Chapter 1

Introduction
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Theory of graphs and its applications have grown exponentially in twentieth century. The development of computer science and optimization techniques accelerated the research activity in the subject. At present this branch enjoys the status of one of the fastest growing fields of research with multifaceted applications ranging from electrical engineering to management science and computer science to social science. Also many branches of mathematics such as matrix theory, statistics, algebra, geometry and topology have closed bonding with graph theory.

There are many potential fields of research in graph theory. Algebraic graph theory, domination in graphs, enumeration of graphs, algorithmic graph theory and labeling of graphs are among worth to mention. The present thesis contains report of investigations concern to the concepts of graph labeling and domination in graphs.

The discussion is spreaded among six chapters. This first chapter is of introductory in nature which provides an overview of the remaining chapters.

Chapter - 2 is aimed to introduce definitions and basic concepts which are essential for the advancement of the topic.

The labeling of discrete structures is one of the emerging areas of research due to its diversified applications. Most of the graph labeling problems trace their origin with graceful labeling that was introduced by A. Rosa in 1960s. The graceful tree conjecture and numerous attempts to settle it, has paved the path for the initiation of many interesting graph labeling problems. Chapter - 3 is focused on graceful and harmonious labelings as well as some variants of these labelings. Some noteworthy results are listed below.

- Splitting graph of $B_{n,n}$ is a graceful graph.
- Splitting graph of $K_{1,n}$ is a graceful graph.
- Shadow graph of $B_{n,n}$ is an odd graceful graph.
- Splitting graph of $B_{n,n}$ is an odd graceful graph.
• Shadow graph of $P_n$ is an odd harmonious graph.

• Splitting graph of $P_n$ is an odd harmonious graph.

• Shadow graph of $B_{n,n}$ is an odd harmonious graph.

• Arbitrary supersubdivision of path $P_n$ is an odd harmonious graph.

• Joint sum of two copies of $C_n$ admits odd harmonious labeling for $n \equiv 0 (mod 4)$.

A weaker version of graceful and harmonious labeling is named as cordial labeling which was introduced by I. Cahit during 1987. This labeling is also known as equitable labeling, the generalization of equitable labeling was also introduced as $k$–equitable labeling by I. Chait in 1990. A brief account of cordial and 3-equitable labeling is provided in Chapter - 4. We prove that,

• Splitting graph of $B_{n,n}$ is a cordial graph.

• Degree splitting graph of $B_{n,n}$ is a cordial graph.

• Alternate triangular snake admits cordial labeling.

• Alternate quadrilateral snake admits cordial labeling.

• Degree splitting graph of $P_n$ is a cordial graph.

• Degree splitting graph of $S_n$ is a cordial graph.

• Degree splitting graph of $H_n$ is a cordial graph.

• Splitting graph of $K_{1,n}$ is 3-equitable graph.

• Splitting graph of $B_{n,n}$ is 3-equitable graph.

• Shadow graph of $B_{n,n}$ is a 3-equitable graph.
Some variants of cordial labeling are also introduced in recent time with added flavour of number theory. The Chapter - 5 contains discussion on some such variants namely prime cordial, divisor cordial and square divisor cordial. We investigate several results in the context of various graph operations such as duplication, switching of a vertex and degree splitting.

In any organizational network (traffic, electrical, communication etc.) often requires a set of nodes which can control or regulate the rest, in view of minimum use of resources. In graph theoretic terminology the problem is to find out a smallest subset $D$ of a vertex set $V$ of a graph $G$ such that every vertex $v \in V$ is either an element of $D$ or has a neighbour in $D$. In this situation we say that $D$ dominates the vertices of $G$ and $D$ is said to be a dominating set of $G$. The cardinality of smallest such set is called the domination number of graph $G$. Although the study of dominating set in graphs has received attention in 1960s, the subject has historical roots dating back in 1862 when de Jaeinsch studied the problem of determining the minimum number of queens which are necessary to dominate all the squares of $n \times n$ chessboard. The concept of domination number of a graph was introduced as ‘coefficient of external stability’ in a book entitled ‘Theory of Graphs and its Applications’ by C. Berge [18] during 1958. The terms ‘dominating set’ and ‘domination number’ were introduced by O. Ore [58] in his book entitled as ‘Theory of Graphs’. The notation $\gamma(G)$ for domination number of graph was introduced during 1977 in a survey paper by Cockayne and Hedetniemi. The concept of domination in graphs is well explored in the book ‘Fundamentals of domination in Graphs’ by Haynes et al. [44]. The same authors have edited a book [43] on some advanced topics on the same issue. Variety of domination models exist in literature and plenty of research papers are available in printed as well as electronic form.

The discussion embodied in last Chapter - 6 is based on domination theory. We have studied domination integrity of graphs which is a measure of vulnerability of graphs. We provide all the basics related to domination integrity of graphs and investigate some results in the context of some graph operations. We also conclude that the domination integrity of graph increases by the expansion of graph network.
Throughout the thesis we pose some open problems and give further scope of research. A list of symbols is given just after the table of contents while bibliography is provided alphabetically at the end in MLA format.

The work reported in the thesis is also published in scholarly, indexed and peer reviewed journals. The reprints of the published papers are provided in annexure.
List of Publications Arising From the Thesis

   ISSN: 2249-3328 (Print), 2319-5215 (Online)

   (http://dx.doi.org/10.5539/jmr.v3n4p21)
   ISSN: 1916-9795 (Print), ISSN 1916-9809 (Online)

   (http://www.scirp.org/journal/ojdm/)
   ISSN : 2161-7635 (Print), 2161-7643 (Online)

   ISSN: 1937 - 1055

   (http://www.veltechuniv.edu.in)
   ISSN: 2231-5330

   (http://ijmsc.com)
   ISSN: 2249-3328 (Print), 2319-5215 (Online)

(http://www.researchmathsci.org)

ISSN: 2279-087X (Print), 2279-088 (Online)


(http://www.malayjournal.org)

ISSN: 2319-3786


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ISSN: 2279-087X (Print), 2279-088 (Online)


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(http://jaem.isikun.edu.tr/index.php)

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ISSN: 2347-1557(Online)


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ISSN: 2070-0237 (Print), 2070-0245 (Online)


The reprints of published papers are provided as an annexure.
Details of the Work Presented in Conferences

1. The paper entitled as "Some New Families of Prime Cordial graphs" was presented in *Mathematics Meet - 2011* at Gujarat University, Ahmedabad, during February 3-5, 2011.

2. The paper entitled as "Some new results on prime cordial labeling" was presented in *The 7th Annual Conference of Academy of Discrete Mathematics and Applications (ADMA)* at National Institute of Technology, Calicut, during June 9-11, 2011.

3. The paper entitled as "Some New Odd Harmonious Graphs" was presented in *State Level Conference of Mathematics Celebrating The National Mathematics Year 2012* at Saurashtra University, Rajkot, on December 22, 2012.

4. The poster entitled "Prime cordial labeling of some wheel related graphs" was presented in *3rd International Conference of Discrete Mathematics (ICDM -2013)* at Karnataka University, Dharwad, during June 10-14, 2013.

5. The paper entitled "Domination Integrity of Shadow Graphs" was presented in *National Seminar on Analysis, Geometry and Applications* at Department of Mathematics, Sardar Patel University, Vallabh Vidyanar, during February 17-18, 2014.

6. The paper entitled "Domination Integrity of Total Graphs" was presented in *The 10th Annual Conference of Academy of Discrete Mathematics and Applications (ADMA)* at Reva Institute of Technology and Management, Banglore, during June 10 - 13, 2014.