CHAPTER 3: HEAT TRANSFER ANALYSIS OF GAS CORE REACTOR

Heat transfer in a nuclear rocket system is a complex engineering design aspect, since in the gas core nuclear reactors the propellant and coolant is same. The operational temperature is much higher compared to the conventional systems, and in the microgravity condition the process is not so favorable to the common operations. In order to analyze the specific process that takes place in space propulsion systems whereas gaseous core model, a numerical heat transfer analysis for the high temperature energy conversation system is need. The general heat transfer process from the fission to the propellant as well as to the system walls will be through conduction, convection and radiation. This study is focused on solving the heat transfer aspects numerically under the microgravity supercritical fission reactions. The reactor is modeled without having any control rods and the pressure will be continuously fluctuating to control and to slow down the neutrons and so that reactor will be under the control.

The idea is to improve the specific impulse of the nuclear rocket by allowing the reactor to generate power at temperatures much higher than the conventional reactors. The greatest challenge to operate gaseous core reactors under these conditions will be controlling the high temperature and as well as the supercritical fission. The wall cooling also needs to be taken care by the external cooling system to idealize the process, which is characterized by the convective flow of a radiating gas (Anghaie, 1996). The cooling is obtained by propellant itself in the reactor core model. The schematic diagram of flow process in gaseous core reactors is shown in the figure 3.1, the fuel is entered into the system with a minute inlet and the propellant gas passes through the reflector walls to absorb heat from the reflector and enters into the reactor chamber. Initially the liquid
hydrogen is pumped into the reactor walls and the absorption of heat from the reflector walls converts the hydrogen into gaseous state. The hydrogen enters into the buffer region through which the convective radiative heat transfer takes place.

Figure 3.1: Gaseous Core Reactor system with Propellant Inlet through radial direction

In this thesis two-dimensional axis-symmetry model was considered to analyses the radiative and convective problem. Two different cases are analyzed to compare the effectiveness of the process. In most of the cases hydrogen is selected as a propellant due to its low molecular weight. In some of the reactor models to maintain criticality even helium is considered due to its inertness. There thermo chemical reactions between the graphite reflector and the hydro have its
own disadvantages. From the experimental results in handling hydrogen at above 3500 K is quite difficult in a closed system (Anghaie, 1986). The facility yet to be developed to conduct experiments in this direction. The idea of analyzing hydrogen and helium is to choose effective fluid with more positive characteristics to operate in space reactors under the given pressure and temperatures ranges. The coupled solution might have given a better idea of even neutron behavior with the propellant.

3.1 NUMERICAL MODELING

The numerical analysis is conducted using computational fluid dynamics, with a commercially available package called ANSYS Fluent. This work is conducted at university of Petroleum and Energy Studies, which has research license for Andy’s fluent. The package works with the fundamental aspects of fluid dynamics by solving continuity, momentum equation, Energy Equation. The detailed explanation is given below with the equations solved to analyses each pentameter and the turbulence models are discussed. The model section based on the conditions while solving the problem is also included in the methodology. The heat transfer model solved in the problem and the boundary conditions used are included. The constant parameters with respect to the fuel and the propellant operational characteristics are added in the appendix d.

3.1.1 THE CONTINUITY EQUATION

The divergence form of the global continuity equations can be obtained by applying the law of conservation of mass to an infinitesimally small volume of fluid fixed in space. It is written in vector form as in equation 3.1
\begin{equation}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{3.1}
\end{equation}

In the Cartesian coordinate system, with \( u, v \) and \( w \) representing the \( x, y \) and \( z \) components, respectively, of the velocity vector \( \mathbf{V} \) and \( \rho \) representing the density of fluid, the above equation becomes

\begin{equation}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \tag{3.2}
\end{equation}

3.1.2 THE MOMENTUM EQUATIONS

The gradient form of the momentum equation can be obtained by applying Newton’s second law of motion to an infinitesimal control volume of fluid fixed in space. This momentum equation is the statement of the conservation of linear momentum of the fluid volume as can be written as

\begin{equation}
\frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = \rho \mathbf{f} + \nabla \cdot \mathbf{\Pi}_{i,j} \tag{3.3}
\end{equation}

In equation 3.3, \( \rho \mathbf{f} \) is the body force per unit volume and \( \mathbf{\Pi}_{i,j} \) is the stress tensor which consists of normal and shearing stresses which in turn are represented by the components of stress tensor as expressed in equation 3.4.
\[ \Pi_{i,j} = -p \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \delta_{ij} \lambda \frac{\partial u_k}{\partial x_k} \]  
\[ (i, j, k = 1, 2, 3) \quad (3.4) \]

Where \( p \) is the pressure and \( \delta_{ij} \) is the Kronecker delta function; \( u_1, u_2 \) and \( u_3 \) represents the three components of the velocity vector \( V \); \( \mu \) is the molecular viscosity coefficient and \( \lambda \) is the second coefficient of viscosity.

The molecular viscosity coefficient and the second viscosity coefficient are related to each other through the coefficient of bulk viscosity \( \kappa \), as hypothesized by Stokes, given in equation 3.5.

\[ k = \frac{2}{3} \mu + \lambda \]  
\[ (3.5) \]

However, the coefficient of bulk viscosity is often negligibly small for Newtonian fluids, yielding equation 3.6

\[ \lambda = -\frac{2}{3} \mu \]  
\[ (3.6) \]

With above relation the momentum equation can be rewritten with substantial derivative notation as equation 3.7.

\[ \rho \frac{DV}{Dt} = \rho \mathbf{f} - \nabla p + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \mu \frac{\partial u_k}{\partial x_k} \right] \]  
\[ (3.7) \]

Or,
\[ \rho \frac{DV}{Dt} = \rho \mathbf{f} - \nabla p + \frac{\partial}{\partial x_j} \left[ \tau_{ij} \right] \]

(3.8)

Where \( \tau_{ij} \) is the viscous stress tensor?

In Cartesian coordinate system, the above equation can be written, with \( u, v \) and \( w \) respectively as the \( x, y \) and \( z \) component of the velocity, as

X-momentum:

\[
\frac{\partial(\rho u)}{\partial t} + \nabla (\rho u \mathbf{V}) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x
\]

(3.9a)

Y-momentum:

\[
\frac{\partial(\rho v)}{\partial t} + \nabla (\rho v \mathbf{V}) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \rho f_y
\]

(3.9b)

Z-momentum:

\[
\frac{\partial(\rho w)}{\partial t} + \nabla (\rho w \mathbf{V}) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z
\]

(3.9c)

In above equations the components of the viscous stress tensor are given by

\[
\tau_{xx} = \lambda (\nabla \mathbf{V}) + 2\mu \frac{\partial u}{\partial x}
\]

(3.10a)
\[ \tau_{xy} = \lambda (\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial v}{\partial y} \]  
(3.10b)

\[ \tau_{xz} = \lambda (\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial w}{\partial z} \]  
(3.10c)

\[ \tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right] \]  
(3.10d)

\[ \tau_{xz} = \tau_{zx} = \mu \left[ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right] \]  
(3.10e)

\[ \tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right] \]  
(3.10f)

### 3.1.3 THE ENERGY EQUATION

The energy equation for viscous internal flows can readily be obtained by applying the law of conservation energy i.e. the first law of thermodynamics to an infinitesimally small volume of fluid fixed in space(Schnitzler, 1986). The energy equation in conservation form is given by equation 3.11.

\[ \frac{\partial}{\partial t} (\rho E) + \nabla (\bar{V} (\rho E + P)) = -\nabla \left[ \sum_{j} h_{j} J_{j} \right] + S_{h} \]  
(3.11)
Where $E_z$ is the total energy per unit volume of fluid and is given by

$$E_z = \rho \left( e + V^2 + \text{potential energy} + \text{vibrational energy} + \ldots \ldots \right)$$

(3.12)

With $e$ as the internal energy per unit mass. The first term of the left hand side of the energy equation 3.11 represents the rate of change of total energy per unit volume of the fluid while the second term on the same side is the energy lost per unit volume by convection through the control surfaces. The term $\frac{\partial Q}{\partial t}$ represents the rate at which heat is supplied to the unit volume of fluid and term $\nabla . q$ denotes the rate at which heat is lost through the control surfaces, per unit volume, by the process of conduction (Brengle, 1992). The heat transfer per unit volume $q$ is related to the temperature gradient by the Fourier Law expressed as

$$q = -k \nabla . T$$

(3.13)

Where $k$ the coefficient of thermal conductivity and $T$ is the temperature and the third and fourth term of the energy equation (3.11) represent the work done on the fluid per unit volume by the body forces and the surface forces respectively.

For a Cartesian coordinate system, the conservation form of the energy equation can be rewritten as
\[
\frac{\partial E}{\partial t} - \frac{\partial Q}{\partial t} - \rho(f_x u + f_y v + f_z w) + \frac{\partial}{\partial x} \left( E, u + pu - u \tau_{xx} - v \tau_{xy} - w \tau_{xz} + q \right) \\
+ \frac{\partial}{\partial y} \left( E, v + pv - u \tau_{xy} - v \tau_{yy} - w \tau_{yz} + q \right) \\
+ \frac{\partial}{\partial z} \left( E, w + pw - u \tau_{xz} - v \tau_{yz} - w \tau_{zz} + q \right) = 0
\]  
(3.14)

In the above equation, the heat flux vector,

\[ q = q_x i + q_y j + q_z k \]  
(3.15)

Where,

\[ q_x = -k \frac{\partial T}{\partial x} \]  
(3.16a)

\[ q_y = -k \frac{\partial T}{\partial y} \]  
(3.16b)

\[ q_z = -k \frac{\partial T}{\partial z} \]  
(3.16c)

3.1.4 THE ENERGY EQUATION FOR HIGH TEMPERATURE GAS

The energy equation and the closure equations given in section 3.1.3 are valid only up to moderate temperatures. In viscous flows it is generally associated with very high temperatures of the order of thousand degrees Celsius (Poston, 2006). As the temperature of the gas is increased to higher values, the assumption of calorifically perfect gas is no longer valid and the gas becomes thermally perfect. A thermally perfect gas is one whose specific heats are functions only of temperature.
Taking into account the effect of high temperatures, the governing energy equation should be modified to accommodate the diffusion terms. The total energy $E_t$ should now include the vibrational, rotational, translational and electronic energies as well (Kammash, 2005). The resultant Energy equation can be given in substantial derivative form by equation 3.21.

$$
\rho \frac{DE_t}{Dt} = -\nabla \cdot q - \nabla \cdot pV + \frac{\partial}{\partial x} (u\tau_{xx}) + \frac{\partial}{\partial y} (u\tau_{yx}) + \frac{\partial}{\partial z} (u\tau_{zx}) + \frac{\partial}{\partial x} (v\tau_{xy}) + \frac{\partial}{\partial y} (v\tau_{yy}) + \frac{\partial}{\partial z} (v\tau_{zy}) + \frac{\partial}{\partial x} (w\tau_{xz}) + \frac{\partial}{\partial y} (w\tau_{yz}) + \frac{\partial}{\partial z} (w\tau_{zx})
$$

(3.17)

Where the heat flux vector $q$ also includes the energy flux due to diffusion and radiation as given by equation 3.22

$$
q = -k\nabla T + \sum \rho_i U_i h_i + q_R
$$

(3.18)

In equation 3.22, the second term represents the energy flux due to diffusion is the summation energy fluxes due to diffusion of all species present in the mixture. The variables $\rho_i$, $U_i$, and $h_i$ respectively are the density, diffusion velocity and the enthalpy of the $i^{th}$ species in the mixture. The term $q_R$ represents the energy transport though the phenomenon of radiation.

3.1.5 THERMODYNAMIC PROPERTIES OF A CHEMICALLY REACTING MIXTURE

For most of the chemically reacting gases, each species in the mixture can be assumed to obey the perfect gas equation of state with negligible intermolecular
forces. Additionally the gas can be assumed to be a mixture of thermally perfect gases. The equation of state for a mixture of perfect gases can be given by

\[
p = \frac{R}{\mathcal{M}} T
\]

(3.19)

Where \( R \) is the universal gas constant (8314.34 J/kg mol K) and \( \mathcal{M} \) is the molecular weight of the mixture. The molecular weight of the mixture in equation (3.24) can be calculated using equation 3.24

\[
\mathcal{M} = \left( \sum_{i=1}^{n} \frac{c_i}{\mathcal{M}_i} \right)^{-1}
\]

(3.20)

In equation (3.20) \( c_i \) is the mass fraction of the \( i \)th species and \( \mathcal{M}_i \) is the molecular weight of each species.

The thermodynamic properties of a mixture of gases in thermo-chemical equilibrium is a function two state variable only viz. Temperature and pressure. The thermodynamic properties of a mixture of perfect gases in thermal equilibrium and chemical non-equilibrium on the other hand are dependent on the mass fraction of each species as well (Dunn, 1991). The specific enthalpy and specific heat of each species in the mixture are given respectively by equations 3.25 and 3.26.

\[
h_i = C_{1,i} T + h_i^0
\]

(3.21)

\[
c_{p,i} = C_{2,i}
\]

(3.22)

Where the coefficients \( C_{1,i} \) and \( C_{2,i} \) for each species is a functions of temperature and \( h_i^0 \) is enthalpy of formation of individual species. The coefficients for the
curve fits of piecewise polynomial variations of specific heats and enthalpy of individual species is readily available in the literature. The enthalpy and specific heat of the mixture of perfect gases in turn are given by equations 3.27 and 3.28.

\[ h = \sum_{i=1}^{n} c_i h_i \]  
(3.23)

\[ c_p = \sum_{i=1}^{n} c_i c_{p,i} \]  
(3.24)

3.1.6 TURBULENT FLOWS

The unsteady Navier-Stokes equations are generally sufficient to solve the turbulent flow field completely in a continuum regime. All levels of turbulence can be captured by the Direct Numerical Simulation (DNS) of the transient Navier-Stokes equations. The DNS require that all length scales of turbulence are resolved, from the smallest eddies to scales of the order of the physical dimensions of the problem under consideration. For the direct numerical simulation, all computations need to be done in three dimensions with grid and time step small enough to capture the small scale motions in a time accurate manner. These requirements put a large demand on the computer resources and such simulations are practically impossible for any real engineering problem with present day computer capabilities. Thus, the present day researches intend to capture the turbulence flow through the time averaged Navier-Stokes equations. In this statistical method, commonly called as Reynolds averaged Navier-Stokes (RANS) equations, the time averaging of flow variables is carried out in order to separate the time-mean quantities from the fluctuations (Marx, 1963). This averaging introduces new variables in the system of equations, thus require additional equations to close the system of equations. The new equations can be
formulated by what is called the *turbulence modelling*. Two types averaging is currently in use viz. classical Reynolds averaging and mass weighted averaging, of which the latter is primarily used for compressible flows.

### 3.1.7 EQUATIONS FOR TURBULENCE

The mass weighted (Favre) averages for any variable \( f \) is given by

\[
f = \overline{f} + f''
\]

(3.25)

Where the mean quantity \( \overline{f} \) and the fluctuating part \( f'' \) are respectively given by

\[
\overline{f} = \frac{\rho \overline{f'}}{\rho}
\]

(3.26)

\[
f'' = \frac{\rho f'''}{\rho}
\]

(3.27)

And the fluctuating part has the property

\[
\overline{\rho f'''} = 0
\]

(3.28)

With mass averages for the dependent variable the Navier-Stokes equations can be written as

Continuity equation:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \overline{u})}{\partial x} + \frac{\partial (\rho \overline{v})}{\partial y} + \frac{\partial (\rho \overline{w})}{\partial z} = 0
\]

(3.29)
Momentum Equations (x-only):

$$\frac{\partial}{\partial t} (\rho \hat{u}) + \frac{\partial}{\partial x} (\rho u \hat{u}) + \frac{\partial}{\partial y} (\rho u \hat{v}) + \frac{\partial}{\partial z} (\rho u \hat{w}) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} (\tilde{\tau}_{xx} - \rho \nu' u')$$
$$+ \frac{\partial}{\partial y} (\tilde{\tau}_{xy} - \rho \nu' v') + \frac{\partial}{\partial z} (\tilde{\tau}_{xz} - \rho \nu' w') \quad (3.30)$$

Where the mean viscous stresses \( \tilde{\tau}_{ij} \) can be given, neglecting the fluctuations in viscosity, as

$$\tilde{\tau}_{ij} = \mu \left[ \left( \frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \hat{u}_k}{\partial x_k} \right] + \mu \left[ \left( \frac{\partial \hat{u}_i^T}{\partial x_j} + \frac{\partial \hat{u}_j^T}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \hat{u}_k^T}{\partial x_k} \right] \quad (3.31)$$

Where \( i, j, k \) are dummy variables representing \( x, y \) and \( z \) directions respectively.

3.1.8 Energy equation:

The energy equation for turbulent flows in compact tensor notation employing Einstein summation convention can be written as

$$\frac{\partial}{\partial t} (\rho c_p \bar{T}) + \frac{\partial}{\partial x_j} (\rho c_p \bar{T} \hat{u}_j) = \frac{\partial \bar{p}}{\partial t} + \bar{u}_j \frac{\partial \bar{p}}{\partial x_j} + u'' \frac{\partial \bar{p}}{\partial x_j}$$
$$+ \frac{\partial}{\partial x_j} \left( k \frac{\partial \bar{T}}{\partial x_j} + \frac{\partial \bar{T}''}{\partial x_j} - c_p \rho \bar{T}' u_j' \right) + \Phi$$

(3.32)

Where
\[ \Phi = \tau_j \frac{\partial u_i}{\partial x_j} = \tilde{\tau}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \tau_j \frac{\partial u_i^\kappa}{\partial x_j} \]  

(3.33)

3.1.9 Turbulence Modelling

Turbulence models to close the Reynolds averaged N-S equations can broadly be divided into two groups depending on whether the model is based on Boussinesq assumption or otherwise. As per Boussinesq assumption, the apparent turbulent shearing stresses are related to the rate of mean strain through an eddy viscosity. For a general Reynolds stress, the Boussinesq assumption gives

\[ -\rho u_i u_j = 2 \mu_T S_{ij} - \frac{2}{3} \delta_{ij} \left( \frac{\mu_T \partial u_k}{\partial x_k} + \rho \bar{k} \right) \]  

(3.34)

Where \( \mu_T \) is the turbulent viscosity? The turbulent kinetic energy \( \bar{k} \) and the rate mean strain tensor \( S_{ij} \) in equation 3.54 are respectively given by

\[ \bar{k} = \frac{u_i u_j}{2} \]  

(3.35)

And

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

(3.36)

The above assumption is commonly known as eddy-viscosity approach. By applying the eddy-viscosity approach to the Favre-averaged Navier Stokes
equations, the dynamic viscosity coefficients in viscous stress tensor (equations 3.10 a-f) is simply replaced by the sum of a laminar and a turbulent component i.e.

$$\mu = \mu_L + \mu_T$$  

(3.37)

In above formulation, the laminar viscosity can be computed using the kinetic theory of gases or by some empirical formulations like the Sutherland’s formula. Similarly, using the Reynolds Analogy the thermal conductivity in equations 3.15 (a-c) can be evaluated as

$$k = k_L + k_T = c_p \left( \frac{\mu_L}{Pr_L} + \frac{\mu_T}{Pr_T} \right)$$  

(3.38)

Where $Pr_L$ and $Pr_T$ are laminar and turbulent Prandtl numbers corresponding to laminar and turbulent viscosities $\mu_L$ and $\mu_T$. Once the value of eddy viscosity $\mu_T$ is known, the Navier Stokes equations for turbulent flows can be solved by adding the eddy viscosity to the laminar viscosity terms.

The turbulence models that use Boussinesq eddy-viscosity assumption are referred as first order models and those not based on this assumption are referred to as second order models. Most of the engineering simulations at present are done with first order models (Goel, 1990). Further the first order models can be classified as zero-equation, one equation and two-equations depending on the number of closure equations.
3.1.10 SHEAR STRESS TRANSPORT (SST) K-\(\omega\) TURBULENCE MODEL

The second model used in this research, primarily to do the turbulence model independence study, is shear stress transport model proposed by Menter (Halloran, 1990). This model overcomes the freestream turbulence intensity dependence of the standard \(k-\omega\) model while retaining the robust near-wall formulation of the standard \(k-\omega\) model. The SST model incorporates the transport of turbulence kinetic energy \((k)\) and specific dissipation rate \((\omega)\).

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + \tilde{G}_k - Y_k + S_k \tag{3.54}
\]

and

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_i}(\rho \omega u_i) = \frac{\partial}{\partial x_j} \left( \Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + D_\omega + S_\omega \tag{3.55}
\]

Modeling the Production of Turbulence

In equation (3.11) the term \(\tilde{G}_k\) represents the production of turbulent kinetic energy and \(G_\omega\) in equation (3.52) represents the generation of \(\omega\), given respectively by equations (3.56) and (3.57)

\[
\tilde{G}_k = \min \left( G_k, 10 \rho \beta ^* k \omega \right) \tag{3.56}
\]

\[
G_\omega = \frac{\alpha}{v_r} \tilde{G}_k \tag{3.57}
\]

with
$$G_k = -\rho u_i u_j \frac{\partial u_j}{\partial x_i}$$ \hspace{1cm} (3.58)

Calculation of the Effective Diffusivity

The effective diffusivities of $k$ and $\omega$ appearing in equations (3.51) and (3.52) is computed using,

$$\Gamma_k = \mu + \frac{\mu_r}{\sigma_k} \hspace{1cm} (3.59)$$

$$\Gamma_\omega = \mu + \frac{\mu_r}{\sigma_\omega} \hspace{1cm} (3.60)$$

Where turbulent Prandtl numbers for turbulence kinetic energy and the specific dissipation rate $\sigma_k$ and $\sigma_\omega$ and the turbulent viscosity $\mu_r$ are computed respectively using the relations 3.61 (a)-(c).

$$\sigma_k = \frac{1}{F_1 / \sigma_{k,1} + (1 - F_1)/\sigma_{k,2}}$$ \hspace{1cm} (3.61a)

$$\sigma_\omega = \frac{1}{F_1 / \sigma_{\omega,1} + (1 - F_1)/\sigma_{\omega,2}}$$ \hspace{1cm} (3.61b)

$$\mu_r = \frac{\rho k}{\omega} \max \left[ \frac{1}{\alpha^*, \alpha_k, \alpha_\omega} \right]$$ \hspace{1cm} (3.61c)
In above equations the coefficient $\alpha^*$ and the blending functions $F_1$ and $F_2$ are the calculated respectively using,

$$
\alpha^* = \alpha_e^* \left( \frac{\alpha_e^* + \text{Re}_f / R_k}{1 + \text{Re}_f / R_k} \right)
$$

(3.62)

$$
F_1 = \tanh(\phi_1^*)
$$

(3.63)

$$
F_2 = \tanh(\phi_2^*)
$$

(3.64)

with

$$
\phi_1 = \min \left[ \max \left( \frac{\sqrt{k}}{0.09 \sigma y}, \frac{500 \mu}{\rho y^2 \omega} \right), \frac{4 \rho k}{\sigma_{\omega,2} D_\omega^* y^2} \right]
$$

(3.64)

$$
D_\omega^* = \max \left[ 2 \rho \frac{1}{\sigma_{\omega,2}} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-10} \right]
$$

(3.65)

and

$$
\phi_2 = \max \left[ \frac{2 \sqrt{k}}{0.09 \sigma y}, \frac{500 \mu}{\rho y^2 \omega} \right]
$$

(3.66)

where $y$ is the distance next to the surface and the $D_\omega^*$ is positive component of the cross diffusion term. Also the coefficient $\alpha_e$ appearing in equation (3.66) is evaluated as
\[ \alpha_{\infty} = F_1 \alpha_{\infty,1} + (1 - F_1) \alpha_{\infty,2} \]  

where, with \( \kappa = 0.41 \)

\[ \alpha_{\infty,1} = \frac{\beta_{1,1}}{\beta_{\infty}} - \frac{\kappa^2}{\sigma_{w,1} \sqrt{\beta_{\infty}^*}} \]  

\[ \alpha_{\infty,2} = \frac{\beta_{1,2}}{\beta_{\infty}} - \frac{\kappa^2}{\sigma_{w,2} \sqrt{\beta_{\infty}^*}} \]

3.1.11 HEAT TRANSFER MODEL

In the reactor core the heat transfer is going to happen through both convection and radiation. The energy equation is solved to account the rise in heat in the propellant region from the fission process. The study flow energy equation is expressed by using non dimensional numbers to account for nussult number for given geometry is expressed.

\[ Nu = F_1 \left( \text{Re, Pr, } M, \frac{\Delta T_{ad}}{T_W - T_b} \right) \]

It is expressed as a function of Reynolds number, Prandtl number, Mach number and total adiabatic stagnation temperatures rise. The energy equation solved to express the heat transfer in the given geometry is written as

\[ \frac{\partial}{\partial t} \left( \rho E \right) + \nabla \cdot \left( \tilde{v} (\rho E + p) \right) = \nabla \cdot \left[ k_{\text{eff}} \nabla T - \sum_j h_j \tilde{J}_j + \left( \tau_{\text{eff}} \cdot \tilde{v} \right) \right] + S_h \]  

(3.71)
The energy accounted by $E$ and the viscous dissipation, sensible enthalpy and diffusion fluxes indicates heat transfer, which can be found from the turbulence model selected.

\[ E = h - \frac{P}{\rho} + \frac{v^2}{2} \]  \hspace{1cm} (3.72 a)

\[ h = \sum_{j} Y_j h_j \]  \hspace{1cm} (3.72 b)

\[ h_j = \int_{T_{ref}}^{T} c_{p,j} dT \]  \hspace{1cm} (3.72 c)

In the solution the boundary condition used at the wall is adiabatic wall with a constant wall temperatures, since in the space heat transfer is through convection and radiation and the reflector temperatures is also maintained with in the specified range. In the pressure based solution the species diffusion equation is also add to the solver in the form of diffusion energy source.

\[ \nabla \cdot \left[ \sum_{j} h_j \overline{J_j} \right] \]  \hspace{1cm} (3.73)

Radiation also included in the solution since the overall heat transfer is accounted as the convection and radiation together in a reactor core. The radiation equations are express below; since the radiative flux is comparatively large the convection and radiation are included as mixed phenomena

\[ Q_{rad} = \sigma \left( T_{max}^4 - T_{min}^4 \right) \]  \hspace{1cm} (3.74)

In the solution surface to surface radiation model is used since the heat transfer through radiation is from the plasma source and which is at the center of the core
and some of the heat is transferred through interactions of the regions and some of it is due to radiation between the fission sources to the buffer region. The equation written for accounting for radiation is in the form of

$$\frac{dI(r,s)}{ds} + (a + \sigma_s)I(r,s) = an^2 \frac{\sigma T^4}{\pi} \int_0^{4\pi} I(r,s')\Phi(s',s)d\Omega$$  \hfill (3.75)

In the solution fixed temperatures conditions are applied to the walls since the reflector temperatures need to be maintained within the limit and variations are more likely to be with propellant temperatures.

$$q = h_f(T_w - T_f) + q_{rad} \quad \hfill (3.76a)$$

$$q_{rad} = \varepsilon_{cw} \sigma(T_w^4 - T_f^4) \quad \hfill (3.76b)$$

In the fixed wall condition the fluid side local heat transfer coefficient is accounted along with the radiative fluxes. In the interface the heat transfer is accounted from solid cells and it can be expressed as

$$q = \frac{K_w}{\Delta n}(T_w - T_s) + q_{rad} \quad \hfill (3.77)$$

The radiative heat transfer equation can be solved based on surface absorption and the emission, absorption and scattering is accounted from

$$\frac{\partial I}{\partial x} + (\sigma + \sigma_s)I(r,s) = an^2 \frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi} \int_0^{4\pi} I(r,s')\Phi(s',s)d\Omega$$  \hfill (3.78)

Wall functions need to be calculated to define heat transfer coefficient to the source, based on the turbulent kinetic energy the energy equation is enabled to account the convection.
\[ h_{eff} = \frac{\rho \epsilon_{p} C_{\mu}^{\frac{1}{4}} k_{p}^{\frac{1}{2}}}{T^*} \]  

(3.79)

The dimensionless temperature need to be calculated to supply \( T^* \) to the above equation

\[
T^* = \frac{(T_w - T_p) \rho \epsilon_{p} C_{\mu}^{\frac{1}{4}} k_{p}^{\frac{1}{2}}}{q} = \left\{ \begin{array}{l}
\Pr y^* + \frac{1}{2} \Pr \frac{C_{\mu}^{\frac{1}{4}} k_{p}^{\frac{1}{2}} U_p^2}{q} U_p^2 (y^* < y_T^*) \\
\Pr \left[ \frac{1}{k} \ln(EY^*) + P \right] + \frac{\rho \epsilon_{p} C_{\mu}^{\frac{1}{4}} k_{p}^{\frac{1}{2}}}{q} \{ \Pr_{T} U_p^2 + (\Pr - \Pr_{T}) U_{T}^2 \} (y^* > y_T^*)
\end{array} \right. \tag{3.80 a}
\]

\[
P = 9.24 \left( \frac{\sigma}{\sigma_{T}} \right)^3 \left[ 1 + 0.28e^{-0.007\sigma} \right] \tag{3.80b}
\]

The major assumption made while considering the radiation model is that the radiation intensity decomposed into series of spherical harmonics. The first term in the equation 3.80 b on right hand side indicates series represented in P1 model. It includes effect of scattering, while solving this equation it assumes all surfaces are diffuse. In the process if accounting radiation heat transfers in the mixed model the solver predicts the localized radiative heat fluxes and calculate the non-dimensional temperatures (Dunn, 1967). The variation in temperatures and pressure inside the reactor is varied in stream wise direction and the values are accounted for the specific heat flux. In calculating the heat fluxes the radial heat conduction is neglected, the propellant gas absorbs and emits radiation. The phase change of
the propellant takes place before entering into the reactor core itself, and the scattering effects of the propellants are neglected.

The axisymmetric form of mass averaged time-dependent compressible Navior-Stokes equations can be considered in the following form.

\[
\frac{\partial \vec{U}_i}{\partial t} + \frac{\partial \vec{F}_i}{\partial z} + \frac{\partial \vec{G}_i}{\partial r} = \frac{\partial \vec{G}_e}{\partial r} + \vec{H}
\]  

(3.81)

Where

\[
\vec{U}_i = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad \vec{F}_i = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho u v \\ (e + P)u \end{bmatrix}, \quad \vec{G}_i = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + P \\ (e + P)v \end{bmatrix}
\]  

(3.82)

Thermal and Viscous Source Terms are

\[
\vec{H} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q \end{bmatrix}
\]  

(3.83)

And

\[
\vec{G}_v = \begin{bmatrix} 0 \\ \mu_T \frac{\partial u}{\partial r} \\ 4 \mu_T \frac{\partial v}{\partial r} - \frac{2}{3} \mu_T \frac{v}{r} \\ \mu_T u + 4 \mu_T v \frac{\partial}{\partial r} q_r - q_r \end{bmatrix}
\]  

(3.84a)
Total viscosity

\[ \mu = \mu_m + \mu_e \]  

(3.85)

Fourier’s Law of conduction heat flux

\[ q_e = -k_e \frac{dT}{dr} \]  

(3.86)

Where

\[ k_e = C_p \left( \frac{\mu_m + \mu_e}{P} \right) \]  

(3.87)

\[ P = \rho RT, \quad e = C_v T, \quad h = C_p T, \quad \gamma = \frac{C_p}{C_v} \]  

(3.88)

The equations that can be obtained by using the above relations by interchanging the terms from equation number 3.85.

\[ P = (\gamma - 1) \left[ e - \frac{1}{2} \rho (u^2 + v^2) \right] \]  

(3.89)

\[ T = (\gamma - 1) \left[ \frac{e}{\rho} - \frac{1}{2} (u^2 + v^2) \right] \]  

(3.90)

\[ \frac{\partial}{\partial t} \left( \frac{\partial U}{\partial t} \right) + \frac{\partial A}{\partial z} + \frac{\partial B_t}{\partial r} + \frac{\partial B_v}{\partial r} = \frac{\partial H}{\partial t} \]  

(3.91a)

Where
\[ A_i = \frac{\partial F_i}{\partial U}, \quad B_i = \frac{\partial G_i}{\partial U}, \quad B_v = \frac{\partial G_v}{\partial U} \]  

(3.91 b)

The equations can be written in terms of nth coefficient to convert them into implicate form to find the Jacobian of \( A_i, B_i, B_v \)

\[ \Delta U^n = \Delta t \left( \frac{\partial U}{\partial t} \right)^n, \quad \Delta H^n = \Delta t \left( \frac{\partial H}{\partial t} \right)^n, \quad \delta U^{n+1} = \Delta t \left( \frac{\partial U}{\partial t} \right)^{n+1} \]  

(3.20)

3.2 GEOMETRIC MODELING AND GRID GENERATION

The problem taken for solving analysing the fluid flow and heat transfer, the geometries are constructed using Catia V5 and then imported into Gambit in using STEP format and scaled according to the dimensions of the problem. In gambit grid is generated, since its state of art pre-processor to support CFD problems. In gambit the interfacing of the mesh surfaces are easy and the quality of the mesh generated is quite attractive. It can accept different CAD models in diversified formats and coordinate miss matching is limited compared to the other solvers. The complex geometries can be created in the form of volumes and can be tightly integrated for the desired shape. The mesh generation part in the gambit modeller functions with automated size function driven tools for mesh generation. It can generate both structured and unstructured mesh with highest quality, the skewness values are below 40 percent and the ratio of elements can be taken as per the requirement of the solution domine.
The mesh generation part include creation of geometry and the specifying solver based on its units selection, mesh generation on the edges and creation of face mesh/ volume mesh. To examine the created mesh and to see the quality of the mesh at various places gambit options and it can be remodifyed. Finally zone assignment is done to set the boundaries, in case of fluid problems the flow boundary conditions, in heat transfer problems thermal boundary conditions can be set.

Figure 3.2: Model Used For Solving Heat Transfer Problem with the Fuel Region

Figure 3.3: Dense Grid Generated Inside the Axisymmetric Core Model
The geometry can be exported in various formats once the reaction and meshing and other things are completed based on solver the format will change. In case of fluent the gambit geometry need to be exported as msh file. In fluent msh file can be read and grid check and boundaries can be verified, if conditions need to be changed fluent have options for it. The model designed in the catia is shown in the figure below. The complete length of the chamber with the throat portion is considered with an axisymmetric model. The diameter of the section is at 3 meters and the throat is designed to have Mach 1. The dense mesh created on the geometry using gambit mesh generator is shown in the figure 3.3, the total number of elements created are 500000, based on grid independent study the number of elements are considered to be an effective, and the results are accurate for the given boundary conditions. At the walls the thermal boundary conditions is given so the grid generated with concentrated grid points.

3.3 SOLVER SELECTION AND BOUNDARY CONDITIONS

The equations governing fluid flow and heat transfer forms an initial boundary value problem, in order to solve such kind of problems we need to solve partial differential equations with the help of boundary conditions by iterative methods. The solver used in calculating the flow variations and heat transfer effects in gas core reactor chamber with various heat generation rates are analysis for different propellant properties. In solver due to the heat generation rate with the given geometry it predicts $p^*$ the pressure field in the flow domain and it solves the continuity, Momentum and Energy Equations. The values are correlated with $p^*$ and the final pressure rise at the throat due to the rise in kinetic energy is obtained from the solver. The velocity correlation is used from the mass flow at the inlet and the $u,v,w$ values are obtained. In case of convective and radiative heat transfer problems with a specific heat generation rate due to the source with moderated heat flux need to be treated as a special case of momentum equation to solve $\phi$ at
general boundary conditions. The solver follows implicit-explicit based finite volume method to discretize the fluid flow equations.

In the physical means of operating gas core reactor is based on nuclear fission and which involves subatomic particles and their behavior. This solution concentrated on solving Navier-Stokes equation on flow domain inside the reactor core, the fission part and the plasma part of the problem is solved in the next chapter to investigate the neutronics and fission energy rate. The idea of conducting numerical experimentation on the current model is to create the behavior of the propellants and to understand the operations conditions to obtain the peak values in case of different enrichments and the results are used to conduct neutronics analysis as an input the reactor core temperatures and pressure conditions are used along with the fuel temperatures maintained in the process. The scope of the problem is limited to deal with operations conditions of the reactor system and modeling uranium hexafluoride for fission cannot be considered in the solver due to the numerical limitations (Jack, 1961).

The fission region is considered as a gaseous uranium flow field and can be contained in the reactor chamber through vortex generation through the radial and axial entries of the fuel at different flow properties. In solution using turbulence model affects the accuracy of the solution, in the current problem k-ε model to obtain the turbulent kinetic energy effects throughout the length scale by using one and two equation models. In the solution the product of effective viscosity and the mean strain rates are replaced by mean momentum equation to with turbulent shear stress, this yield to faster convergence and the accurate solution. Diffusion equations are solved to calculate the convective and radiative heat transfer rates under high temperatures conditions. This problem considers rossland radiation model to obtain the flow patterns through radiation. The multiple inlets are provided for hydrogen to enter the chamber and fixed wall temperatures is used
to maintain the reflector temperatures within the upper limit it varied with the multiple cases and the consideration are made from Van Booman model to correlate reflector temperatures limitations for graphite. In one of the case the upper limit is set to be 2200k and in other case it was limited to 1900k. Since the reactor criticality affects by reflector temperatures selection. In figure 3.4 the detailed approach followed in solving N-S equations on a flow domain is explained using a flow chart. In the problem the heat transfer analysis between fissioning gas and propellant analyzed, for identifying the behavior in the core chamber a convective and radiative model is solved for different operating conditions. The consideration of different heat fluxes for hydrogen and helium and their operating pressure are tabulated below. In the analysis the solution is obtained based on set solution parameter and can be initialized on a modeled grid and the values are compared with the exact solution or experimental predictions.

In case of solving numerical heat transfer problems using convection and radiative flow domain, the geometrical modeling and grid generation plays an important role. In the current work a 2D axisymmetric model is developed with a structured grid capable of handling complete flow domain. Boundary conditions are specified on each edge of the computational 2D domain, the material properties are specified for the propellants used in the flow path. The variation of physical properties as per the change in temperature taken as a piecewise polynomial order and the coefficients are add to the case. The numerical procedure and the solution algorithms are presented in the figure 3.4, the starting values for the flow field in a given domain are supplied as intial conditions. The residuals can be thought of as a measure of how much the solution to a given transport equation deviates from exact and we monitor average residuals for the each transport equation solved. The convergence criteria depends on the solution methods incorporated with the transport equations.
Figure 3.4: Method of Solving Navier-Stokes Equations
The solutions are obtained by various trails to reach desired effectiveness and the result should be as accurate as possible. The initial boundary value problem is solved by iterations through partial differential equations applicable to the physics of the problem using a numerical method. Specific boundary conditions need to be identified with in the solver limitations based on the system application. If the solution gets converged with in the given limits results need to be verified across the different parameters. Most possible errors in numerical investigations are related to the discretization, geometry modeling. If the solution is not accurate the process of reconsidering the geometry or changing the boundary conditions will optimize the quality of the solution. Sometimes iterative convergence error occurs due to the limits of the solution and the CFL number is varied to reach convergence.

Table 3.1: Boundary Conditions Considered for Solution

<table>
<thead>
<tr>
<th>Type</th>
<th>Propellant</th>
<th>Thermal Boundary Condition $K(T_{wall})$</th>
<th>Heat Generation Rate $U$</th>
<th>U-C-F Enrichment %</th>
<th>Mass Flow rate of Propellant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Hydrogen</td>
<td>1900</td>
<td>1000 MW/m$^3$</td>
<td>50%</td>
<td>4.2 Kg/s</td>
</tr>
<tr>
<td>Case 2</td>
<td>Helium</td>
<td>1850</td>
<td>1000 MW/m$^3$</td>
<td>50%</td>
<td>4.2 Kg/s</td>
</tr>
<tr>
<td>Case 3</td>
<td>Hydrogen</td>
<td>1600</td>
<td>620 MW/m$^3$</td>
<td>30%</td>
<td>3.6 Kg/s</td>
</tr>
<tr>
<td>Case 4</td>
<td>Helium</td>
<td>1600</td>
<td>620 MW/m$^3$</td>
<td>30%</td>
<td>3.6 Kg/s</td>
</tr>
<tr>
<td>Case 5</td>
<td>Hydrogen</td>
<td>1200</td>
<td>280 MW/m$^3$</td>
<td>5%</td>
<td>3 Kg/s</td>
</tr>
<tr>
<td>Case 6</td>
<td>Helium</td>
<td>1200</td>
<td>280 MW/m$^3$</td>
<td>5%</td>
<td>3 Kg/s</td>
</tr>
</tbody>
</table>
Physical modeling errors are taken care after every solution, with the expected variations and the trends in the plots obtained are cross verified. In case of CFD problems the physical modeling errors can occur in considering the specific dimensions in geometry preparations. The second possibility is thought he boundary conditions selected and the models chosen. The only method of validation in case of geometry is through literature and comparing the dimensions with the work. The truncation error is generally visible in the solution because the partial differential equations are solved using approximate methods.

The problem is solved using the tabulated boundary conditions the geometry is considered from the parameters discussed through literature and the conditions are applied as per the need of the solution. There are six cases analyzed using CFD to identify the GCR core behavior, in that two different propellants are chosen. Most commonly used rocket propellant with less molecular weight is hydrogen and due to the inertness and in terms of higher heat handling capacity fluid as Helium as considered. The analysis is conducted based on the heat generation rate obtained in the reactor with the fuel enrichment and the peak values are chosen for conducting neutronics analysis. In case one, hydrogen is considered as propellant and the fuel enrichment is chosen at 50 % for generation rate of 100 MW/m$^3$ is selected in the similar manner the analysis is conducted on Helium gas for the same boundary conditions. The mass flow rate selected for all the cases kept similar to compare the Temprature and pressure variations in the core chamber with respect to the fission heat generation rate. Mixed boundary condition is used with the ideal gas density properties, and the thermal conductivity and viscosity are varied with the piecewise polynomial profile so that different zone are having different values as per the change in generation rate. The thermal boundary condition is set to the reflector wall to limit the Temprature, with in the desired ratio.
3.4 RESULTS AND DISCUSSIONS

Numerical heat transfer is an interesting study in nuclear reactors which operates at high pressure and high temperature conditions. In order to achieve higher rate of energy conversion from fission to the maximum Ve, the propellant selection and investigation of heat transfer in a GCR core can develop an idea for reactor operations. In case of GCR core propellant selection and the rate at which heat transfer taken place are investigated at different heat generation rates. An axisymmetric model was solved with a throat designed to operate at Mach 1, the stream lines of the flow domain is shown in the figure 3.4. The flow is smooth at the wall and the temperatures limitation applicable with a thermal boundary condition, back flow is visible near the vortex region to separate the propellant and the gaseous fuel from mixing.

![Figure 3.5: The Stream Lines of the Flow Pattern inside the core](image)

In case one the analysis is conducted for the GCR core which is operated with the 50% enriched gaseous fuel, the heat generation rate in case of GCR is obtained from the design parameters. The analysis is conducted using a pressure based solver for examining the core behaviour and the heat transfer between the fuel gases to the propellant. Radiative and convective heat transfer is considered with a temperature limitation to the core containment, since the propellant takes the heat from the walls and enters the reactor chamber.
Figure 3.6: Static Temperature in Variation in case 1 with Hydrogen Propellant

Figure 3.7: Total tempratures Variation in Case 1 with Hydrogen Propellant
In the case 1 Hydrogen is considered as a propellant and the static temperature and total temperature variations in case of 1000 MW/m³ generation rate. The maximum temperature at the core is reaching 10000 K and which is considered as a core reaction temperature in case of solving 50 % enriched fuel in neutronics. The temperature rise in the reactor core can be explain with the help figure 3.5, at the inlet hydrogen enters the core at 2000k and the it occupies the buffer region and the overall temperature rise for the working fluid is ranging up to 10000k. The idea of using hydrogen is take dual advantage as coolant to the reflector walls and as well as the rocket propellant. The variation in total temperature is also presented in the plot 3.6, which indicates the stagnation points and their temperature changes due to the reverse flow in the flow domain. The total loss in source temperature to the propellant is within the range of 300 K and the heat transfer is effective in case 1.

Figure 3.8: Enthalpy Change in Case 1 for Hydrogen Propellant
Figure 3.9: Total Energy Variation in case 1 with Hydrogen Propellant

Figure 3.10: Total Pressure Variation in Case 1 for Hydrogen case 1
The enthalpy change in the reactor system with respect to source is indicated in the figure 3.7; the variation is in j/kg due to the back flow from the source and the maximum enthalpy this variation is occurring due to the sensitiveness of the wall function in heat transfer analysis. The thermal boundary condition applied to the reflectors walls is to maintain Temperature constant, an isothermal boundary condition with a specified temperature based on the reflector wall temperature limitation is applied. In case 1 the temperature is limited to 2000 K, in reality it affects the criticality and the neutron reflection. But in case of highly enriched reactor cores the operational temperatures are very high and the propellant circulation rate in the buffer region also creates viscous heating. The energy levels are very much high at the center of the core, the radial position if we consider the maximum energy levels are at 6.38 MJ/kg, since the source energy levels are high the buffer region takes more heat from the fission. In case of radiation heat transfer the total radiation source temperature is at 10000 K and it is limited at the wall due to thermal boundary condition.

![Figure 3.11: Radiation Temperature Variation in Case 1 with Hydrogen](image)
The velocity magnitude in the GCR core for the case 1 is shown in the figure 3.12, the velocity variation is from 1.32 m/s to 11.2 m/s, at the throat maximum velocity is obtained. The stream lines of the propellant in a high temperature conditions for the case 1 is shown in the figure 3.13.
The core is exposed to the heat from the fission source and the total heat transfer is a result of both convection and radiation. The convective heat fluxes are the result of fluid flow near the source and the radiative heat changes with the radial distance. The flow velocity changes with the increase in temperature with the fluid and it becomes supersonic flow and it reaches Mach 1 at the throat. Since the ratio of the throat to the nozzle is considered as 1:36 in terms of Mach number. The incident radiation profile along the core is plotted in the figure 3.13 and which indicates the sudden rise in the radiation levels when the fluid reaches the source and the variation are depends on the scattering and the absorption of the total radiation emitted from the source. In case of gases the absorption coefficients is very less since the density of the high temperature gas is very less, as a same time the scattering coefficient is high.

Figure 3.14: Incident Radiation in GCR from the source in Case 1
In case 2 the boundary conditions are similar as case one and the mass flow inlet applied with a 3 kg/s mass flow rate and propellant is changed to helium to see the effectiveness of the heat transfer and the behavior of the core. In most of the gas cooled nuclear reactors helium is preferred as a coolant due to its inertness and due to less absorption coefficient. So that reaction with the core material is limited, in the current case graphite is used as a reflector material. In the solution two different thermal boundary conditions are used, in spite of source the isothermal wall limits the heat flow in the radial direction and the distribution of the heat for the propellant in the buffer region is comparatively low in case 2. The static temperature variation is visible in the graph 3.14, the variation the static temperature behavior in the core between two different propellants is related to their thermal conductivity.

Figure 3.15: Static Temperature in Case 2 with Helium Propellant
Figure 3.16: Total Temperature in Case 2 with Helium as a Working Fluid

Figure 3.17: Enthalpy Change in J/kg for Case 2
The total temperature variation and the static temperature variation there is likely to have a temperature difference both the working fluids. In case of hydrogen the thermal conductivity is high and the inlet temperature is kept similar for both the working fluids. In case of helium the possibility of allowing the working fluid at lower temperature can result better heat transfer. In case of enthalpy change the total heat content from the source is indicated at the core and the variation is visible along the buffer region. The total variation in the enthalpy and energy content between Inlet and to the outlet is in the range of 1.04 MJ/kg to 9.40 MJ/kg, some parts of the buffer region is also showing the limited values due to the stagnation in the flow created by the pressure difference. The total energy is high at the fission source and the energy distribution in the throat and the buffer region is comparatively limited. The maximum energy levels maintained inside the core is in the range of 1.62 MJ/kg to 6.38 MJ/kg, in the reservoirs region there is a bypass flow which also takes away the heat content.

Figure 3.18: Total Energy Variation in J/Kg in Case 2 with Helium
Figure 3.19: Radiation Temperature in case 2 with Helium

Figure 3.20: Velocity magnitude for the case 2 with Helium
The radiative heat transfer needs to be effective since there is no physical containment that differentiates the fuel flow and the propellant circulation region. The pressure difference govern the flow and the vortex is created to contain the fuel inside the chamber, the radiative heat contours are shown in the figure 3.18 the maximum temperature variation in case of helium is reaching 9760 K. It indicates that the source temperature is much higher and the heat transfer is effective. If we compare the similar operating conditions with the different working fluids in terms of effectiveness, hydrogen shows better performance since the absorption coefficient is comparatively low with the helium. The velocity variation is shown in the figure 3.20, the maximum velocity attained by the helium is in the reservoir region and it is reaching 12.0 m/s. The pressure variation in the case 2 is visible in the figure 3.21, the inlet pressure conditions of the working fluid is at 2.54 Mpa and the heat generation in the chamber is also increasing the total pressure to 7.42 Mpa.

Figure 3.21: Total Pressure Variation for Case 2 with Helium
The incident radiation in the case two is comparatively less with the case 1 since the absorption coefficients and the scattering coefficients of both the working fluids are varying. Since both are gases the variation is limited, but the overall change in density in the core is affecting the radiation. The incident radiation with respect to change in position along the core is plotted in the figure 3.22, at the inlet the variation is little effective and at the source it is reaching to peak and in the throat region it is low. The maximum incident radiation at the source is reaching at 3e03 MW/m³, which is trice from reactor heat generation rate at the interface. The solution is limited by constrains used in the solution, since the direct fission source cannot be modeled in the computational fluid dynamics approach. The incident radiation is limited at the wall boundaries due to the isothermal boundary condition and the solver limits the heat transfer.

Figure 3.22: Incident Radiation in Case 2 for Helium
The thermal boundary conditions applied to replicate the physical phenomena are likely to create the heat generation rate by constant heat generation rate at the source and the constant temperature at the wall boundaries. Due which the pressure variation is also occur only at the center of the core and the wall are maintained limited variations. The total pressure variation in along the core between the case 1 and case 2 are plotted in the figure 3.22 to compare the effectiveness of the working fluid inside the core. In case 1 the pressure variation is high at the source due to the high heat transfer rate and the variation in fluid density. The pressure variations are plotted in the figure 3.23 to understand the behavior of the working fluids along the length of the core.

Figure 3.23: Total Pressure Variation in Case 1 and Case 2 along the Core
Figure 3.24: Total Temperature Variation along the Length of the Core

Figure 3.25: Static Pressure Variation in Case 1 and Case 2
The hydrodynamics of the fluid motion affects the behavior of the working fluid under high temperature and pressure conditions. In hydrogen the molecular weight is less and the thermal conductivity is comparatively high and which is likelier to be preferred as a rocket propellant. The total temperature variation and the static pressure variation in both the cases are shown in the figure 3.24 and the 3.25, the thermal conductivity and the viscosity are varied with respect to the temperature since the piecewise polynomial coefficients are selected from the graphs shown in the appendix D. The incident radiation is compared in the graph shown in the figure 3.26, in case of hydrogen the incident radiation is comparatively low since the convective heat fluxes are more and the remaining heat is transferred to the radiation. In case of helium the radiative heat fluxes are high so and the convective heat fluxes are comparatively low.

Figure 3.26: Incident Radiation for case 1 and Case 2
In case 3 the propellant selected for solving 30 % enriched GCR core with a uniform density distribution of the fuel along the core. The heat generation rate at the interface is considered as constant with 620 Mw/m² with an isothermal boundary condition. The temperature is limited to 1600 K at the core reflector wall to improve the buffer region fluids kinetic energy; also it supports more neutron reflection behavior of the graphite material at lower temperatures. The inlet temperature reduced to 1800 K due to the variation in pressure and the pre heating temperature limitation. The convective and radiative fluxes are considered to generate the heat and to evaluate the heat transfer between the propellant and the fuel gas. The static temperature variation is shown in the figure 3.27, the maximum temperature attained in the fission region is at 7800 K, and the maximum temperature maintained is the propellant from the source is limited to 7250 K.

Figure 3.27: Static Temperature in Case 3 with Hydrogen Propellant
Figure 3.28: Total Temperature in Case 3 with Hydrogen Propellant

Figure 3.29: Total Enthalpy in Case 3 with Hydrogen Propellant
The total temperature variation along the stagnation points in the flow region is shown in the figure 3.28, in case 1 and 2 the maximum temperature raise is due to the higher energy content in the fission. The variation in the fuel enrichment is greatly affecting the total heat content and the generation rate. The advantage in using moderated generation rates with an affective temperature does help in maintain the GCR chamber within the limits and eventually the better thrust conditions can attain. In case 3 with the hydrogen propellant the maximum enthalpy maintained near the fuel surface is in the range of 8.73 MJ/kg. The ionization of propellant is a major problem in case of high temperature GCR core; many fuel inlet channels are added in operation at the throat region. In the current case the propellant flow is effectively maintained with in the targeted mass flow rate.

Figure 3.30: Radiation Temperature in Case 3 with Hydrogen Propellant
The heat transfer from the radiation can be compared for the current case from the figure 3.30, the maximum radiation temperature in the flow stream can be found at the source. The absorption coefficient $t$ of the hydrogen gas is low, as at the same time the scattering coefficient is high. In GCR core to get the advantage of high energy fission the heat transfer need to be affectively high through radiation so that the mixing for propellant and fuel can be minimized. The maximum temperature attained is at 7460K with an initial propellant temperature of 4020, with convective heat transfer. It indicates the radiative heat flux is higher by a fraction two compared from convective heat flux. Since the criticality of the reactor chamber is maintained at the designed value and pressure variations are limited due to the continuous generation of heat.

Figure 3.31: Total Energy in Case 3 with Hydrogen Propellant
Figure 3.32: Velocity Vectors in Case 3 with Hydrogen Propellant

Figure 3.33: Static Pressure in Case 3 with Hydrogen Propellant
In the boundary conditions of all the case the solution is obtained at the constant mass flow rate with the addition of flow channels in the reservoir region to compensate for the ionization of the propellant. The velocity magnitude is represented in the figure 3.32 and static pressure variations are shown in the figure 3.33. The maximum velocity obtained in the 30% enriched GCR core with a maximum temperature is at 12.4 m/s. The static pressure variations are from the input pressure of 1.84 Mpa is used and the pressure rise due to the fission and the wall flux is at 6.62 Mpa. At the higher pressure incident radiation scattering will be limited; in the current case the incident radiation is showing higher value along the core at 600Mw/m2.

![Incident Radiation in Case 3 with Hydrogen Propellant](image-url)

Figure 3.34: Incident Radiation in Case 3 with Hydrogen Propellant
In case 4, helium is selected for the same boundary conditions as the 30% enriched GCR core model heat generation temperature. The uniform fuel gas density is considered in the core to investigate the effectiveness of the heat transfer by the propellants. The behavioral changes that are going to be observed in the analysis is depends on the mode of heat transfer and the flux distribution along the radial length. In case 4 the heat generation rate is in the range of 620 MW/m$^3$, the maximum temperature reached with the calculated heat generation rate is shown in the below figure 3.35, it indicates the static temperature variation along the core. The maximum temperature at the center of the core is observed to at 7680 K with an inlet temperature of 1600 K. The isothermal temperature condition is enforced while calculating the below profile to improve neutronics in GCR core.

Figure 3.35: Static Temperature in Case 4 with Helium Propellant
Figure 3.36: Total Temperature in Case 4 with Helium Propellant

Figure 3.37: Total Enthalpy in Case 4 with Helium Propellant
In case 4 the difference is visible with the maximum temperature maintained in the core and eventually the kinetic energy generation is low. The energy levels indicate in the figure 3.39 shows the maximum energy at the interface between the fuel and the propellant with 5.67 MJ/kg, which is maintained to keep self-sustained fission with neutron criticality. In order to improve the specific impulse of the rocket system which operates with 30 % nuclear fuel enrichment the exhaust velocity need to be high. In the current case the maximum energy that is available at 5.04 MJ/kg can be directly covered in the nozzle. In the current case the maximum heat transfer is through radiation along the core, the figure 3.38 indicates the maximum radiation temperature between the sources to the propellant with 7150 K of interface temperature.

Figure 3.38: Radiation Temperature in Case 4 with Helium Propellant
Figure 3.39: Total Energy in Case 4 with Helium Propellant in J/kg

Figure 3.40: Velocity Magnitude in Vector form for Case 4
In order to maintain unique fuel composition the pressure need to be maintained in the reactor core; in case of helium absorption of heat at higher temperatures are more affective. In the middle section the propellant absorbs the heat to improve enthalpy of a buffer region fluid, where at the propellant surrounded in the wall region is adopted as a coolant. The wall cooling mechanism is more affective in the current case, since helium itself is an effective coolant in gas cooled nuclear reactors. The thermal wall condition applied in the current case to limit the reflector temperature is at 1600 K, so the buffer region fluid gets benefited more to improve the specific impulse. The mass flow rate is also varied along the core due to the variation in the core enrichment, in case 1 and 2 there is no targeted mass flow rate.

Figure 3.41: Static Pressure in Case 4 with Helium Propellant
In order to contain the nuclear fuel within the reactor chamber, without mixing with the propellant, a mechanism which is governed by the pressure difference is adopted. The static pressure variation shown in the figure indicates the maximum variation of the pressure in the current case with the 1.84 Mpa of inlet to the pressure rise of 6.54 Mpa. The idea is to improve the radiation heat transfer, so that the fuel is contained without mixing. The result obtained with the current case solution for incident radiation is shown in the figure 3.42, and which indicates the maximum incident radiation is at 5.8 MW/m$^2$ along the length of the core. This indicates the generation of energy from the GCR operates with the above mentioned values can capable of production 1200 Mwth energy to effective thrust with a moderated specific impulse.

Figure 3.42: Incident Radiation in Case 4 with Helium Propellant
In case 3 and case 4 the analysis is conducted to understand the behavior of two different propellants which are more suitable to use in gas core nuclear rockets. The inlet pressures are similar in both the cases and the variation starts from the source position as shown in the figure 3.43. In case 3 the value reaches slightly high compared with the helium propellant case. In variation is not so high in terms of throat functionality and still the ratio of 1:36 of Mach can be maintained but only difference visible is due to the rise in temperature in the fluid and the reverse flow in the reservoir region. In case of hydrogen the heat addition at the constant pressure is visible, so that is more likely to be as an isobaric process and the variation is happening at reservoir due to change in cross section. This pressure helps to maintain the optical thickness of the uranium gaseous fuels in the core.
Figure 3.43: Static Pressure in case 3 and Case 4

Figure 3.44: Total Temperature in Case 3 and Case 4
In case 3 and 4 the total temperature variation is indicating the difference in source temperature and the propellant at the reservoir region. The difference in both the cases is very limited since the generation rate is reduced by a fraction of 0.5 and that clearly shows the difference in incident radiation also. In case 3 the maximum incident radiation at the source is reaching to 6.8MW/m$^2$ and whereas in case of helium, since the convective heat flux is dominant the incident radiation is limited to 5.67 Mw/m$^2$. The total pressure variation trends show the maximum pressure rise in both the case at a value of 6.25 Mpa is attained in GCR core. The problem in maintain the higher radiative heat fluxes in GCR core is related to the reflector temperature limitation due to scattering of radiation.
In case 5, hydrogen is selected as a propellant for the 5 % enriched GCR core, to investigate the propellant behavior. The constant heat generation rate considered for the analysis is at 280 MW/m³, the material properties are based on piecewise polynomial and the scattering coefficients and absorption coefficients are considered for the higher temperature hydrogen from the experimental data at higher temperatures above 3500, the plots are added in the appendix D. The static temperature variations in the case five are shown in the figure below, which is changing from 1600 K to 5960 K. In the current case the inlet temperature of the hydrogen is at 1600 K with a specific mass flow rate of 3 kg/s. The variation in the temperature at the core is reaching around 4400 K and the maximum temperatures shown in the figure is in the fuel gas.
Figure 3.47: Static Temperature Variation in Case 5 with Hydrogen

Figure 3.48: Total Temperature Variation in Case 5 with Hydrogen Propellant
Figure 3.49: Total Enthalpy in Case 5 with Hydrogen Propellant

The total temperature variation shown in the figure 3.48 indicates the available temperature for the buffer region and the variation is uniform along the core with the maximum temperature of 4700 K. Since the propellant starts expending after the throat region the maximum pressure and temperature available is converted into the kinetic energy as the expense of loss of pressure in the supersonic nozzle used in the nuclear rocket. The total energy variation indicates the difference in source energy levels to the propellant energy in the buffer region. In order to analyze the thermodynamic performance of the gas core fission reactors, the total enthalpy change and the energy variation described the efficiency of the conversion system along with the thermodynamic cycle.
Figure 3.50: Radiation Temprature in Case 5 with Hydrogen Propellant

Figure 3.51: Total Energy Variation in Case 5 with Hydrogen Propellant
Figure 3.52: Velocity Magnitude Vector Plot for Case 5 with Hydrogen

The radiation temperature shown in the figure 3.50 indicates the maximum temperature attained by the radiation in the GCR core at 5060 K. The resection exposed to the incident radiation is direct and the counters are showing the maximum temperature at the wall in the radial direction also. Which need to be absorbed by the propellant, in case of obtaining the constant heat generation rate the free energy function is considered from the data, from the desired values of the composition mentioned in the table 4.2 the heat of formation is calculated. The operating pressure is obtained from the standard free energy formation from the fission in the range of 0.1 Mpa to 2.5 Mpa. Based on the energy release and the heat addition to the propellant operation pressure can be raised. The equilibrium compositions mentioned in the GCR analysis is with the U-C-F percentage variation and the attainable fission energies with the formed atomic ratios. The mole fraction of the condensed species is referred as a pure compound.
In case 5 the operating pressure obtained is in the range of 1.53 Mpa to 4.62 Mpa and the maximum pressure is obtained at the throat section of the GCR chamber. The variation in the radiative and the convective heat fluxes changes the behavior at the exit of the reactor. Whereas the radiative heat fluxes increases the wall temperature at the center of the core, due to less absorption coefficients of the hydrogen used in the reactor chamber. The incident radiation is shown in the figure below which indicates the uniform distribution of the temperature compared to the other cases in the current analysis. This is due to the reduction the source temperature due to the less enriched GCR core.
In case 6 Helium is considered as a propellant and the heat generation rate is considered for the 5% enriched gaseous fuel source and the same heat generate used in the case five with 280 Mw/m³. The behavior of the propellant is examined under the similar operating conditions for both the cases. In case of hydrogen the thermal conductivity is high and the rate at which heat is transferred through convection is creating higher temperature in the flow stream. In the current case the static temperature and total temperature variations are shown in the figures below. In case of helium as a working fluid the total temperature variation is visible from the inlet temperature to the maximum value of 4650 K at the source. Whereas the stagnation temperature is variation is high and which is touching 4900 K due to the back flow temperature from the pressure outlet.
Figure 3.55: Static Temperature in Case 6 Helium as a Working Fluid

Figure 3.56: Total Temperature in Case 6 with Helium as a Working Fluid
Figure 3.57: Enthalpy Variation in Case 6 with Helium Working Fluid in J/kg

The total enthalpy variation is presented in the figure, the maximum rise in enthalpy is occurring at the source with a 7.50MJ/kg. The variation in the enthalpy shows the uniform distribution of atomic density along the core, so that the pre fission conditions are met. A calculation of the actual atomic ratios at every location as per variation in the fuel gas density is not possible with the current solver. Since the resulting fuel gas mixtures with change in atomic density affects fission rate, so that total heat generation varies along the length of the core. Due to the limitations with the computational fluid dynamics solvers the uncoupled analysis need to be done as per the fission energy to heat levees inside the core and the constant generation rate need to be considered. This method of solving the problem needs a coupled solver which can predict the atomic variations as per the fuel gas mixture changes in the length of the core. This is beyond the scope of this particular research.
Figure 3.58: Total Energy Variation in Case 6 with Helium as a Working Fluid

Figure 3.59: Radiative Temperature in Case 6 with Helium as a Working Fluid

Figure 3.60: Vector Plot of Velocity Magnitude in Case 6 with Helium
The radiation temperature variation is shown in the figure, the maximum temperature is at the source and it is reaching 4400K since the inlet temperature of the propellant is comparatively low. In case six the inlet temperature maintained is at 1600 K, due to the limitation of the reflector wall temperature at 1200 K. The thermal boundary condition applies at the reflector wall is limited to 1600K as an iso thermal wall temperature condition. The velocity of the propellant at the reservoir region can be obtained from the figure 3.60, in the current case inlet mass flow rate is given at 3m/s and the targeted mass flow also calculate din the solution. In case gas core nuclear reactors which are used in propulsion applications the usage of propellant and the fuel gas should be optimum for given conditions. In case of hydrogen the molecular weight of the propellant is low so that more volume can be carried in the mission along with the pay load. In case of helium the limited propellant need to be used to obtain long distance travel.

Figure 3.61: Static Pressure Variation in Case 6 with Helium as Working Fluid
The static pressure variation in the current case is shown in the current figure 3.61, in case of helium propellant with 280 Mw/m$^3$ heat generation rate is used. The static pressure variation is in the range of 1.54 Mpa to 4.54 Mpa, the maximum pressure is attained at the source wall and the region distributed with the convective heat flux. The buffer region is maintained in the pressure range of 3.94 Mpa and the throat is having higher pressure. The incident radiation profile is shown along the length of the core at the fuel interface in the figure 3.62. The maximum incident radiation in the case of helium is at 8.5 Mw/m$^2$, in case of hydrogen the variation is uniform along the core. The variation is peak in case of helium due to less scattering coefficients in the flow field. The incident radiation is less at the reservoir and the throat region.

![Incident Radiation along Core in case 6 with Helium](image)

Figure 3.62: Incident Radiation along Core in case 6 with Helium
Figure 3.63: Static Pressure Variation in Case 5 and Case 6

Figure 3.64: Total Temperature Variation Case 5 and Case 6
In the case 5 and 6 the minimum enriched GCR core is considered, which can be practically implemented within the current means of the technology. The temperature and pressure profiles in both the cases indicate the material requirements to handle high temperatures and the core dimensions chosen for the fission is more comparable. In case 5 the static pressure is becoming constant after the reservoir region since the heat addition is comparatively after the mean free path length. The obtained results are comparable with the Dam, 1996 and the selection of graphite thickness and the core dimensions are more productive in heat transfer and the effective core pressure development. The total pressure and temperature profile between both the cases are shown in the figure 3.64.

Figure 3.65: Total Pressure Variation Case 5 and Case 6
The total pressure variation in both the cases maintains similar trends, whereas the case 5 the back flow pressure high so that the fall in pressure at the reservoir region can be observed from the figure 3.65. The above accomplishments clearly demonstrate the numerical model for convective and radiative heat transfer who has the ability to develop better specific impulse and the power density. The comparison of incident radiation in both the cases shows that the difference in absorption coefficients and the scattering do effect the radiative heat transfer at the core so that the difference in fluxes are Cleary justified. The power densities obtained at the core is in the range of 100 W/cc at the given generation rate. The analysis shows that the generation rate is strongly dependent on gas temperature. This behavior is due to the limitation on the wall temperature and the radiative heat fluxes dominate the phenomena.

Figure 3.66: Incident Radiation in Case 5 and Case 6
The enthalpy rebalancing is visible in all the cases and comparison of various heat generation rates with the two different propellants in six cases gives an effective idea over the convective radiative heat fluxes along the fuel wall. The behavior of the propellant is more important to investigate the neutronics and the heat removal process and core walls cooling are more important in case of super critical reactors. The results compared in all the cases are plotted between the hydrogen cases and the helium cases in the figure 3.67-3.73, it indicates helium behavior at higher temperature is more suggestible for GCR core and whereas at lower temperatures the heat transfer is not so effective. Helium is more advisable in case graphite core due to less reactions with the wall.

Figure 3.67: Pressure Variation along the Non-Dimensional Length
Figure 3.68: Bulk Temperature Variation along the Non-Dimensional Length

Figure 3.69: Density Variation in Hydrogen Used GCR Core for Different Cases
Figure 3.70: Radiative Heat Fluxes along the Non-Dimensional Length

Figure 3.71: Convective Heat Fluxes along the Non-Dimensional Length
Figure 3.72: Pressure Variation along the Non-Dimensional Length

Figure 3.73: Bulk Temperature along the Non-Dimensional Length
Figure 3.74: Propellant Gas Density variations along the Core Temperature

Figure 3.75: Radiative Heat Flux Variation in Hydrogen Propellant GCR
The total temperature variation and the total pressure variations are compared in the figure 3.74 and 3.55, due to the flow filed with high Reynolds numbers the thin layer Navier-Stoke equations are used in the solver with a k-epsilon turbulence model. In all the cases convective and radiative heat fluxes are obtained, in all the cases the results shows in the below figures that the radiation heat transfer is mode dominant GCR core. The bulk temperatures and the pressure variations are shown in the figures, which show when there is a change in the behavior of the fuel gas inside the core reactivity changes with the neutron multiplications factor. Because of above reasons there are few portions where the pressure profile started fluctuating. This depression in the pressure in the reservoir region indicates the back flow of the fluid due to the change in resection and due to the throat area. Convective heat flux in case of 50 % enriched GCR core is very high in both the case.
In the heat transfer analysis of a GCR the variation of entropy due to heat generation at various operating parameters is compared for the six cases in the figure 3.77. The maximum entropy generated in the higher enriched fuel core with the highest temperature and pressure variations with higher power density. The results obtained in this work on hydrogen and helium propellants convective and radiative heat transfer indicates the behavior of the core at various conditions. In all the cases radiation intensity dominated the convective heat fluxes. Which indicate the core operation without allowing the propellant and fuel gas mixing, along the center of the core in an axisymmetric model; Iso thermal boundary condition is more effective a constant heat generation source and to maintain the wall temperature with in the considered limit.