Chapter-2

A CLASS OF RATIO TYPE ESTIMATORS FOR ESTIMATING THE POPULATION MEAN USING AUXILIARY INFORMATION

2.1 Introduction

The use of auxiliary information in survey sampling at the estimation state has an eminent role to increase the precision of estimators available in the literature. In this context ratio, product and regression methods of estimation are good examples [see Singh, S. (2003)]. When the population mean $\bar{X}$ of the auxiliary variable $x$ known, a large number of estimators of the population mean $\bar{Y}$ of the study variable $y$ has been suggested by several researchers, for instance, see Murthy (1967), Cochran (1977), Sukhatme et al. (1984), Singh, H.P. (1986), and the references cited therein. It is to be pointed out that the estimators reported in the literature have the same mean square error $(MSE)/minimum MSE$ (to the first degree of approximation) as that of the usual regression estimator.
In this chapter we have suggested a class of estimators for population mean in simple random sampling using auxiliary information on an auxiliary variable with its bias and mean square error (MSE) formulae under large sample approximation. Asymptotic optimum estimator (AOE) in the suggested class is contained with its properties under large sample approximation. We have shown that suggested class of estimators is more efficient than usual unbiased, usual ratio, usual product, usual linear regression, Srivenkataramana (1980) (Bandyopadhyay (1980)) dual to ratio, Srivastava (1974) (Prasad (1989)), Shivastava (1967) and Sharma and Tailor (2010) ratio-cum-dual to ratio estimators/class of estimators. In the support of the theoretically results we give an empirical study.

The objective of this chapter is to develop a class of ratio type estimator of the population mean $\bar{Y}$ of the study variable $y$ using auxiliary information on, auxiliary variable $x$ under simple random sampling without replacement (SRSWOR).

2.2 Reviewing estimators

Consider a finite population $U = (U_1, U_2, \ldots, U_N)$ containing $N$ units. Let $y$ and $x$ denote the study variable and the auxiliary variable taking values $y_i$ and $x_i$ respectively on the unit $U_i$ ($I = 1, 2, \ldots, N$) of popalatics U. Let $s$ denote a typical sample of size $n$ which was chosen by simple random sampling without replacement (SRSWOR) form the population $U$. We
assume simple random sample for convenience, since in many problem of this type will often be more purposeful see Meeden (1995, p. 71). It is well known that the sample mean \( \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \) is an unbiased estimator of the population mean \( \bar{Y} \) of the study variable \( y \) whose variance is

\[
MSE(\bar{y}) = Var(\bar{y}) = \frac{1-f}{n} S^2_y = \frac{1-f}{n} \bar{Y}^2 C^2_y, \tag{2.2.1}
\]

where \( f = \frac{n}{N} = \frac{N-1}{N} \sum_{i=1}^{N} (y_i - \bar{Y})^2 \) and \( C^2_y = \frac{S^2_y}{\bar{Y}^2} \).

When the population mean \( \bar{X} \) of the auxiliary variable \( x \) is known and the two variables \( y \) and \( x \) are positively correlated, the classical ratio estimator for the population mean \( \bar{Y} \) is defined by

\[
t_R = \frac{\bar{y}}{x}, \tag{2.2.2}
\]

where \( x = n^{-1} \sum_{i=1}^{n} x_i \).

In the same situation, Bandyopadhyay (1980) and Srivenkataramana (1980) suggested a dual to ratio estimator for the population mean \( \bar{Y} \) is

\[
t_d = \frac{\bar{y} \bar{x}^*}{\bar{X}}, \tag{2.2.3}
\]

where \( \bar{x}^* = (\bar{X} - n \bar{X})(N - n)^{-1} \).

To the first degree of approximation, the mean squared errors of the estimator \( t_R \) and \( t_d \) are respectively given by

\[
MSE(t_R) = \frac{1-f}{n} \bar{Y}^2 \left[ C^2_y + C^2_x (1-2K) \right] \tag{2.2.4}
\]
\[ MSE(t_d) = \frac{(1-f)}{n} \bar{Y}^2 \left[ C_y^2 + g(g - 2K)C_x^2 \right] \]  

where \( g = n(N - n)^{-1}, \ C_x^2 = \left( S_x^2 / \bar{X}^2 \right), \ S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2, \ K = \rho(C_y / C_x), \ \rho = \left( S_{xy} / (S_x S_y) \right) \) and \( S_{xy} = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})(y_i - \bar{Y}) \).

We note that the estimators \( \bar{y}_r, t_k \) and \( t_d \) are inferior to the usual regression estimator defined as

\[ \bar{y}_r = \bar{y} + \hat{\beta}(\bar{X} - \bar{x}), \]

where \( \hat{\beta} = (s_{xy} / s_x^2) \), \( s_{xy} = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \) and \( s_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{X})^2 \).

Then mean square error of the regression estimator \( \bar{y}_r \) to the first degree of approximation is given by

\[ MSE(\bar{y}_r) = \frac{(1-f)}{n} S_y^2 (1 - \rho^2) = \frac{(1-f)}{n} \bar{Y}^2 C_y^2 (1 - \rho^2) \]  

2.3 The class of estimators and properties

We consider the following class of estimators for population mean \( \bar{Y} \) of the study variable \( y \) as

\[ t = \lambda_1 \bar{y} \left( \frac{\bar{X}}{\bar{x}} \right)^\alpha + \lambda_2 \bar{y} \left( \frac{\bar{x}}{\bar{X}} \right), \]

where \( \alpha \) being scalar takes values (0, 1, -1) for designing the different estimators and \( (\lambda_1, \lambda_2) \) are suitably chosen constants to be determined such that \( MSE \) of \( t \) is minimum.
To obtain the bias and \( MSE \) of suggested class of estimators \( t \) we write

\[
\bar{y} = \bar{Y}(1 + e_0),
\]

\[
\bar{x} = \bar{X}(1 + e_1),
\]

such that

\[
E(e_0) = E(e_1) = 0
\]

and

\[
\begin{align*}
E(e_0^2) &= \frac{(1 - f) C_y^2}{n} \\
E(e_1^2) &= \frac{(1 - f) C_x^2}{n} \\
E(e_0 e_1) &= \frac{(1 - f) KC_x^2}{n}
\end{align*}
\]  \hspace{1cm} 2.3.2

Now we express suggested class of estimators \( t \) defined at (2.3.1) in terms of \( e \)’s as

\[
t = \lambda_1 \bar{Y}(1 - e_0)(1 - e_1)^\alpha + \lambda_2 \bar{Y}(1 + e_0)(1 + ge_1)
\]  \hspace{1cm} 2.3.3

Now we express \( |e_1| < 1 \) so that the term \((1 + e_1)^\alpha\) is expandable in term of power series. Thus by expanding the right hand side of (2.3.3) multiplying out and neglecting terms of \( e \)’s having power greater than two we have

\[
t = \bar{Y} \left[ \lambda_1 \left\{ 1 + e_0 - ae_1 - ae_0 e_1 + \frac{\alpha(\alpha + 1)}{2} e_1^2 \right\} + \lambda_2 \left\{ 1 + e_0 - ge_1 - ge_0 e_1 \right\} \right]
\]

or

\[
(t - \bar{Y}) = \bar{Y} \left[ \lambda_1 \left\{ 1 + e_0 - ae_1 - ae_0 e_1 + \frac{\alpha(\alpha + 1)}{2} e_1^2 \right\} + \lambda_2 \left\{ 1 + e_0 - ge_1 - ge_0 e_1 \right\} - 1 \right] \]  \hspace{1cm} 2.3.4
Taking expectation of both sides of (2.3.4) we get the bias of \( t \) to the first degree of approximation as

\[
B(t) = \bar{Y} \left[ (\lambda_1 + \lambda_2 - 1) + \frac{(1 - f)}{n} C_x^2 \left\{ \frac{\lambda_2}{2} (\alpha - 2K + 1) - \lambda_2 gK \right\} \right]
\]

2.3.5

Squaring both sides of (2.3.4) and neglecting terms of \( e \)'s having power greater than two we have

\[
(t - \bar{Y})^2 = \bar{Y} \left[ 1 + \lambda_1^2 \left\{ 1 + 2e_0 - 2\alpha e_1 + e_0^2 - 4\alpha e_1 e_0 + \alpha(2\alpha + 1)e_1^2 \right\} + \lambda_2^2 \left\{ 1 + 2e_0 - 2ge_1 + e_0^2 - 4ge_1 e_0 + g^2 e_1^2 \right\}
\]

\[
+ 2\lambda_1 \lambda_2 \left\{ 1 + 2e_0 - (\alpha + g)(e_1 + 2e_0 e_1) + \frac{\alpha}{2} (\alpha + 2g + 1)e_1^2 + e_0^2 \right\}
\]

\[
- 2\lambda_1 \left\{ 1 + e_0 - \alpha e_1 - \alpha e_0 e_1 + \frac{\alpha(\alpha + 1)}{2} e_1^2 \right\}
\]

\[
- 2\lambda_2 \left\{ 1 + e_0 - ge_1 - ge_0 e_1 \right\} \right].
\]

2.3.6

Taking expectation of both sides of (2.3.6) we get the \( \text{MSE} \) of suggested class of estimators \( t \) to the first degree of approximation as

\[
\text{MSE}(t) = \bar{Y}^2 \left[ 1 + \lambda_1^2 A + \lambda_2^2 B + 2\lambda_1 \lambda_2 C - 2\lambda_1 D - 2\lambda_2 E \right]
\]

2.3.7

where

\[
A = \left[ 1 + \frac{1 - f}{n} \left\{ C_y^2 + \alpha C_x^2 (2\alpha - 4K + 1) \right\} \right],
\]

\[
B = \left[ 1 + \frac{1 - f}{n} \left\{ C_y^2 + gC_x^2 (g - 4K) \right\} \right],
\]

\[
C = \left[ 1 + \frac{1 - f}{n} \left\{ C_y^2 + C_x^2 \left( \alpha \frac{2}{\alpha} \left( \alpha - \frac{4(\alpha + g)K}{\alpha} + 2g + 1 \right) \right) \right\} \right]
\]
D = \left[ 1 + \frac{1-f}{n} \frac{\alpha C^2}{2} (\alpha - 2K + 1) \right],

E = \left[ 1 - \frac{1-f}{n} gKC^2 \right].

Differentiating (2.3.7) with respect to \( \lambda_1 \) and \( \lambda_2 \) equating to zero we have

\[
\begin{bmatrix}
A & C \\
C & B
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix}
= \begin{bmatrix}
D \\
E
\end{bmatrix}.
\]

2.3.8

After simplification of (2.3.8) we get the optimum values of \( \lambda_1 \) and \( \lambda_2 \) as

\[
\lambda_1 = \frac{(BD - CE)}{(AB - C^2)} = \lambda_{10},
\]

\[
\lambda_2 = \frac{(AE - CD)}{(AB - C^2)} = \lambda_{20}.
\]

2.3.9

Thus the resulting minimum \( MSE \) of suggested class of estimators \( t \) is obtained as

\[
MSE_{\min}(t) = \left[ 1 - \frac{(BD^2 + AE^2 - 2CDE)}{(AB - C^2)} \right].
\]

2.3.10

Thus we state the following theorem.

**Theorem 3.1**: to the first degree of approximation,

\[
MSE_{\min}(t) \geq \left[ 1 - \frac{(BD^2 + AE^2 - 2CDE)}{(AB - C^2)} \right].
\]

with equality holding if

\[
\lambda_1 = \lambda_{10} \text{ and } \lambda_2 = \lambda_{20}.
\]
2.4 Particular case \((\lambda_1 + \lambda_2 = 1)\)

If we set \((\lambda_1 + \lambda_2 = 1)\) in (2.3.1) we get another class of estimators for the population mean \(\bar{Y}\) as

\[
T_1 = \lambda_1 \bar{y} \left( \frac{X}{\bar{X}} \right)^{\alpha} + (1 - \lambda_1) \bar{y} \left( \frac{x^*}{\bar{X}} \right) \tag{2.4.1}
\]

Putting \((\lambda_1 + \lambda_2 = 1)\) in (2.3.5) and (2.3.7) we get the bias and \(MSE\) of \(T_1\) respectively as

\[
B(T_1) = \frac{(1 - f)}{2n} \bar{y} \left[ \lambda_1 \alpha (\alpha - 2K + 1) - 2gK \right] \tag{2.4.2}
\]

\[
MSE(T_1) = \bar{Y}^2 \left[ 1 + B - 2E + \lambda_1^2 (A + B - 2C) - 2\lambda_1 (B - C + D - E) \right] \tag{2.4.3}
\]

The bias and \(MSE\) of \(T_1\) are minimized for

\[
\lambda_1 = \frac{(B - C + D - E)}{(A + B - 2C)} = \lambda_1^* \tag{2.4.4}
\]

Thus the resulting bias and \(MSE\) to \(T_1\) to the first degree of approximation are respectively given by

\[
B_n(T_1) = \frac{(1 - f)}{2n} \bar{y} \left[ \frac{(B - C + D - E)\alpha (\alpha - 2K + 1)}{(A + B - 2C)} \right] - 2gK \tag{2.4.5}
\]

\[
MSE_{min}(T_1) = \bar{Y}^2 \left[ 1 + B - 2E - \frac{(B - C + D - E)^2}{(A + B - 2C)} \right] \tag{2.4.6}
\]

\[
= \frac{(1 - f)}{n} S^2 \left( 1 - \rho^2 \right) = MSE(\bar{Y}_n).
\]

We now established the following theorem.
**Theorem 4.1:** To the first degree of approximation,

\[
MSE_{\text{min}}(T_i) = \bar{Y}^2 \left[ 1 + B - 2E - \frac{(B - C + D - E)^2}{(A + B - 2C)} \right]
\]

with equality holding if

\[\lambda_i = \lambda_i^*.\]

### 2.5 Efficiency comparisons

From (2.2.1), (2.2.4), (2.2.5) and (2.2.7) we have

\[MSE(\bar{y}) - MSE(\bar{y}_v) = \frac{(1-f)}{n} S^2 \rho^2 \geq 0, \quad 2.5.1\]

\[MSE(t_R) - MSE(\bar{y}_v) = \frac{(1-f)}{n} Y^2 C^2 (1 - K)^2 \geq 0, \quad 2.5.2\]

\[MSE(t_d) - MSE(\bar{y}_v) = \frac{(1-f)}{n} Y^2 C^2 (g - K)^2 \geq 0, \quad 2.5.3\]

If follows from (2.5.1) to (2.5.3) that the regression estimator \(\bar{y}_v\) is more efficient than \(\bar{y}, t_R\) and \(t_d\).

We note that the class of estimators \(t\) reduces to the following class of estimators of the population mean \(\bar{Y}\) as

\[t_1 = \lambda_1 \bar{y} \left( \frac{X}{\bar{x}} \right)^{\alpha} \quad \text{for} \quad (\lambda_1, \lambda_2) = (\lambda_1, 0), \quad 2.5.4\]

\[t_1^{(i)} = \lambda_1 \bar{y} \left( \frac{X}{\bar{x}} \right) \quad \text{for} \quad (\lambda_1, \lambda_2, \alpha) = (\lambda_1, 0, 1), \quad 2.5.5\]

\[t_2 = \lambda_2 \bar{y} \left( \frac{X^*}{\bar{x}} \right) \quad \text{for} \quad (\lambda_1, \lambda_2) = (0, \lambda_2), \quad 2.5.6\]
\[ t_{(a)} = \bar{y} \left( \frac{\bar{y}}{\bar{X}} \right)^{\lambda} \] for \( (\lambda_1, \lambda_2) = (1, 0) \) \hfill 2.5.7

\[ t_{ST} = \lambda \bar{y} \left( \frac{\bar{X}}{\bar{X}} \right) + (1 - \lambda) \left( \frac{\bar{X}}{\bar{X}} \right) \] for \( (\lambda_2, \alpha) = (1 - \lambda_1, 1) \) \hfill 2.5.8

It is to be mentioned that the class of estimators \( t_{(1)}, t_{(a)} \) and \( t_{ST} \) are due to Srivastava (1974) and revisited by Prasad (1989), Srivastava (1967) and Sharma and Tailor (2010) respectively.

To the first degree of approximation, the \( MSEs \) to \( t_1, t_{(1)}, t_2, t_{(a)} \) and \( t_{ST} \) are respectively given by

\[ MSE(t_1) = \bar{Y}^2[1 + \lambda_1^2A - 2\lambda_1D], \] \hfill 2.5.9

\[ MSE(t_{(1)}) = \bar{Y}^2[1 + \lambda_1^2A - 2\lambda_1D^*], \] \hfill 2.5.10

\[ MSE(t_2) = \bar{Y}^2[1 + \lambda_2^2B - 2\lambda_2E], \] \hfill 2.5.11

\[ MSE(t_{(a)}) = \bar{Y}^2[1 + A - 2D], \] \hfill 2.5.12

\[ MSE(t_{ST}) = \frac{(1 - f)}{n} \bar{Y}^2 \left[ C_2 + (1 - g)\lambda_1 + g \right] \left( (1 - g)\lambda_1 + g - 2K \right), \] \hfill 2.5.13

which are respectively minimized for

\[ \lambda_1 = \frac{D}{A} = \lambda_{10}^*, \text{ (say)} \] \hfill 2.5.14

\[ \lambda_1 = \frac{D^*}{A^*} = \lambda_{10}^{(1)}, \text{ (say)} \] \hfill 2.5.15

\[ \lambda_2 = \frac{E}{B} = \lambda_{20}^*, \text{ (say)} \] \hfill 2.5.16
\[ \alpha = K, \]  
\[ \lambda_i = \frac{K - g}{1 - g} = \lambda_{r}, \text{(say)} \]  

where

\[ A^* = \left[ 1 + \frac{1 - f}{n} \left( C_y^2 + C_x^2(3 - 4K) \right) \right] \] and \[ D^* = \left[ 1 + C_x^2(1 - K) \right]. \]

Thus the resulting minimum \( MSEs \) of \( t_1, t_1^{(l)}, t_2, t_{(\alpha)} \) and \( t_{ST} \) are respectively given by

\[ MSE_{\min}(t_1) = Y^2 \left( 1 - \frac{D^2}{A} \right), \]  
\[ MSE_{\min}(t_1^{(l)}) = Y^2 \left( 1 - \frac{D_{*}^2}{A} \right), \]  
\[ MSE_{\min}(t_2) = Y^2 \left( 1 - \frac{E^2}{B} \right), \]  
\[ MSE_{\min}(t_{(\alpha)}) = \frac{(1 - f)}{n} Y^2 C_y^2 (1 - \rho^2) = MSE(\bar{Y}_a), \]  
\[ MSE_{\min}(t_{ST}) = \frac{(1 - f)}{n} Y^2 C_y^2 (1 - \rho^2) = MSE(\bar{Y}_a). \]

From (2.5.7), (2.3.10), (2.5.22) and (2.5.23) we have

\[ MSE(\bar{Y}_a) \text{ or } MSE_{\min}(T) \text{ or } MSE_{\min}(t_{(\alpha)}) \text{ or } MSE_{\min}(t_{ST}) - MSE_{\min}(t) \]

\[ = Y^2 \left[ B - 2E - \frac{(B - C + D - E)}{(A + B - 2C)} + \frac{(BD^2 - 2CDE + AE^2)}{(AB - C^2)} \right] \]

\[ = Y^2 \left[ B(D - A) + C(C - D) + E(A - C) \right] \geq 0, \]
which shows that the proposed class of estimators \( t \) is more efficient than the usual regression estimator \( \bar{y}_{lr} \) (or \( T_1 \) or \( t_{(\alpha)} \) or \( t_{ST} \)) and hence it is more efficient than \( \bar{y}, t_R \) and \( t_d \).

Now from (2.5, (5.12), (5.19) and (5.21) we have

\[
MSE(t_{(\alpha)}) - MSE_{\text{min}}(t_1) = \bar{y}^2 \left(\frac{A-D}{A}\right)^2 \geq 0, \tag{2.5.25}
\]

\[
MSE(t_d) - MSE_{\text{min}}(t_2) = \bar{y}^2 \left(\frac{B-E}{B}\right)^2 \geq 0, \tag{2.5.26}
\]

It is observed from (2.5.25) and (2.5.26) that the estimators \( t_1 \) and \( t_2 \) are respectively better than Srivastava (1967) class of estimators \( t_{(\alpha)} \) and dual to ratio estimator \( t_d \) envisaged by Srivenkataramana (1980) Bandyopadhyay (1980). As the usual ratio estimator \( t_R \) is a member of Srivastava (1967) class of estimators \( t_{(\alpha)} \) for \( \alpha = 1 \), therefore the estimator \( t_t \) is also more efficient than the ratio estimator \( t_R \). Usual product estimator \( t_p = \bar{y} (\bar{x}/\bar{X}) \) is also the member of the Srivastava class of estimators \( t_{(\alpha)} \) for \( \alpha = -1 \), so the estimator \( t_1 \) is also more efficient than usual product estimator \( t_p \) which is due to Robson (1957) and revisited by Murthy (1964).

From (2.3.10), (2.5.14) and (2.5.16) we have
\[
MSE_{\min}(t_1) - MSE_{\min}(t) = \bar{y}^2 \frac{(AE - CD)^2}{A(AB - C^2)} \geq 0, \quad 2.5.27
\]

\[
MSE_{\min}(t_2) - MSE_{\min}(t) = \bar{y}^2 \frac{(BD - CE)^2}{B(AB - C^2)} \geq 0. \quad 2.5.28
\]

From (2.5.27) and (2.5.28) we note that the proposed class of estimators \( t \) is more efficient than the classes of estimators \( t_1 \) and \( t_2 \). It is to be mentioned that for \( (\lambda_1, \alpha) = (\lambda_1, 1) \) the class of estimators \( t_1 \) reduces to the Srivastava (1974) and Prasad (1989) estimator \( t_1^{(i)} \), as the proposed class of estimators \( t \) is more efficient than \( t_1 \) and hence it is more efficient than the estimator \( t_1^{(i)} \).

**2.6 Empirical study**

To judge the merits of the proposed class of estimators \( t \) over \( \bar{y}, t_R, t_d, t_1, t_1^{(i)}, t_2, T_1, t_{(\alpha)}, t_{ST} \) and \( \bar{y}_r \), we consider the five population data sets. The descriptions of population data sets are as follows

**Population I**: [Kadilar and Cingi (2006)]

- \( y \): level of apple production
- \( x \): the number of apple trees (1 unit = 1000 trees)

\( N = 104, n = 20, \bar{y} = 625.37, C_y = 1.866, C_x = 1.653, \rho = 0.865. \)
**Population II:** [Cochran (1977), p. 172]

\[ y: \text{production in bushels of peaches} \]
\[ x: \text{peach trees in an orchard} \]

\[ N = 256, \ n = 100, \ \bar{y} = 56.47, \ C_y = 1.42, \ C_x = 1.40, \ \rho = 0.89. \]

**Population III:** [Koyuncu and Kadilar (2009)]

\[ y: \text{number of teachers} \]
\[ x: \text{number of students} \]

\[ N = 923, \ n = 180, \ \bar{y} = 436.4345, \ C_y = 1.72, \ C_x = 1.86, \ \rho = 0.95. \]

**Population IV:** [Murthy (1967), p. 126-130]

\[ y: \ (x - \bar{X})^2 \]
\[ x: \text{geographical area} \]

\[ N=128, \ n=30, \ \bar{y} =11.9076, \ \bar{X} =5.5923, \ C_y=1.5288, \ C_x = 0.6171, \ \rho = 0.5782. \]

**Population V:** [Das (1988)]

\[ y: \text{number of agricultural labourers for 1961} \]
\[ x: \text{number of agricultural labourers for 1971} \]

\[ N = 278, \ n = 30, \ \bar{y} = 39.0680, \ C_y = 1.4451, \ C_x = 1.6198, \ \rho = 0.7213. \]

We have computed the percentage relative efficiencies (PREs) of different estimators (●) with respect to usual unbiased estimator \( \bar{y} \) by using following formula
for five population data sets described earlier and findings are summarized in Table 2.6.1.

Table 2.6.1 PREs of different estimators with respect to $\bar{y}$

<table>
<thead>
<tr>
<th>Estimator</th>
<th>PRE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>100.00</td>
</tr>
<tr>
<td>$t_d$</td>
<td>147.15</td>
</tr>
<tr>
<td>$t_2$</td>
<td>154.72</td>
</tr>
<tr>
<td>$t_R$</td>
<td>396.50</td>
</tr>
<tr>
<td>$t_1^{(r)}$</td>
<td>412.70</td>
</tr>
<tr>
<td>$y_b \left( or T_1 \ or \ t_{(\alpha)} \ or \ t_{ST} \right)$</td>
<td>397.18</td>
</tr>
<tr>
<td>$t \rightarrow t_3 \ for \ (\alpha = 1)$</td>
<td>412.84</td>
</tr>
<tr>
<td>$t \rightarrow t_4 \ for \ (\alpha = -1)$</td>
<td><strong>491.10</strong></td>
</tr>
</tbody>
</table>

Table 2.6.1 clearly exhibits that the estimator $t_3$ and $t_4$ which are members of suggested class of estimator $t$ performing better than the usual unbiased estimator $\bar{y}$, usual ratio estimator $t_R$, dual to ratio estimator $t_d$ [Bandyopadhyay (1980) and Srivenkataramana (1980)], usual regression estimator $y_b$, class of estimator $T_1$, class of estimator $t_{(\alpha)}$ [Srivastava (1967)], ratio-cum-dual to ratio estimator $t_{ST}$ [Sharma and Tailor (2010)], class of
estimators $t_2$ and class of estimators $t_i^{(1)}$ [Srivastava (1974) and Prasad (1989)] in the sense of having largest $PREs$ for all the population data sets. Thus suggested class of estimators is to be preferred in practice. However this conclusion cannot be extrapolated due to limited empirical study.

2.7 Conclusion

We have suggested a class of estimators for population mean of a study variable using in formation on an auxiliary variable in simple random sampling without replacement (SRSWOR). The properties of suggested class of estimators have been investigated under large sample approximation. Asymptotic optimum estimator has been identified with its mean square error formula. It has been shown that the suggested class of estimators is more efficient than some existing estimators such as usual unbiased estimator, usual ratio estimator, usual product estimator, usual regression estimator, dual to ratio estimator due to Bandhyopadhya (1980) and Srivenkataramana (1980), class of estimators due to Srivastava (1974) and Prasad (1989), class of estimators due to Srivastava (1967), ratio-cum-dual to ratio estimator due to Sharma and Tailor (2010)] and other estimators. An empirical study is carried out to throw light on the performance of the suggested class of estimators over already existing estimators. Further empirical studies carried out in this article clearly reflect the usefulness of the proposed class of estimators in practice.