CHAPTER 4

MODELING

4.1 INTRODUCTION

The Al alloy (1101) / Alumina particulate composites, Al (LM4) alloy/Alumina silicate particulate composites and Al (LM4) alloy/Alumina Silicate Short fiber composites were modeled using Micro Mechanics approach with the assistance of FEA software package ANSYS. Empirical model were also developed for all the combinations of above composites by using weibull statistical analysis for initial estimations.

4.2 FINITE ELEMENT ANALYSIS

The characterization of composite materials is fundamental to their reliable use. One of the methods (Hashin, 1983) is to determine the bounds on effective mechanical and thermal properties of particulate composites which assume the constituents to be isotropic and homogeneous. The bounds provide a range, as well as maximum error range, in predicting homogenized effective properties. In certain situations, the upper and lower bounds of the effective properties can be far apart and the technique is good only for linear properties. Constituent level numerical analysis techniques are time consuming to set-up, analyze as well as interpret the results. Composite micromechanics equations are simplified equations that are based upon the mechanics of materials approach. In this work, micromechanics equations are used to predict effective composite mechanical properties in an average sense
and the calculated values were used for Finite Element Analysis. In general Finite Element Analysis involves three stages of activity:

1. Pre-processing
2. Processing
3. Post Processing

Preprocessing involves the preparation of data, such as nodal coordinates, connectivity, boundary conditions, loading and material information. The processing stage involves stiffness generation, stiffness modification, solution of equations, resulting in the evaluation of nodal variables. The post processing stage deals with the presentation of results. Typically, the deformed configuration, mode shapes, temperature and stress distribution are computed at this stage. A complete finite element analysis is a logical interaction of the three stages. In this investigation, the FE Modelling and analysis was performed using ANSYS.

4.2.1 FEA for particulate MMC

The development of micromechanics equations for the particulate composites follows along the same lines as those for the continuous fiber reinforced composites (Mital et al 1996). In the case of particulate reinforced composites, the particles in various shapes and sizes are dispersed uniformly in the matrix material. Similar to the case of continuous fiber reinforced composites, where the fibers are assumed to be arranged in a regular array pattern like square or hexagon, the particulate material is assumed to be dispersed uniformly as spherical particles with a diameter that is the average value of the range of diameters in a cubic lattice. The distance between the neighboring particles is computed from the overall particle volume fraction.
The following assumptions have been made in the micromechanics of particulate composites:

- Each phase of the composite can be described by the continuum mechanics. Hence, the input parameters are moduli, Poisson's ratios of the individual phases.
- The interface between the particle and the binder has been assumed to be a perfect bond.
- The properties of individual phases are assumed to be isotropic.

This approach, represented by a set of simplified equations, satisfies the force equilibrium in all directions, and is able to capture the mechanics involved in the problem. This fact has been previously verified by the authors for continuous fiber reinforced composites and the comparison between the micromechanics predictions with the experimental values have shown good agreement. Elastic modulus, Shear Modulus and Poisons ratios of particulate metal matrix composites were calculated using the following equations and used as input for the Finite Element Analysis. The mass density of the particulate composite is given by a rule of mixture type of equation (equation 4.1):

$$\rho_{pc} = V_p \rho_p + (1-V_p) \rho_m$$  \hspace{1cm} (4.1)

where $V_p$ is the volume fraction of the particles, subscripts pc, p and m stand for particulate composite, particle and the matrix respectively. The normal modulus of the particulate composite is given by equation (4.2):
The shear modulus of the particulate composite is also given by a similar expression (equation 4.3):

$$G_{pc} = \frac{V_p^{0.67}G_m}{1-V_p^{0.33} \left(1 - \frac{E_m}{E_p}\right)} + (1-V_p^{0.67})G_m$$

Since, the constituents were assumed to have isotropic properties; the resulting particulate composite will also have isotropic properties. Therefore, the Poisson's ratio can be computed from the usual relationship (equation 4.4)

$$\nu_{pc} = \frac{E_{pc} - 2G_{pc}}{2G_{pc}}$$

### 4.2.2 FEA for Short fiber MMC

Equations used for describing the elastic and thermal characteristics of a lamina are, in general based on micromechanics formulations. An understanding of the interaction between various constituents is also useful in delineating the failure modes in the fiber reinforced composites. A thin lamina containing randomly oriented discontinuous fibers exhibits planar isotropic behaviour. The properties are ideally the same in all directions in the plane of the lamina. For such a lamina tensile and shear modulus and poisons ratio are calculated from the following equations (4.5), (4.6) and (4.7).
\[
E_{\text{random}} = \frac{3}{8} E_{11} + \frac{5}{8} E_{22} \quad (4.5)
\]
\[
G_{\text{random}} = \frac{1}{8} E_{11} + \frac{1}{4} E_{22} \quad (4.6)
\]
\[
\nu_{\text{random}} = \frac{E_{\text{random}}}{2G_{\text{random}}} - 1 \quad (4.7)
\]

where,

\[E_{\text{random}}\] - Elastic Modulus of randomly oriented discontinuous fiber reinforced Composite

\[G_{\text{random}}\] - Shear Modulus of randomly oriented discontinuous fiber reinforced Composite

\[\nu_{\text{random}}\] - Poisson's ratio of randomly oriented discontinuous fiber reinforced Composite

\[E_{11}\] - Longitudinal Elastic Modulus

\[E_{22}\] - Transverse Elastic Modulus

\[
E_{11} = \frac{1 + 2(l_f / d_f)\eta_n \nu_f}{1 - \eta_n \nu_f} E_m \quad (4.8)
\]

where

\[
\eta_n = \frac{E_f}{E_m} - 1 \quad (4.9)
\]

\[
E_{22} = \frac{1 + 2\eta_T \nu_f}{1 - \eta_T \nu_f} E_m \quad (4.10)
\]

where

\[
\eta_T = \frac{E_f}{E_m} - 1 \quad (4.11)
\]
Equations (4.8) to (4.11) are derived from the Halpin-Tsai equations with the following assumptions:

- Fibers are uniformly distributed throughout the matrix.
- Fiber cross section is circular
- Perfect bonding exists between fibers and matrix.
- The matrix is free of voids.
- The lamina is initially in a stress-free state.
- Both fibers and matrix behave as linearly elastic materials.

FEA model was developed and meshed with solid 46 elements using ANSYS as shown in Figure 4.1.

![FEA model](image)

**Figure 4.1 FEA model**

One end is fixed to constrain all translational and rotational degrees of freedom and the other end is subjected to uniaxial tensile load. Load values are taken from test results. FEA results are compared with experimental results and are presented in Chapter 6, 7 and 8.
4.3 WEIBULL MODELING

Although many papers as per the literature focus on the mechanical behavior of Al alloy matrix composites (AMCs), only few of them deals with the reliability of the composites. Generally, Aluminium alloy composites demonstrate various brittle fracture modes and then the strength distribution of the composites normally exhibits scatter in fracture strength depending on volume fraction of particulate and the fabrication process and condition. Therefore, the composites materials have not been frequently used as structural material due to their low reliability. As a consequence, the reliability and statistical strength analysis are more and more necessary for quality assurance. Most statistical strength analysis has been discussed using Weibull distribution function equation (Layden 1983, Chi 1984 and Martineau 1984). This equation is based on the theory in which the fracture is controlled by the weakest defect of all the defects in the materials, and is the so-called “Weakest link theory”. The statistical analysis was performed for tensile and impact strength data of stir cast Al alloy (1101) / Alumina particulate composites, Al (LM4) alloy/Alumina silicate particulate composites and squeeze cast Al (LM4) alloy/Alumina Silicate Short fiber composites in order to predict the reliability.

The term reliability in terms of probability of survival is used for the assessments of functional performances of a part under current service condition and in definite time period. The tensile strength of the composite materials was scattered widely because of their anisotropy. Hence safe life i.e reliability is an important parameter for this type of structure. Reliability means that “a material can be used without failure”. Weibull distribution is being used to model extreme values such as failure times and fracture strength. Two popular forms of this distribution are two-parameter and three-
parameter Weibull distributions. The (cumulative) distribution function of the three parameter Weibull distribution is given as follows (Hallinan 1993).

\[ F(x; \gamma, \alpha, \beta) = 1 - e^{- \left( \frac{x - \gamma}{\alpha} \right)^\beta}, \gamma \geq 0, \alpha \geq 0, \beta \geq 0 \]  

(4.12)

where \( \gamma, \alpha, \) and \( \beta \) are the location, scale and shape parameters, respectively. When \( \gamma = 0 \) in Equation (4.12) the distribution function of the two-parameter Weibull distribution is obtained. In this study, the two-parameter Weibull distribution, which can be used in fracture strength studies, will be considered. When \( \gamma = 0 \), the distribution function can be written as follows:

\[ F(x; \alpha, \beta) = 1 - e^{- \left( \frac{x}{\alpha} \right)^\beta}, \alpha \geq 0, \beta \geq 0 \]  

(4.13)

In the context of this study, \( F(x; \alpha, \beta) \), represents the probability that the fracture strength is equal to or less than \( x \). Using the equality \( F(x; \alpha, \beta) + R(x; \alpha, \beta) = 1 \), the reliability \( R(x; \alpha, \beta) \), that is, the probability that the fracture strength is at least equal to \( x \), is defined as (Dodson 1994)

\[ R(x; \alpha, \beta) = e^{- \left( \frac{x}{\alpha} \right)^\beta}, \alpha \geq 0, \beta \geq 0 \]  

(4.14)

The parameter \( \alpha \) and \( \beta \) of the distribution function \( F(x; \alpha, \beta) \) are estimated from observations. The methods usually employed in the estimation of these parameters are method of linear regression, method of maximum likelihood, and method of moments. Among these methods, linear regression is still common among practitioners, and hence was used for parameter estimation in this study. This method is based on transforming equation (4.13) into \( 1 - F(x; \alpha, \beta) = \exp \left( (x / \alpha)^\beta \right) \) and taking the double logarithms of both sides. Hence, a linear regression model in the form \( Y = mX + r \) is obtained:
\[
\ln\left[\ln\left(\frac{1}{1 - F(x; \alpha, \beta)}\right)\right] = \beta \ln(x) - \beta \ln(\alpha) \tag{4.15}
\]

\(F(x; \alpha; \beta)\) is unknown in equation (4.15) and, therefore, it is estimated from observed values: order \(n\) observations from smallest to largest, and let \(x(i)\) denote the \(i\)th smallest observation (\(i=1\) corresponds to the smallest and \(i = n\) corresponds to the largest). Then a good estimator of \(F(x(i); \alpha; \beta)\) is the Median Rank (M.R) of \(x(i)\):

\[
M.R = \left[\frac{(i - 0.3)}{(n + 0.4)}\right] \tag{4.16}
\]

When linear regression, based on least squares minimization, is applied to the paired values

\[
(X, Y) = \left[\ln(x(i)), \ln\left(\ln\left(\frac{1}{1 - F(x(i); \alpha, \beta)}\right)\right)\right] \tag{4.17}
\]

For the model in equation (4.15), the parameter estimates \(\alpha\) and \(\beta\) are obtained. In order to compute \(\alpha\) and \(\beta\), first, they are ordered from the smallest to the largest and \((X, Y)\) values are computed (Taljera 1981). Then applying linear regression to these \((X, Y)\) values, the linear regression model with the regression lines for various fractional volumes were obtained.

Apart from this, a detailed microstructure and SEM images were taken to understand the failure mechanism of composite material and are discussed in Chapter 6, 7 and 8.