APPENDIX 1

NELDER–MEAD SIMPLEX ALGORITHM

A simplex method for finding a local minimum of a function of several variables has been devised by Nelder and Mead (1965). It is the most popular direct search method since it does not require the calculation of derivatives. Nelder–Mead simplex (NMS) method is based on the comparison of the function values at the $n+1$ vertices for $n$-dimensional decision variables. The calculation at each time will generate a new vertex for the simplex. If this new point is better than at least one of the existing vertices, it replaces the worst vertex. The simplex vertices are changed through reflection, expansion, shrinkage and contraction operations in order to find an improving solution. The modifications in NMS algorithm are stated in (Tomick et al 1995) and the implementation procedure is clearly given in (Mathews and Fink 2004). NMS is applied to all power dispatch problems and their results are compared with results obtained using EAs.

For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. The process generates a sequence of triangles (which might have different shapes), for which the function values at the vertices become smaller and smaller. The size of the triangle is reduced and the coordinates of the minimum point are found. It is effective and computationally compact. Figure A1 shows a schematic of the Nelder–Mead algorithm in two dimensions and its explanation is as follows.
Figure A1.1 Illustration of the Nelder–Mead simplex algorithm

1. Initial triangle $BGW$: Let $f(x, y)$ be the function, that is, to be minimized. To start, we have three vertices of a triangle: $V_k = (x_k, y_k), \ k = 1, 2, 3.$ The function $f(x, y)$ is then evaluated at each of the three points: $z_k = (x_k, y_k), \ for \ k = 1, 2, 3.$ The subscripts are then reordered so that $z_1 \leq z_2 \leq z_3.$ We use the notation $B = (x_1, y_1), G = (x_2, y_2)$ and $W = (x_3, y_3)$ to help remember that $B$ is the best vertex, $G$ is good (next to best) and $W$ is the worst vertex.

2. Midpoint of the good side: The construction process uses the midpoint of the line segment joining $B$ and $G$. It is found by averaging the coordinates.

$$M = \frac{B + G}{2} = \left(\frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2}\right)$$

3. Reflection using the point $R$: The function decreases as we move along the side of the triangle from $W$ to $B$, and it decreases as we move along the side from $W$ to $G.$ Hence, it is feasible that $f(x, y)$ takes on smaller values at points that lie away from $W$ on the opposite side of the line between $B$ and $G.$ We choose a test point $R$ that is obtained by ‘reflecting’ the triangle through the side $BG.$ To determine $R,$ we first find the midpoint $M$ of the side $BG.$ Then draw the line segment from $W$ to $M$ and call its length $d.$ This last segment is extended a distance $d$ through $M$ to locate the point $R.$ The vector formula for $R$ is $R = M + (M - W) = 2M - W$
4. Expansion using point E: If the function value at $R$ is smaller than the function value at $W$, then we have moved in the correct direction towards the minimum. Perhaps the minimum is just a bit farther than the point $R$. So we extend the line segment through $M$ and $R$ to the point $E$. This forms an expanded triangle $BGE$. The point $E$ is found by moving an additional distance $d$ along the line joining $M$ and $R$. If the function value at $E$ is less than the function value at $R$, then we have found a better vertex than $R$. The vector formula for $E$ is

$$E = M + \eta(M - W)$$

where $\eta$ is called the expansion coefficient having the value greater than one.

5. Contraction using point C: If the function values at $R$ and $W$ are the same, another point must be tested. Perhaps the function is smaller at $M$, but we cannot replace $W$ with $M$ because we must have a triangle. Consider the two midpoints $C_1$ and $C_2$ of the line segments $WM$ and $MR$, respectively. The point with the smaller function value is called $C$, and the new triangle is $BGC$. Note that the choice between $C_1$ and $C_2$ might seem inappropriate for the two-dimensional case, but it is important in higher dimensions

$$C_1, C_2 = M \pm k(M - W)$$

where $k$ is called the contraction coefficient having the value less than one.

6. Shrink towards $B$: If the function value at $C$ is not less than the value at $W$, the points $G$ and $W$ must be shrunk towards $B$. The point $G$ is replaced with $M$, and $W$ is replaced with $S$, which is the midpoint of the line segment joining $B$ with $W$

$$S = \left( \frac{B + W}{2} \right) = \left( \frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right).$$
### APPENDIX 2

Table A1  Data for Case 2 with three fuel options

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APPENDIX 3

START

Create an initial population

Evaluate P(t)
Evaluate fitness of each population member and perform fitness scaling, if necessary

Create new offspring using selection and mutation operations
Evaluate C(t)

Gen = Gen +

Maximum Generation

NO

YES

STOP

Figure A3.1 Flowchart of GA
Figure A3.2 Flowchart of EP
Evaluation of fitness function

If fitness value < pbest in history, Current = New best

Set the best amongst all the pbests as the gbest

Calculate particle velocity

Update particle position

Iteration ++

STOP

Initialization

Iteration < Max. Iteration

Figure A3.3 Flowchart of PSO
Initialize the parameters $p, S, N_o, N_e, N_{re}, C_p, \theta (i,j,k), V_i$

Reproduction counter loop $k=k+1$

Chemotactic loop counter, $j=j+1$

Chemotactic step for each Bacteria count, $i=i+1$

Swim: moving in one direction Tumble: changing the direction

$\begin{align*}
V_{id}^{new} &= \omega V_{id}^{old} + c_i \phi_k \left( \theta_{i,j,k}^{new} - \theta_{i,j,k}^{old} \right) \\
\theta_{i,j,k}^{new} &= \theta_{i,j,k}^{old} + V_{id}^{new}
\end{align*}$

$\begin{align*}
i < S & \quad \text{YES} \\
\text{YES} & \quad \text{NO}
\end{align*}$

$\begin{align*}
V_{id}^{new} &= \omega V_{id}^{old} + c_i \phi_k \left( \theta_{i,j,k}^{new} - \theta_{i,j,k}^{old} \right) \\
\theta_{i,j,k}^{new} &= \theta_{i,j,k}^{old} + V_{id}^{new}
\end{align*}$

$\begin{align*}
\text{YES} & \quad \text{NO}
\end{align*}$

$\begin{align*}
\text{YES} & \quad \text{NO}
\end{align*}$

Print $J_{best}$

**Figure A3.4 Flowchart of BFA**
Figure A3.5 Flowchart of NSGA-II algorithm