{a^3} \Pi_A(k, t) - iP_{\Pi_A}(k, t) \right),
\]
\[
B_s(k, t) = e^{i \frac{\Omega_-(t)dt}{a^6}} \left( \frac{\Omega_-(t)}{a^3} \Pi_B(k, t) + iP_{\Pi_B}(k, t) \right),
\]
\[
B_s^\dagger(k, t) = e^{-i \frac{\Omega_-(t)dt}{a^6}} \left( \frac{\Omega_-(t)}{a^3} \Pi_B(k, t) - iP_{\Pi_B}(k, t) \right).
\]

The Hamiltonian \( H_f \) in the coordinate representation is:

\[
H_f(t) = \int \frac{d^3k}{(2\pi)^3} \left[ \left( \frac{\Omega_+}{a^6} \right)^2 \Pi_A^2(k, t) + P_{\Pi_A}^2(k, t) + \left( \frac{\Omega_-}{a^6} \right)^2 \Pi_B^2(k, t) + P_{\Pi_B}^2(k, t) \right],
\]

where

\[
\left( \frac{\Omega_+}{a^6} \right)^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6} (4\alpha(t) - 1),
\]
\[
\left( \frac{\Omega_-}{a^6} \right)^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6} (4\beta(t) - 1).
\]

Since the \( \frac{\Omega_+}{a^6} \) and \( \frac{\Omega_-}{a^6} \) are time dependent, the Hamiltonian \( H_f \) in (4.60) represents a time dependent harmonic oscillator.

The time evolution of a wave function \( \chi(t) \) under the action of a Hamiltonian \( H(t) \) is simply

\[
H(t)\chi(t) = i \frac{d}{dt}\chi(t).
\]

From the form of \( H_f \) given in eq(4.73), it is clear that it is the direct sum of two independent Hamiltonians \( H_A(t) \) and \( H_B(t) \) for each of the \( A_s \) and \( B_s \) modes. Therefore, the wave function \( \chi(t) \) for the Hamiltonian \( H_f \) is just the sum of two wavefunctions \( \chi_A(t) \) and \( \chi_B(t) \) where they evolve independently. From here on we consider only \( H_A(t) \). Similar expression will follow for \( H_B(t) \).
Fourier decomposing the wave function $\chi_A(t)$ and the Hamiltonian $H_A(t)$, we get

$$H_A(k, t)\chi_A(k, t) = \frac{i}{dt} \chi_A(k, t). \quad (4.77)$$

In the coordinate space representation $(\Pi_A, P_A)$, the wave functions $\chi_A(k, t)$ can be represented by Gaussian wave function

$$\chi_A(k, t) = L_A(t)e^{[-W_A(t)\Pi_A]}\chi_A(k, 0). \quad (4.78)$$

A time derivative gives

$$i\frac{\partial}{\partial t}\chi_A(k, t) = \left(\frac{\dot{L}_A}{L_A} - i\Pi_A^2\dot{W}_A(k, t)\right)\chi_A(k, t), \quad (4.79)$$

while the eq (4.76) gives

$$i\frac{\partial}{\partial t}\chi_A(k, t) = \frac{1}{2a^6}\left[\Omega_A^2\Pi_A^2 - \frac{\partial^2}{\partial \Pi_A^2}\right]\chi_A(k, t). \quad (4.80)$$

From these two equations we obtain

$$\frac{\dot{L}_A}{L_A} = \frac{\dot{\chi}_A(k, t)}{\chi_A(k, t)} + \Pi_A^2\dot{W}_A(k, t) \quad (4.81)$$

and

$$W_A(t) = -ia^3/2\frac{\dot{\chi}_A(k, t)}{\chi_A(k, t)}. \quad (4.82)$$

Thus the evolution equation is

$$\ddot{\chi}_A(k, t) + 3\frac{\dot{a}}{a}\dot{\chi}_A(k, t) + \frac{\Omega_A^2}{a^6}\chi_A(k, t) = 0. \quad (4.83)$$

In a similar fashion the evolution equation for $\chi_B(k, t)$ is

$$\ddot{\chi}_B(k, t) + 3\frac{\dot{a}}{a}\dot{\chi}_B(k, t) + \frac{\Omega_B^2}{a^6}\chi_B(k, t) = 0. \quad (4.84)$$

We consider a conformally flat FRW metric

$$ds^2 = a(\tau)^2(d\tau^2 - dx^2), \quad (4.85)$$
by employing the scaled time

\[ d\tau = a(t)^{-1} dt. \quad (4.86) \]

The equations of motion for the wave functions \( \chi_A(k, t) \) and \( \chi_B(k, t) \) given above can be transformed into ones that resemble damped harmonic oscillators with time dependent frequencies.

\[
\frac{1}{a^2(\tau)} \frac{d^2}{d\tau^2} \chi_A(k, \tau) + \frac{2}{a^3(\tau)} \frac{da}{d\tau} \frac{d}{d\tau} \chi_A(k, \tau) + \frac{\Omega_-}{a^6} \chi_A(k, \tau) = 0, \quad (4.87)
\]

\[
\frac{1}{a^2(\tau)} \frac{d^2}{d\tau^2} \chi_B(k, \tau) + \frac{2}{a^3(\tau)} \frac{da}{d\tau} \frac{d}{d\tau} \chi_B(k, \tau) + \frac{\Omega_+}{a^6} \chi_B(k, \tau) = 0. \quad (4.88)
\]

A change of variable

\[ u_A = a \chi_A, \quad (4.89) \]

and similarly for \( \chi_B(k, \tau) \) transforms (4.87) and (4.88) to

\[-u''_A + (E - V_1(\eta, \tau)) u_A = 0, \quad (4.90)\]

and

\[-u''_B + (E + V_2(\eta, \tau)) u_B = 0, \quad (4.91)\]

where prime denotes differential with respect to \( \tau \) and

\[ E = -\frac{(\omega_\phi + \omega_\psi)^2}{4a^6}, \quad (4.92) \]

\[ V_1(\eta, \tau) = -\frac{1}{a(\tau)} \frac{d^2 a}{d\tau^2} - [A|\eta|^2], \quad (4.93) \]

\[ V_2(\eta, \tau) = \frac{1}{a(\tau)} \frac{d^2 a}{d\tau^2} - [A|\eta|^2], \quad (4.94) \]

where \( A = \frac{\lambda(\omega_\phi + \omega_\psi)^2}{8\omega_\phi \omega_\psi} \), \( \omega_\phi \) and \( \omega_\psi \) are given by the equations (4.22) and (4.23) respectively.

We now have time evolution equations in the Schrödinger-like form described in chapter 3. Thus, we know that by calculating the reflection “R” and the transmission coefficients “T” across the potentials \( V_1(\eta, \tau) \) and \( V_2(\eta, \tau) \), we can get the number of particles and antiparticles produced by \( \text{sinh}^2(r) = |\nu|^2 = \frac{R}{T} \). We see from the equations (4.90) and (4.91) that
the baryons encounter a potential barrier whereas the anti-baryons encounter a potential well.

We assume an oscillating inflaton background

$$\eta = \Lambda(\tau) \sin(m\tau), \quad (4.95)$$

where $\Lambda = |\Lambda| e^{i\xi}$ is complex and $m$ is the inflaton mass. In the general case, the amplitude $\Lambda$ would be a function of time, since the expansion of the universe would cause the oscillating inflaton to lose energy and hence decrease its amplitude and frequency, giving a phenomenological approximation $\Lambda = \Lambda_0 e^{\tau/\tau_1}$, where $\tau_1$ is the damping scale. In this analysis, we shall assume $\Lambda$ is almost constant allowing the inflaton oscillations to fall in a resonance band causing a parametric amplification/suppression of the particle/antiparticle modes.

With this assumption of the form of the classical background inflaton field, the evolution equations (4.90) and (4.91) are given by

$$u''_A + \left( -\frac{a''}{a} + E - \frac{B}{2} + \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \quad (4.96)$$

and

$$u''_A + \left( -\frac{a''}{a} + E + \frac{B}{2} - \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \quad (4.97)$$

where $B = |\Lambda|^2 A$. These are exact equations for the evolution of the baryon and anti-baryon fields in an expanding universe.

We first consider the case of constant expansion, to reduce (4.96) and (4.97) to the Mathieu equations

$$u''_A + \left( E - \frac{B}{2} + \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \quad (4.98)$$

and

$$u''_A + \left( E + \frac{B}{2} - \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \quad (4.99)$$

From the theory of Mathieu equations and parametric resonance we see that the frequency

$$\omega^2_k = \frac{2k^2 + m^2_\phi + m_\psi^2 + 2\sqrt{k^2 + m^2_\phi} \sqrt{k^2 + m^2_\phi}}{4} = \left(\frac{n}{2}\omega\right)^2, \quad (4.100)$$
must be half integer multiples of a lowest frequency \( \omega \). By applying the theory of parametric resonance from the chapter 3, we get \( \omega \propto k \). Now we make another approximation, by assuming that \( m^2_{\phi}, m^2_{\psi} \ll \omega^2_k \), we find

\[
\ddot{u}_A + (E - B\sin^2(m\tau))u_A = 0, \tag{4.101}
\]

and

\[
\ddot{u}_B + (E + B\sin^2(m\tau))u_B = 0, \tag{4.102}
\]

where

\[
E = \frac{k^2}{\rho^2}, \quad B = \lambda|\Lambda|^2. \tag{4.103}
\]

In the region of broad resonance, we replace the oscillating potential near its zeros with an asymptotically flat potential of the form

\[
|\eta|^2 = |\Lambda|^2\sin^2(m\tau) \simeq 2|\Lambda|^2\tanh^2(m\frac{\tau - \tau_i}{\sqrt{2}}). \tag{4.104}
\]

In each instability region, we have

\[
\ddot{u}_A + (\kappa^2 - 2\lambda|\Lambda|^2\tanh^2(m\frac{\tau - \tau_i}{\sqrt{2}}))u_A = 0, \tag{4.105}
\]

and

\[
\ddot{u}_B + (\kappa^2 + 2\lambda|\Lambda|^2\tanh^2(m\frac{\tau - \tau_i}{\sqrt{2}}))u_B = 0. \tag{4.106}
\]

We will calculate the transmission and reflection coefficients of the above equations, which will then provide the amount of particle production for each case.

First we will solve the differential equation (4.106) governing the evolution of the antiparticle wave function [59]. We define:

\[
\kappa^2 = \frac{k^2}{\rho^2}; \tag{4.107}
\]

and a change of variable as before

\[
y = \rho(\tau - \tau_i), \tag{4.108}
\]

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gives
\[ \frac{d^2u_B}{dy^2} + \left[ \kappa_2^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2}\tanh^2(y) \right] u_B = 0. \] (4.109)

The solution to the above equation is given by
\[ u_B = \frac{1}{\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)} \left( e^x + e^{-x} \right)^{-1} \left( e^{\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)}(\tau - \tau_i)} + e^{-\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)}(\tau - \tau_i)} \right)} \] (4.110)

where \( a, b \) and \( c \) are given by
\[ a = -4i \left( \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)} - \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4} \right)} \right) + \frac{1}{2} \] (4.111)
\[ b = -4i \left( \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)} + \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4} \right)} \right) + \frac{1}{2} \] (4.112)
\[ c = -4i \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)} + 1. \] (4.113)

By using the properties of hypergeometric function \((\tau - \tau_i) \to \infty\) we get
\[ u_B = \frac{\mu_2}{\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2 \right)}} e^{-\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \frac{1}{4} \right)}}(\tau - \tau_i) + \frac{\nu_2}{\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \frac{1}{4} \right)}} e^{\sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \frac{1}{4} \right)}}(\tau - \tau_i) \] (4.114)

The amount of particle production for \( B_s(k, t) \) modes is given by
\[ n_{2k} = |\mu_{2k}|^2 = \left( \frac{\cosh(\pi \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4} \right)})}{\sinh^2(\pi \sqrt{\left( \frac{2\lambda|\Lambda|^2}{\rho^2} + \frac{1}{4} \right)})} \right)^2. \] (4.115)

In the similar fashion, the equation (4.105) can be solved using the following definition
\[ \kappa_1^2 = \frac{k^2 - (2\lambda|\Lambda|^2)}{\rho^2}, \rho^2 = \frac{m^2}{2} \] (4.116)

along with a change of variable given by:
\[ y = \rho(\tau - \tau_i). \] (4.117)

Hence, we get the evolution equation for \( u_A \) is
\[ \frac{d^2u_A}{dy^2} + \left[ \kappa_1^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2}\sech^2(y) \right] u_A = 0. \] (4.118)
The transmission coefficient [60] for this potential barrier is given by

\[ |T|^2 = \frac{\sinh^2(\pi \kappa_1)}{\left(\cos(\pi \sqrt{\frac{2(\lambda|\Lambda|^2)}{\rho} + \frac{1}{4}}) + \sinh^2(\pi \kappa_1)\right)^2}. \] (4.119)

The amount of particle production for \( A_s(k,t) \) modes is given by

\[ n_{1k} = |\nu_{1k}|^2 = \frac{\cos^2(\pi \sqrt{\frac{2(\lambda|\Lambda|^2)}{\rho} + \frac{1}{4}})}{(\sinh^2(\pi \kappa_1))^2}. \] (4.120)

The total integrated baryon asymmetry of the universe is given by

\[ N_B - N_{\bar{B}} = \int_0^\infty dk k^2 n_{1k} - \int_0^\infty dk k^2 n_{2k}. \] (4.121)

### 4.3 Results and Discussions.

![Figure 4.1](image)

Figure 4.1: Fig. 1. shows the variation of particles (dashed line) and antiparticles (solid lines) for \( \frac{2\lambda|\Lambda|^2}{m^2} = .25 \) as a function of comoving wave number \( k \)
Figure 4.2: Fig. 1. shows the variation of particles (dashed line) and antiparticles (solid lines) for \( \frac{2|\lambda|\Lambda^2}{m^2} = .26 \) as a function of comoving wave number \( k \).

We plot the co-wave number dependent number of baryons and anti-baryons in figure 1. and figure 2. for various values of the parameter \( \frac{2|\lambda|\Lambda^2}{m^2} \). As illustrated in the figures, the baryon asymmetry depends on the parameter \( \frac{2|\lambda|\Lambda^2}{m^2} \). We see that as the value of \( \frac{2|\lambda|\Lambda^2}{m^2} \) decreases the value of asymmetry decreases. This can be explained as in a rapidly expanding universe, \( \Lambda \) decreases and makes \( \frac{2|\lambda|\Lambda^2}{m^2} \) smaller, while, in a slowly expanding universe, \( \Lambda \) changes very slowly, therefore, the inflaton field stays in the instability bands for a longer time and thus gives rise to sufficient asymmetry.

On the other hand, for large amplitude oscillations, the parameter \( \frac{2|\lambda|\Lambda^2}{m^2} \) can be very large. In this case, resonance occurs for a broad range of values of the momentum and the amplification of the baryon modes and the suppression of the anti-baryon modes can become more efficient. Thus this parameter \( \frac{2|\lambda|\Lambda^2}{m^2} \) governs the range of momentum modes undergoing parametric amplification. From our model we have generated a baryon asymmetry in an entirely non-perturbative fashion. The approximations made are only at the end of the derivations and have been used to illustrate the methodology. It is to be noticed that the
equations (4.96-4.97) are exact for an oscillating inflaton field in an FRW Universe. It is entirely possible to solve these equations to incorporate the effects of the expansion of the Universe on the generation of baryon asymmetry. Since we are relying on parametric resonance for enhancement/suppression of the particle/anti-particle creation, the restriction to the lowest instability band of the Mathieu equation is a reasonable one.