CHAPTER 5

MODELLING AND IMPLEMENTATION

5.1 INTRODUCTION

Effective problem formulation is fundamental to the success of all analyses. Problem formulation involves decomposition of the analytic problem into appropriate dimensions such as structures, functions, mission areas, command echelons, and modeling of systems. Problem formulation is an iterative process that evolves over the course of the study. It is essential even for small studies or where time is short – it will save time later and help ensure quality the problem formulation is a key part. In this research work the problem formulation is based on voltage stability improvement and real power loss minimization incorporating FACTS devices.

5.2 STATIC MODEL OF SVC

A variable susceptance $B_{SVC}$ represents (Ambriz-Perez 2000) the fundamental frequency equivalent susceptance of all shunt modules making up the SVC. This model is an improved version of SVC models. The circuit shown in Figure 5.1 is used to derive the SVC's nonlinear power equations and the linearised equations required by Newton's load flow method.

In general, the transfer admittance equation for the variable shunt compensator is

$$I_{SVC} = jB_{SVC} V_j$$

(5.1)
and the reactive power is

$$Q_{SVC} = -V_j^2 B_{SVC}$$ \hspace{1cm} (5.2)

In SVC susceptance model the total susceptance $B_{SVC}$ is taken to be
the state variable, therefore the linearised equation of the SVC is given by

$$
\begin{bmatrix}
\Delta P_j \\
\Delta Q_j
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & \theta_j
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_j \\
\Delta B_{SVC}/B_{SVC}
\end{bmatrix}
$$ \hspace{1cm} (5.3)

![Figure 5.1 Variable susceptance model of SVC](image)

Figure 5.1 Variable susceptance model of SVC

At the end of iteration $i$ the variable shunt susceptance $B_{SVC}$ is
updated according to

$$B_{SVC}^{(i)} = B_{SVC}^{(i-1)} + (\Delta B_{SVC} B_{SVC})^{(i)} B_{SVC}^{(i-1)}$$ \hspace{1cm} (5.4)

This changing susceptance value represents the total SVC
susceptance which is necessary to maintain the nodal voltage magnitude at
the specified value (1.0 p.u. for this work).

5.3 \hspace{1cm} \text{STATIC MODEL OF TCSC}

TCSC is a series compensation component which consists of a
series capacitor bank shunted by thyristor controlled reactor (Kazemi and
Badrzadeh 2004). The basic idea behind power flow control with the TCSC is
to decrease or increase the overall lines effective series transmission impedance, by adding a capacitive or inductive reactance correspondingly. The TCSC is modeled as variable reactance shown in Figure 5.2. The equivalent reactance of line \( X_{ij} \) is defined as:

\[
Z_{ij} = R_{ij} + X_{ij}
\]

![Diagram of TCSC](image)

**Figure 5.2** Variable reactance model of TCSC

\[
X_{ij} = -0.8X_{\text{line}} \leq X_{\text{TCSC}} \leq 0.2X_{\text{line}}
\]  \hspace{1cm} (5.5)

where, \( X_{\text{line}} \) is the transmission line reactance, and \( X_{\text{TCSC}} \) is the TCSC reactance. The level of the applied compensation of the TCSC usually varies between 20% inductive and 80% capacitive.

### 5.4 STATIC MODEL OF SSSC

The SSSC can be operated without an external energy source as reactive power source with and fully controllable independent of transmission line current for the purpose of increasing or decreasing the overall reactive voltage drop across the transmission line and thereby controlling the electric power flow (Gyugyi et al 1997; Zhang and Zhang 2006; Motie birjandi and Sabzawari 2010).
The widely used power injection model of SSSC requires modification of the jacobian matrix and makes the Newton-Raphson Load Flow (NRLF) coding more complex. A new circuit elements based model of SSSC is utilized to control the line power flows and bus voltage magnitudes for voltage stability limit improvement. The new model of the SSSC changes only the bus admittance matrix and consequently reduces the coding of load flow problem incorporating SSSC simple. This converter performs the main function of injecting a controllable series voltage. The basic configuration of SSSC is depicted in Figure 5.3. The model of SSSC is also shown in Figure 5.4.

The real and reactive powers exchanged with the line by the series voltage inserted by SSSC are modeled as a negative resistance and reactance
connected in parallel. The negative resistance represents injection of real power and the reactance may be either capacitive or inductive depending on whether reactive power is delivered or absorbed.

The complex power exchanged by the series converter with the line is expressed as

$$S_c = V_{sc}I$$  \hspace{1cm} (5.6)

where $V_{sc}$ is the complex voltage injected by the converter and $I$ the current through the line given by

$$I = \frac{V_i \angle \delta_i + V_{sc} \angle \gamma - V_j \angle \delta_j}{Z_1}$$  \hspace{1cm} (5.7)

The active and reactive powers exchanged with the line are modeled as resistance and reactance associated as in equations (5.8) and (5.9).

$$R = \frac{V_{sc}^2}{P_{se}}$$  \hspace{1cm} (5.8)

$$X = \frac{V_{sc}^2}{Q_{se}}$$  \hspace{1cm} (5.9)

The elements $R$ and $X$ can be calculated directly using the following equations.

$$R = \frac{V_{sc}Z_1}{V_i \sin(\delta_i - \delta_j - \gamma) + V_j \sin \gamma}$$  \hspace{1cm} (5.10)

$$X = \frac{V_{sc}Z_1}{V_i \cos(\delta_i - \delta_j - \gamma) - V_j \cos \gamma + V_{sc}}$$  \hspace{1cm} (5.11)
The resistance R and the reactance X representing the effect of series voltage are transformed into their equivalent series combination. This makes the line simple with only series connected elements of the line ($X_L$) and the $R_{SSSC}$ and $X_{SSSC}$ denoting the SSSC.

$$R_{SSSC} = \frac{RX^2}{R^2 + X^2} \quad (5.12)$$

$$X_{SSSC} = \frac{XR^2}{R^2 + X^2} \quad (5.13)$$

### 5.5 LINE QUALITY PROXIMITY INDEX

Voltage stability can be assessed in a system by calculating the line based voltage stability index. Mohamed et al (1989) derived four line stability factors based on a power transmission concept in a single line. The value of line index shows the voltage stability of the system. The value close to unity indicates that the respective line is close to its stability limit and value much close to zero indicates light load in the line. An index was extracted in order to verify since this index is the most suitable one. The formulation begins with the power equation in a power system. Figure 5.5 illustrates a single line of a power transmission concept.

![Figure 5.5 Single line concept of power transmission](image)

**Figure 5.5 Single line concept of power transmission**
The power equation can be derived as;
\[
\frac{X}{\sqrt{V_i^2}} Q_i - Q_i + \left(\frac{X}{\sqrt{V_i^2}} P_i^2 + Q_j\right)
\] (5.14)

The line stability factor is obtained by setting the discriminant of the reactive power roots at bus 1 to be greater than or equal to zero thus defining the line stability factor, LQP as,
\[
LQP = 4 \left(\frac{X}{V_i^2}\right) \left(\frac{X}{V_i^2} P_i^2 + Q_j\right)
\] (5.15)

In this research work, this type of index is used to assess the proximity of voltage stability.

5.6 IMPLEMENTATION OF DE ALGORITHM

5.6.1 Representing an Individual

Each individual in the population is defined as a vector containing the values of control parameters including the size of the SSSC, TCSC and SVC.

5.6.2 Number of Individuals

There is a trade-off between the number of individuals and the number of iterations of the population and each individual fitness value has to be evaluated using a power flow solution at each iteration, thus the number of individuals should not be large because computational effort could increase dramatically. Individuals of 10, 20 and 50 are taken as an appropriate population sizes.
5.6.3 Feasible Region Definition

There are several constraints in this problem regarding the characteristics of the power system and the desired voltage profile. Each of these constraints represents a limit in the search space. Therefore the DE algorithm has to be programmed so that the individual can only move over the feasible region. For instance, the test system considered has 4 transmission lines with tap changer transformer. These lines are not considered for locating SSSC, leaving 37 other possible locations for the SSSC. In terms of the algorithm, each time that an individual’s new position includes a line with tap setting transformer, the position is changed to the geographically closest line (line without transformer). Finally, in order to limit the sizes of the SSSC units, the restrictions of level of compensation is applied to the individuals. The optimal parameter values of differential evolution algorithm shown in Table 5.1.

The proposed algorithms are run for 10 times and the best results and corresponding initial values are shown as optimal values of algorithm parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Individuals</td>
<td>50</td>
</tr>
<tr>
<td>Cross Over Constant</td>
<td>0.6</td>
</tr>
<tr>
<td>Scaling Constant</td>
<td>0.3</td>
</tr>
<tr>
<td>Number of Iterations</td>
<td>100</td>
</tr>
</tbody>
</table>
5.6.4 Integer DE

For this particular application, the position of individuals is determined by an integer number (line number). Therefore the individuals’ movement is approximated to the nearest integer numbers. Additionally, the location number must not be a line with tap setting transformer. If the location is line with tap setting transformer, then the individual component regarding position is changed to the geographically closest line without a tap setting transformer.

5.7 IMPLEMENTATION OF SFL ALGORITHM

Step 1: Select m, the number of memplexes and n, the number of frogs in each memplexes. Total frogs P = m x n. Generate required population (X_i), i=1 to P, by random generation. Evaluate the fitness f(X_i) of each frog and arrange them in ascending order of fitness.

Step 2: According to the fitness value, arrange the frogs in to memplexes (The first frog goes to the first memplex, the second frog goes to the second memplex, the m th frog goes to the m th memplex, the (m + 1) th frog goes back to the first memplex, and so forth.). Find the position of frogs with the best, worst fitnesses identified as X_b and X_w respectively and the global best X_g for all m-memplexes.

Step 3: Improving worst frog position: The local exploration is implemented in each memplexes, i.e., the worst performance frog (X_w) in the memplex is updated according to the following modification rule: Di = rand x (X_b - X_w), i=1 to m.
Accept $D_i$ if it is within $D_{\text{min}}$ and $D_{\text{max}}$, i.e., $D_{\text{min}} < D_m < D_{\text{max}}$, otherwise set to minimum or maximum limits of $D_i$. ‘rand’ is the random number generated between 0 and 1. The new position of the frog is updated as

$$X_{w}^{\text{new}} = X_{w}^{\text{old}} + D_i ; (D_{\text{min}} < D_m < D_{\text{max}}).$$

Then recalculate fit of this frog.

**Step 4:** If the fitness of $X_{w}^{\text{new}}$ is more than the fitness of $X_{w}^{\text{old}}$ then accept the $X_{w}^{\text{new}}$. Else generate new $D_i$ value with respect to global $X_g$:

$$D_i = \text{Rand} \times (X_h - X_w).$$

Accept $D_i$ if it is $D_{\text{min}}$ and $D_{\text{max}}$, otherwise set to minimum or maximum limits of $D_i$. The new position is computed by

$$X_{w}^{\text{new}} = X_{w}^{\text{old}} + D_i.$$ Again compute fitness of this frog.

If the fitness of $X_{w}^{\text{new}}$ is more than the fitness of $X_{w}^{\text{old}}$ then accept the $X_{w}^{\text{new}}$. Else randomly generate the new frog in place of $X_w$ within the acceptable frog limits.

**Step 5:** Repeat step 3 and 4 for all memeplexes. This completes one iteration. Now shuffle the frogs as per step 2.

**Step 6:** Repeat algorithm until the solution criterion is met or maximum number of iterations are completed. The solution criterion is $|X_{w}^{\text{new}} - X_{w}^{\text{old}}| < \epsilon$, where $\epsilon$ is the convergence tolerance. Stop. The optimal values of SFLA parameters are given in Table 5.2.

The proposed algorithms are run for 10 times and the best results and corresponding initial values are shown as optimal values of algorithm parameters.
Table 5.2 Optimal values of SFLA parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of frogs</td>
<td>50</td>
</tr>
<tr>
<td>Number of memeplexes</td>
<td>5</td>
</tr>
<tr>
<td>Number of frogs per memeplexes</td>
<td>10</td>
</tr>
<tr>
<td>No. of iterations</td>
<td>200</td>
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</table>

5.8 COMMON OBJECTIVES IN A POWER SYSTEM

- Active power cost minimization
- Active power loss minimization
- Minimum control shift
- Minimum number of controls scheduled

Examples of the associated equality and inequality constraints are:

- Limits on all control variables
- Power flow equations
- Generation/load balance
- Branch flow limits
- Bus voltage limits
- Active and reactive reserve limits
- Generator MVar limits
- Corridor (transmission interface) limits
- Generator MW limits
- Parameter limits of FACTS devices

5.9 FORMULATION

The objective of this work is to improve the voltage stability limit by minimizing real power loss, sum of load bus voltage deviation and sum of
line voltage stability index. An augmented objective function is formed with the three objective components and weights.

5.9.1 Objective Function

The objective function of this work is to find the optimal rating and location of TCSC and SVC which minimizes the real power loss, voltage deviation and maximizes the voltage stability limit. Hence, the objective function can be expressed as:

\[ F = \min \left\{ P_L + wV_D + (1-w)LQP \right\} \]  \hspace{1cm} (5.16)

where \( w \) is the weighing factor for voltage deviation and LQP index and is set to 10.

5.9.2 Real Power Loss Minimization (\( P_L \))

The total real power of the system can be calculated as follows

\[ P_L = \sum_{k=1}^{N_L} G_k [V_i^2 + V_j^2 - 2V_iV_j \cos(\delta_i - \delta_j)] \]  \hspace{1cm} (5.17)

where, \( N_L \) is the total number of lines in the system; \( G_k \) is the conductance of the line \( k \); \( V_i \) and \( V_j \) are the magnitudes of the sending end and receiving end voltages of the line; \( \delta_i \) and \( \delta_j \) are angles of the end voltages.

5.9.3 Load Bus Voltage Deviation Minimization

Bus voltage magnitude should be maintained within the allowable range to ensure quality service. Voltage profile is improved by minimizing the deviation of the load bus voltage from the reference value (it is taken as 1.0 p.u. in this work).
\[
V_D = \sum_{k=1}^{N_{PO}} \left| (V_k - V_{ref}) \right|
\]  
(5.18)

where \( V_k \) is the voltage of a specified load bus, \( V_{ref} \) is the voltage of the reference bus and \( N_{PO} \) is the total number of load buses in the proposed test system.

### 5.9.4 Line Voltage Stability Index Minimization

Voltage stability limit of a power system is increased by minimizing voltage stability index value. The indicator takes values between 0 (no-load) and 1 (full load). The line based stability index (LQP) is given as:

\[
LQP = \sum_{i=1}^{N_L} LQP_i
\]  
(5.19)

where \( N_L \) is the total number of lines in the proposed test system.

### 5.9.5 Constraints

The minimization problem is subject to the following equality and inequality constraints

#### 5.9.5.1 Equality constraints

**Load Flow Constraints**

\[
P_{Gi} - P_{Di} - \sum_{j=1}^{N_{ei}} V_i V_j Y_{ij} \cos (\delta_{ij} + \gamma_i - \gamma_j) = 0
\]  
(5.20)

\[
Q_{Gi} - Q_{Di} - \sum_{j=1}^{N_{ei}} V_i V_j Y_{ij} \sin (\delta_{ij} + \gamma_i - \gamma_j) = 0
\]  
(5.21)

where

\[ P_{Gi} = \text{ Real power generated power at bus } i \]
\[ P_{Di} = \text{Real power demand at bus } i \]
\[ Q_{Gi} = \text{Reactive power generated power at bus } i \]
\[ Q_{Di} = \text{Reactive power demand at bus } i \]
\[ V_{i} = \text{Voltage magnitude at bus } i \]
\[ V_{j} = \text{Voltage magnitude at bus } j \]
\[ Y_{ij} = \text{Admittance of the line conductor between bus } i \text{ and } j \]
\[ \hat{o}_{ij} = \text{Load angle of the line between bus } i \text{ and } j \]
\[ \gamma_{i} = \text{Magnitude of injected voltage at bus } i \]
\[ \gamma_{j} = \text{Magnitude of injected voltage at bus } j \]

### 5.9.5.2 Inequality constraints

**Reactive Power Generation Limit of SVCs**

\[ Q_{SVCi}^{\text{min}} \leq Q_{SVCi} \leq Q_{SVCi}^{\text{max}}, i\in N_{\text{SVC}} \quad (5.22) \]

**Reactance Limits of TCSCs**

\[-0.8X_{ij} \leq X_{TCSC} \leq 0.2X_{ij}, k\in N_{\text{TCSC}} \quad (5.23)\]

**Voltage Constraints**

\[ V_{i}^{\text{min}} \leq V_{i} \leq V_{i}^{\text{max}}, i\in N_{\text{B}} \quad (5.24) \]

**Transmission line flow limit**

\[ S_{i} \leq S_{i}^{\text{max}}, i\in N_{\text{I}} \quad (5.25) \]

where

\[ Q_{SVCi} = \text{Reactive power generation of } i \text{th SVC } (i = 1, 2\ldots N) \]
\[ N_{\text{SVC}} = \text{Number of SVC connected to the system} \]
\[ X_{\text{TCSC}_k} = \text{Reactance of the } k \text{ th TCSC (} k=1, 2\ldots N \) \]

\[ N_{\text{TCSC}} = \text{Number of TCSC connected to the system} \]

\[ V_i = \text{Voltage magnitude of bus } i (i = 1, 2\ldots N_B) \]

\[ S_i = \text{Transmission line flow of the } i \text{ th line (} i = 1, 2\ldots N_L \) \]

### 5.10 CONTINGENCY RANKING

Contingencies such as unexpected line outages often contribute to voltage collapse blackouts. These contingencies generally reduce or even eliminate the voltage stability margin. To maintain security against voltage collapse, it is desirable to estimate the effect of contingencies on the voltage stability margin. Action can then be taken to increase the margin so that likely contingencies do not cause blackout. Contingency Screening and Ranking (CS&R) is one of the important components of on-line voltage stability assessment. The purpose of contingency screening and ranking is to rapidly and accurately determine which contingencies may cause power system instability according to their severity. Suitable preventive control actions can be implemented considering contingencies that are likely to affect the power system performance.

### Table 5.3 Contingency ranking

(a) For IEEE 14 bus system

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<td>0.5795</td>
<td>1</td>
<td>0.6132</td>
<td>3</td>
<td>0.5869</td>
<td>1</td>
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<td>0.3826</td>
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<td>0.4185</td>
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<td>10</td>
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<td>0.3239</td>
<td>15</td>
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<td>15</td>
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### (b) For IEEE 30 bus system

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<th>SVSI</th>
<th>L&lt;sub&gt;mn&lt;/sub&gt;</th>
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### (c) For IEEE 57 bus system

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<th>SVSI</th>
<th>L&lt;sub&gt;mn&lt;/sub&gt;</th>
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### (d) For IU-NTPS 23 bus system

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<td>12</td>
<td>0.2287</td>
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<td>16</td>
<td>0.2164</td>
<td>12</td>
<td>0.2115</td>
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All the probable line outages of the test system are considered one at a time and the corresponding LQP values are calculated. The line whose outage leaves the system with highest value of LQP index is identified as the most critical line. The step by step procedure for contingency ranking is given below.

**Step 1:** Load flow is carried out for a particular operating condition and the total system losses and reactive power generation are calculated.
Step 2: Line outages are considered one at a time and the corresponding reactive power generation and losses are obtained by running load flow

Step 3: The reactive power generation and losses corresponding different line outages are sorted in descending order.

The most critical line is the line with rank 1 whose outage results in the highest value of reactive power generation and losses (highly stressed condition). The contingency ranking of lines with their respective LQP index values are compared with other line stability indices are given in Table 5.3 for the top 5 credible contingencies.

5.11 CONCLUSION

This chapter highlights the detailed models of SVC, TCSC and SSSC to make the formulation of FACTS devices which are related to this research work. The formulation of LQP index is also presented. Implementation of DE and SFLA and their relevant optimal values of parameters are presented. The various objective functions and constraints regarding this research work were demonstrated. Finally the contingency ranking and its procedure is also discussed in this chapter.