Chapter 5

Summary and Conclusions

The quantum mechanics (QM) is a very important branch of physics which explains behavior of microscopic particles. The Schrödinger equation (SE) plays the main role in the development of the QM. The solutions of the SE play an idealistic role for constructing a well behaved physical models in different branches of physics. The solutions of the SE can be obtained if we know the type of interaction potential exists among the particles within the physical system. Depending upon the behavior of these interactions, we come across various types of interaction potentials in nature. These may be simple or complex. As such the solutions of the SE can’t be found exactly for all types of interaction potentials. Till now, we just have a few simple physical systems, for which the SE can be solved exactly, such as a simple harmonic oscillator, Coulomb potential, step potential, pseudoharmonic and Mie potential. It is well known that the solutions of the SE for different types of physical systems exist in the nature are difficult to obtain. Therefore, analysis of the SE for complex systems is still an open problem for future research.

In a quest to solve the SE for various physical systems, various analytical, numerical and approximation techniques are developed. The researchers built the solutions for different physical systems by employing such techniques. The SE is studied for different physical potentials in lower as well as in higher dimensions, but even today it is the most fundamental equation to study the behavior of elementary particles. The research work is going on for applying the SE on many newly discovered physical systems such as the quark-antiquark systems, quantum dots and wires.

As, in general, the complexity of the underlying interaction potential increases with the increase in dimensions, so most of studies on the solutions of the SE are confined in lower dimensions i.e. upto 3D. But recently, many researchers extended the scope of solutions of the SE in N-dimensional space. These higher dimensional stud-
ies present the general treatment of the problems and can also provide the energy spectra of a given systems in lower dimensions. In particular, the multidimensional studies play the important role in the study of dynamics of quarks in particle physics and to explore the effect of gravity in astrophysics.

Keeping in view many interesting applications of the SE, the subject matter of present thesis was to investigate the solutions of the SE for the Coulomb perturbed potential (CPP) and to explore its applications in some newly investigated phenomena such as the quark-antiquark systems and quantum dots. In brief, the main outcomes of the present work are as follows.

5.1 Conclusions

In the introductory chapter one, a brief description of the role of the SE and its applications for various interaction potentials is highlighted. We have also discussed the solutions of the SE in lower as well as in higher dimensions for anharmonic potential models. A brief account of some important and widely employed analytic and approximation methods to obtain the solutions of the SE for a variety of physical models is given. We also introduced, in brief, a numerical method i.e. the finite difference method (FDM) in MATLAB programming for verifying the analytic and approximation solutions.

In chapter 2, we have obtained the approximate solutions of the Coulomb perturbed potential (CPP) via an appropriate ansatz to the wavefunction by employing the power series method (PSM) in higher dimensions i.e. in N-dimensional space. Here we found the recursion relation among series coefficients and the convergence condition for the given series and also discussed the inter-dimensional degeneracies in multidimensional space. The normalization constants $a_0$ and $a_1$ for the even and odd series wavefunctions respectively, are also computed. The same problem is also solved analytically by applying the Virial Theorem (VT). To check the efficacy of the obtained results by the PSM and VT methods, the problem was also solved numerically using the FDM in MATLAB programming. A close perusal of these results (cf. (Table 1)), reveals that the obtained analytic results using the PSM are in good agreement with the numerical results in lower dimensions and the results of other studies, however there is a small deviation in the results obtained via the VT and the exact one. This difference may be attributed to the averaging of kinetic and potential energies terms in VT. From this study, it is to be noted that, one can obtain the results in lower dimensions with good accuracy from the energy spectra in higher dimensional space.

In chapter 3, we considered the CPP to act as the interaction potential for the
heavy quarkonia ($Q\bar{Q}$). The mass spectra of heavy $Q\bar{Q}$ such as charmonium and bottomonium are obtained using the PSM in 3-dimensions. The calculated analytic results are compared with the numerical results (cf. Tables (3.1) and (3.2)) and found that results are in good agreement with each other and also with the results of other studies. But the drawback of the PSM is that, it gives a constraint condition on $n$ (cf. Eq.(3.12)) i.e. we can’t vary the $n$ freely. However, the constraint condition on $n$ does not appear, if the problem of heavy ($Q\bar{Q}$) is solved by employing the asymptotic iteration method (AIM). The AIM method is then employed for obtaining the energy spectra of heavy $Q\bar{Q}$ system asymptotically. From this study, we derived the mass spectra of heavy $Q\bar{Q}$ system in 3D. The problem is also analyzed numerically. It is found that, these results are in good agreement with other theoretical and experimental results (cf. Tables (3.3) and (3.4)).

Finally in chapter 4, we presented another application of the CPP to the quantum dots. If we consider the linear term to be zero in the CPP, then it acts as the confine potential for the quantum dots. Firstly, we solved the N-dimensional radial SE for the CPP by employing Ciftci and Hall Method. Then we reduced the general results (cf. Eq.(4.11)) in 3-dimensions. After that, we obtained the ground state energy spectra of 2-electron GaAs quantum dots in 2-dimensions in a magnetic field. Here, we also applied the AIM method for calculating the ground state energy spectra of a 2-electron InGaAs quantum dots in 2-dimensions in a magnetic field asymptotically. These obtained results for GaAs and InGaAs quantum dots conform the results of other studies (cf. Tables (4.2) and (4.3)).

5.2 Future Scope of the Present Study

In the present thesis work, a modest attempt has been made to explore the possibility of using the CPP as a potential candidate to get deep insight of physical problems. Therefore, the present study can further be enhanced on the following fronts:

- Current research trends indicate that, we can’t neglect the effect of higher dimensions on our daily life. So, one may develop the connections between the physical systems and higher dimensional space and see the effect of these extra dimensions on the properties of the physical systems.

- One may employ the Coulomb perturbed potential for obtaining other characteristics of heavy quarkonia such as mass spectra for the fine spectral terms and the decay constants of the heavy flavored mesons.

- One may apply the AIM method for asymptotic study of the other interaction potentials of physical relevance to further check its efficacy.
• One may study the spectral properties of some other quantum dots within the framework of the Coulomb perturbed potential as the confine potential.

• One may also analyze the applications of the Coulomb perturbed potential in other branches of physics where Coulomb potential is perturbed by some external forces.