CHAPTER 3

MODEL ORDER REDUCTION OF THE INTERVAL SYSTEM

3.1 INTRODUCTION

Industrial processes such as distillation column, gasifier system are modelled as higher order systems, since they have a large number of state variables. State variables are estimated as an interval number, because it affects by external disturbance, aging and measurement uncertainty. Hence it varies within a bound around a nominal value. Such processes are modelled as a higher order interval system. The main objective of this research work is to state the feasibility of controlling the large scale interval system via the control laws derived for the model with reduced dimensions.

A controller design is always difficult when the order is too high with interval coefficients. To overcome this problem, a model order reduction scheme is proposed in this research work. The proposed model reduction technique is based on truncation, where the numerator polynomial is reduced by retaining the initial time moments and the markov parameter. The denominator polynomials are reduced by using the differentiation technique. Each method has its own advantages and disadvantages. But the validity of the method is based on the closeness of the response that is with ISE and IAE values.
3.2 IMPLEMENTATION OF MOR

![Flowchart for model order reduction](image_url)

**Figure 3.1 Flowchart for model order reduction**

Figure 3.1 shows the sequence of operations, which has to be performed for model order reduction. Initially, a process has to be modelled as higher order model without eliminating any variables which affect the system performance. When the order of the system is high, then using the model order reduction technique, a reduced order model will be obtained. To validate the model order reduction technique, a controller is designed for the lower order model and is used to control the higher order model. The simulated output shows that the higher order model response and reduced order model response are close to each other. From the time response and frequency response plot the characteristics of the higher and reduced order models give similar responses.
3.3 BIASED MODEL ORDER REDUCTION TECHNIQUE

In this method, the denominator polynomial is reduced by using truncation technique and the numerator polynomial is reduced by keeping the initial time moments and Markov parameters. Hence, the key features of the higher order system are matched with the reduced order model by retaining its dominant poles. It is a computationally simple method compared to the Routh - Pade approximation and $\gamma$-$\delta$ method. But the major limitation is that, it can be applied only when the system model has the numerator polynomial, whose order is exactly one less than that of the denominator polynomial. If a system model has the order difference between the numerator and the denominator of more than one, any of the parameter, such as Markov parameter or time moment may not be retained in the reduced order model.

In the proposed model order reduction technique, the denominator polynomial is reduced by using the truncation technique, and the numerator polynomial is reduced by retaining the initial time moment and the markov parameter.

Let us consider the $n^{th}$ order interval model given by Equation (3.1).

$$G_s(s) = \left[ \frac{B_{n-1}, \bar{B}_{n-2}}{A_s, \bar{A}_s} \right] s^{n-1} + \left[ \frac{B_{n-2}, \bar{B}_{n-3}}{A_s, \bar{A}_s} \right] s^{n-2} + \ldots \left[ \frac{B_1, \bar{B}_0}{A_s, \bar{A}_s} \right] s + \left[ \frac{B_0, \bar{B}_0}{A_s, \bar{A}_s} \right]$$ (3.1)

If the system has ‘k’ number of dominant poles, then the reduced $k^{th}$ order model is represented by Equation (3.2).

$$R_s(s) = \left[ \frac{b_{k-1}, \bar{b}_{k-2}}{a_k, \bar{a}_k} \right] s^{k-1} + \left[ \frac{b_{k-2}, \bar{b}_{k-3}}{a_k, \bar{a}_k} \right] s^{k-2} + \ldots \left[ \frac{b_1, \bar{b}_0}{a_k, \bar{a}_k} \right] s + \left[ \frac{b_0, \bar{b}_0}{a_k, \bar{a}_k} \right]$$ (3.2)
3.3.1 Denominator Polynomial Reduction

The denominator polynomial is reduced by using the direct truncation method. The order of the reduced model depends on the number of dominant polynomials present in the higher order model.

If the higher order model has only one dominant pole, then the denominator polynomial is determined using Equation (3.3).

\[ D_1(s) = (A_0, \bar{A}_0) + \frac{(n-1)C_{n-1}}{n-1}(A_1, \bar{A}_1)s \]  \hspace{1cm} (3.3)

If two dominant poles are available in the process model, then the higher order model is deduced as the 2\textsuperscript{nd} order model. Its denominator is obtained, from Equation (3.4).

\[ D_2(s) = (A_0, \bar{A}_0) + \frac{(n-1)C_{n-2}}{n-2}(A_1, \bar{A}_1)s + \frac{(n-2)C_{n-2}}{nc_{n-2}}(A_2, \bar{A}_2)s^2 \]  \hspace{1cm} (3.4)

Similarly if the system has ‘k’ number of dominant poles, then the denominator polynomial of the k\textsuperscript{th} reduced order model is obtained using Equation (3.5).

\[ D_k(s) = \left\{ \sum_{i=1}^{k} [A_{i-1}, \bar{A}_{i-1}] \left( \frac{(n-i+1)C_{n-k}}{nC_{n-k}} \right) s^{i-1} \right\} \]  \hspace{1cm} (3.5)

where k=1,2,3,….

3.3.2 Numerator Polynomial Model Reduction

The numerator polynomial of the reduced order model N_k(s) is obtained by keeping the initial time moments and the Markov parameter.
Hence, the reduced order model has the time moment and Markov parameter contribution as represented by Equation (3.6).

\[ N_k(s) = N_{kt}(s) + N_{km}(s) \quad (3.6) \]

where \( N_{kt}(s) \) is the numerator term from the time moments
\( N_{km}(s) \) is the numerator term from the markov parameter and
\( k = t + m \) (k is the reduced model order).

If the higher order model has two (even) dominant poles, then it can be reduced up to the second order model with only one Markov parameter. ie,

\( k = 2, \quad m = 1 \) and \( t = 1 \)

If a system has an odd number of dominant poles then more number of markov parameters should be obtained. ie, If the system has three (odd) dominant poles,

ie,

\( k = 3, \) then \( m = 2 \) and \( t = 1. \) Always more number of Markov parameters should be retained in the reduced order model.

\[ N_r(s) = T_1 + T_2 s + \ldots + T_n s^{k-m-1} + M_m s^{k-m} \ldots + M_2 s^{k-2} + M_1 s^{k-1} \quad (3.7) \]

The time moment for the reduced order model is calculated from the Equation (3.8).

\[ T_i(s) = \left( \begin{array}{c} a_0 \\overline{a_0} \\ \overline{B_{i,1}} \overline{B_{i,1}} \end{array} \right) \left( \begin{array}{c} B_{i,0} \overline{B_{i,1}} \\ \overline{A_{i,0}} \overline{A_{i,1}} \end{array} \right) \quad (3.8) \]

The Markov parameters for the reduced order model are determined by using the Equation (3.9).
\[
\begin{align*}
\left[ M_n, \overline{M}_n \right] &= \frac{I}{(A_n, \overline{A}_n)} \sum_{i=1}^{\mathcal{m}} \left( B_{n-i}, \overline{B}_{n-i}\right) \left( a_{k-(n-i)}, \overline{a}_{k-(n-i)} \right) \\
&\quad - \sum_{j=0}^{\mathcal{m}-i} \left( M_j, \overline{M}_j \right) \left( A_{k-(n-j)}, \overline{A}_{k-(n-j)} \right)
\end{align*}
\] (3.9)

With an initial Markov parameter \((M_0, \overline{M}_0) = (0,0)\)

### 3.4 ILLUSTRATION FOR MODEL ORDER REDUCTION (MOR)

To study the effectiveness of the proposed model order reduction procedure a higher (seventh) order model is obtained from the literature proposed by Saini et al (2010) is reduced to a second order model.

**Example 3.1:** This example represents a higher order system considered by Saini (2010);

Numerator polynomial of the system is

\[
[1.9, 2.1]s^6 + [24.7, 27.3]s^5 + [157.7, 174.3]s^4 + [541.9, 599.02]s^3 + [929.9, 102.7.8]s^2 \\
+ [721.81, 7.79]s + [187.055, 206.745]
\] (3.10)

Denominator polynomial of the system is

\[
D(s) = [0.95, 1.05]s^2 + [8.779, 9.703]s^6 + [52.2, 57.8]s^5 + [182.8, 202.1]s^4 + [429.02, 474.1]s^3 + \\
[572.4, 6327]s^2 + [325.2, 359.5]s + [57.35, 63.389]
\] (3.11)

System model is given in Equation (3.12)

\[
G(s) = \frac{N(s)}{D(s)}
\] (3.12)

Model coefficient varies due to various uncertainties and parametric perturbations.
Poles for lower bound model are 1.8345±i2.71, -1.370±i3.4733, -1.782, -0.7209, -0.3142

Poles for upper bound model are -1.4235 ± i3.4603, -1.7900±i2.7415, -1.776, -0.7209, -0.3141

Zeros for lower bound model are -3.5, -0.5, -1.4950 + 0.1224i, -1.4950 - 0.1224i

Zeros for upper bound model are -3.493, -1.546, -1.459, -3.0 ± i3.99

Dominant poles for lower bound model are -0.7209, -0.3142

By retaining the above dominant poles some of the key features of higher order model are preserved.

Numerator polynomial is derived by retaining one initial time moment and one Markov parameter of the higher order model. Since the higher order model has two dominant poles, the lowest possible reduced order model will have the order two. (i.e.) k =2, m =1, t=1.

The second order reduced model denominator is obtained using the Equation (3.4).

\[ D_2(s) = [27.26, 30.13] s^2 + [92.937, 102.7] s + [57.35, 63.78] \] (3.13)

The numerator \( N_2(s) \) is calculated by retaining the initial Markov parameter and time moment. Initial time moment is obtained using Equation (3.14).
\[ \begin{bmatrix} T_1 \n T_1 \end{bmatrix} = \begin{bmatrix} 169.24 \n 229.64 \end{bmatrix} \]  \hspace{1cm} (3.14)

Markov parameter is calculated by using the Equation (3.9)

\[ [M_1, M_1] = \frac{1}{[A_4, A_4]} \left( B_3 \overline{B_3} (a_{\omega, \omega}, a_{\omega, \omega}) - 0^* (A_3, \overline{A_3}) \right) \]  \hspace{1cm} (3.15)

\[ [M_1, M_1] = \begin{bmatrix} 49.32 \n 66.6 \end{bmatrix} \]  \hspace{1cm} (3.16)

Hence the reduced 2nd order numerator is obtained using Equations (3.14) to (3.16) is.

\[ N_2(s) = \begin{bmatrix} 49.32 \n 66.6 \end{bmatrix} s + \begin{bmatrix} 169.24 \n 229.64 \end{bmatrix} \]  \hspace{1cm} (3.17)

Hence the 2nd order reduced order system using Equation (3.13) and (3.17) is given in Equation (3.18).

\[ G_2(s) = \frac{\begin{bmatrix} 49.32 \n 66.6 \end{bmatrix} s + \begin{bmatrix} 169.24 \n 229.64 \end{bmatrix}}{\begin{bmatrix} 27.26 \n 30.13 \end{bmatrix}s^2 + \begin{bmatrix} 92.937 \n 102.72 \end{bmatrix}s + \begin{bmatrix} 57.352 \n 63.789 \end{bmatrix}} \]  \hspace{1cm} (3.18)

### 3.5 STABILITY ANALYSIS OF THE INTERVAL SYSTEM

The Kharitonov polynomials of the numerator polynomial of Equation (3.12) are

\[ K_{n1} = 1.9s^6 + 24.7s^5 + 174.3s^4 + 599.02s^3 + 929.96s^2 + 721.81s + 206.75 \]
\[ K_{n2} = 2.1s^6 + 27.3s^5 + 157.7s^4 + 541.98s^3 + 1027.8s^2 + 797.79 s + 187.06 \]
\[ K_{n3} = 2.1s^6 + 24.7s^5 + 157.7s^4 + 599.02s^3 + 1027.8s^2 + 721.81s + 187.06 \]
\[ K_{n4} = 1.9s^6 + 27.3s^5 + 174.3s^4 + 541.98s^3 + 929.96s^2 + 797.79 s + 206.75 \]  \hspace{1cm} (3.19)

The Kharitonov polynomials of the denominator polynomial of Equation (3.12) are
\[ K_{di} = 0.95s^7 + 8.779s^6 + 57.8s^5 + 202.1s^4 + 429.02s^3 + 572.4s^2 + 359.5s + 63.38 \]
\[ K_{d2} = 1.05s^7 + 9.703s^6 + 52.2s^5 + 182.8s^4 + 474.1s^3 + 632.7s^2 + 325.2 + 57.35 \]
\[ K_{d3} = 1.05s^7 + 8.779s^6 + 52.2s^5 + 202.1s^4 + 429.1s^3 + 572.4s^2 + 325.2 + 63.38 \]
\[ K_{d4} = 0.95s^7 + 9.703s^6 + 57.8s^5 + 182.8s^4 + 429.02s^3 + 632.7s^2 + 359.5s + 57.35 \]

(3.20)

Using Equations (3.17) and (3.18), the Kharitonov rectangle was drawn for a higher order model and the stability is analyzed.

Similarly for the reduced order model, each 4 Kharitonov polynomial for the denominator and the numerator is obtained.

The characteristic equation of the higher order model with unity feedback is

\[ 1 + GH(s) = [0.95, 1.05]s^7 + [10.967, 11.8]s^6 + [76.9, 85.1]s^5 + [340.5, 376.4]s^4 \]
\[ + [970.92, 1073.12]s^3 + [1502.3, 1660.5]s^2 + [1046.9, 1157.29]s + [244.4, 270.1] \]

(3.21)

The second order reduced model characteristic equation is given in Equation (3.22).

\[ 1 + G_rH(s) = [27.26, 30.13]s^2 + [142.26, 169.32]s + [226.59, 293.429] \]

(3.22)

Using the interval analysis, the reduced order model is rearranged and represented by Equation (3.23).

\[ 1 + G_rH(s) = s^2 + [4.72, 6.21]s + [7.52, 10.764] \]

(3.23)

Figure 3.2 (a) shows the Kharitonov rectangle of the stable higher order model. It passes through seven quadrants. Hence, the order of the higher order model is seven. Also, it encircles the origin in the anticlockwise direction; so the system is completely stable. The encirclement of the origin and number of quadrants passed by the Kharitonov rectangle, are ensured by observing Figure 3.2 (b) and 3.2 (c).
Figure 3.2 (a) Kharitonov rectangles for the higher order system

Figure 3.2 (b) Enlarged view of Figure 3.2 (a)
Figure 3.2 (c) Enlarged view of Figure 3.2 (a)

Figure 3.3 shows the Kharitonov rectangle of reduced order model. It has been drawn between the real axis and imaginary axis of the Kharitonov edge polynomials for various frequencies. Figure 3.3 shows the Kharitonov rectangle for the reduced order model. The rectangle passes through two (first and second) quadrants and encircles the origin in the anticlockwise direction. Hence, according to the zero exclusion principle, the reduced order model is stable and the order of the model is 2, since it passes through 2 quadrants. The anticlockwise encirclement of origin shows stability of the system.
Figure 3.3 Kharitonov rectangles for the reduced order model

To study the steady state and transient state behavior of the system, the step response and impulse response have been obtained, for both the higher and reduced order model. Figure 3.4 shows the Step response of the Kharitonov polynomial of the higher and reduced order models. Among the five responses, two are drawn for the higher order model, and the remaining three are for reduced order model. The reduced order responses are within the higher order responses.
Figure 3.4 Comparison of the step responses of the higher order and reduced order models

To study the system with sudden disturbance, the impulse response is obtained. Figure 3.5 shows the impulse response of the Kharitonov polynomial of the higher and reduced order models. Similar to the step response analysis, the impulse response has been obtained for both the higher and reduced order models. Here also reduced order model response curves are within the higher order model response curves.

Figure 3.5 Comparison of the impulse response of the higher order and reduced order models
To study the system under various operating frequencies, the bode response and nyquist plot is obtained. Figure 3.6 shows the bode response of the Kharitonov polynomial of the higher and reduced order models. Similar to the step, impulse response plots, the Bode response and nyquist response (Figure 3.7) of the reduced order model is closer to that of the higher order model.

**Figure 3.6** Comparison of the Bode response of the higher order and reduced order models

**Figure 3.7** Comparison of the Nyquist response of the higher order and reduced order models
Table 3.1(a) & (b) shows that the proposed method reduces the higher order model effectively with less error compared to the other methods. Compared to the Bandyopadhyay and Saini (2010) methods, the integral squared error of the proposed method is very low.

### Table 3.1(a) Model order reduction method error analysis for example 3.1

<table>
<thead>
<tr>
<th>Model order reduction Method</th>
<th>Reduced model</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lower bound model</td>
</tr>
<tr>
<td>Proposed method</td>
<td>(\frac{[49.32666/s+169.2422964]}{[27.263013/s^2+92.93710272/s+[57.35263.789]})</td>
<td>0.808</td>
</tr>
<tr>
<td>Saini method</td>
<td>(\frac{[3647.4297/s+2717.2932]}{[61.56899/s^3+2557.3471/s+[833.87.67]})</td>
<td>0.062</td>
</tr>
<tr>
<td>Bandyopadhyay</td>
<td>(\frac{[1.161.84/s+0.270.53]}{s^2+0.520.83/s+[0.083.016]})</td>
<td>2.259</td>
</tr>
</tbody>
</table>

**Example 3.2:**

Let us consider a fourth order interval system given by Equation 3.24 is considered as higher order system.

\[
G_4(s) = \frac{[12.14/s^4+220.240/s^3+800.900/s+1100.1200]}{[1.1.2/s^4+16.18/s^3+90.100/s^2+160.180/s+110.120]} \quad (3.24)
\]

The 2\(^{nd}\) order model obtained using the biased model reduction method is given by Equation 3.25.

\[
G_2(s) = \frac{[150.233.2/s+1100.1200]}{[15.16.66/s^2+80.90/s+110.120]} \quad (3.25)
\]
Table 3.1(a) Model order reduction method error analysis for example 3.2

<table>
<thead>
<tr>
<th>Model order reduction Method</th>
<th>Reduced model</th>
<th>ISE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lower bound model</td>
<td>upper bound model</td>
</tr>
<tr>
<td>Proposed method</td>
<td>[ \frac{[1502332]s + [11001200]}{[1516.66]s^2 + [8090]s + [110120]} ]</td>
<td>0.0108</td>
<td>0.4215</td>
</tr>
<tr>
<td>Saini method</td>
<td>[ \frac{[116184]s + [27.53]}{s^2 + [52.783.98]s + [8.316]} ]</td>
<td>0.98</td>
<td>1.67</td>
</tr>
<tr>
<td>Bandyopadhyay</td>
<td>[ \frac{[116184]s + [0.270.53]}{s^2 + [1.771.8]s + [1.11.2]} ]</td>
<td>0.487</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Figure 3.8 shows that the closed loop response of fourth order model and second order reduced model for a unit step command.

![Figure 3.8](image1.png)  
**Figure 3.8 (a) Step response of higher order and reduced order model**

![Figure 3.8](image2.png)  
**Figure 3.8 (b) Bode response of higher order and reduced order model**

It is observed from the Figure 3.8 (a), that both the higher order model and lower order model response are closer to each other in transient and steady state. Also time response specification of higher order model such as settling time, rise time, peak time and peak over shoot are matches with lower order model response parameter.
Figure 3.8(b) shows the Bode response for higher order and lower order model. Hence by using this method in time domain and frequency domain, the approximated model features resembles.

**Figure 3.9 (a)** Kharitonov rectangle for higher order model  

**Figure 3.9 (b)** Kharitonov rectangle for Reduced order model

Kharitonov rectangle as shown in Figure 3.9 (a), (b) explains about the stability of family of transfer function represented by the Equation 3.24. From Figure 3.9 (a) and Figure 3.9 (b), it is inferred that both higher order model and reduced order model are stable, since both Kharitonov rectangles are encircle the origin and also by counting the number of quadrant passed by the rectangle, the order of the system can be identified.

### 3.6 CONTROLLER DESIGN FOR THE INTERVAL SYSTEM

Controller tuning is an essential preliminary procedure for almost all the industrial process control systems. Despite the significant developments in advanced process control schemes such as predictive control, internal model control, and sliding mode control, PID controllers are still widely used in industrial control applications, because of their structural simplicity, reputation and easy implementation. The merits of the PID controller are as follows:
(i) Obtainable in a variety of structures such as academic PID, series PID, parallel PID and IMC-PID (Vijayan & Panda 2011)

(ii) Provides optimal and robust performance

(iii) Supports online/offline tuning and retuning based on the process performance requirement,

Many researchers proposed PID tuning rules, to control various stable and unstable systems by different schemes to enhance closed loop performance (Åström & Hägglund 1984, 2006; O'Dwyer 2003). For stable systems, the PID controller offers a viable result for both reference tracking and disturbance rejection. However, for unstable systems, it can only effectively work either for reference tracking or for disturbance rejection. The proportional and derivative kick in the controller also results in a large overshoot and large settling time.

In process control applications, the PID and modified structured PID are still widely used in industrial control systems where reference tracking and disturbance rejection are major tasks.

3.6.1 Conventional PID Controller

Industrial PID controllers are usually available in a packaged form and to perform well with the industrial process problems, but the PID controller needs optimal tuning. Figure 3.8 shows the block diagram of a simple closed loop control system. In this structure, the controller \( G_c(s) \) has to provide closed loop stability, smooth reference tracking, shape the dynamic and the static qualities of the disturbance response (Johnson & Moradi 2005).
Figure 3.10 General block diagram of a closed loop system

In process industries, the PID controller is used to improve both the steady state as well as the transient response of the plant. In Figure 3.10 $G_p(s)$ represents the process under control and $G_c(s)$ is the controller. The main objective of this system is to make $Y(s) = R(s)$. In this framework, the controller continuously adjusts the value of $U_c(s)$, until the error $E(s)$ is zero irrespective of the disturbance signal $D_1(s)$ and/or $D_2(s)$.

The closed loop response of the system with the set point $R(s)$, disturbance on the supply side $D_1(s)$ and load disturbance $D_2(s)$ is expressed as,

$$Y(s) = \left[ \frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)} \right] R(s) + \left[ \frac{1}{1+G_p(s)G_c(s)} \right] G_p(s)D_1(s)$$

$$+ \left[ \frac{1}{1+G_p(s)G_c(s)} \right] D_2(s) \tag{3.26}$$

where, the complementary sensitivity function, and the sensitivity function of the above loop are represented in Equations (3.27) and (3.28) respectively.

$$T(s) = \frac{Y(s)}{R(s)} = \left[ \frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)} \right]$$

$$\tag{3.27}$$
\[ S(s) = \frac{1}{1 + G_p(s)G_c(s)} \]  

(3.28)

The final steady state response of the system for the set point tracking and the disturbance rejection is presented by Equations (3.29) to (3.30).

\[ y_R(\infty) = \lim_{t \to \infty} y_R(s) = \lim_{t \to \infty} \left[ \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)} \right] \frac{A}{s} = A \]  

(3.29)

\[ y_{D1}(\infty) = \lim_{t \to \infty} \left[ \frac{G_p(s)}{1 + G_p(s)G_c(s)} \right] \frac{L_1}{s} = 0 \]  

(3.30)

\[ y_{D2}(\infty) = \lim_{t \to \infty} \left[ \frac{1}{1 + G_p(s)G_c(s)} \right] \frac{L_2}{s} = 0 \]  

(3.31)

where, \( A \) = amplitude of the reference signal and \( L \) = disturbance amplitude.

To achieve a satisfactory \( y_R(\infty) \), \( y_{D1}(\infty) \) and \( y_{D2}(\infty) \), it is necessary to have optimally tuned PID parameters.

In this research work, a noninteracting form of a PID controller structure is considered. For real control applications, the feedback signal is the sum of the measured output and the measurement noise component. A low pass filter is used with the derivative term to reduce the effect of the measurement noise. The PID structure is given in Equation (3.32)

\[ G_c(s) = K_p \left[ 1 + \frac{1}{T_i s} + \frac{T_d s}{T_d s + N} \right] \]  

(3.32)

where \( K_p / T_i = K_i \), \( K_p * T_d = K_d \), \( N \) = filter constant.
Figure 3.11 Parallel form of the PID structure

Figure 3.11 shows the parallel form of the PID controller. The output signal from the controller is shown in Equation (3.33).

\[
U_c(s) = K_p e(t) + K_i \int_0^s e(t) \, dt + \left( \frac{K_d}{T_f s + 1} \right) \frac{de(t)}{dt}
\]  

(3.33)

The major limitation of the classical PID structure, is that a step change in the reference input \(R(s)\) will cause an immediate sharp change in the control signal \(U_c(s)\). This abrupt change in the controller output is represented as the proportional and/or derivative kick. These kick effects rapidly change the command signal to the actuator, which controls the entire operation of the plant.

The above discussed PID controller is tuned in numerous ways, since a properly tuned controller gives best robust performance. In the recent literature, controllers are designed using various optimization algorithms such as GA, PSO, BFO and hybrid algorithms. In this research work an Improved Particle Swarm Optimization (IPSO) is adopted to get the better robust performance.

### 3.6.2 IPSO Based Controller Tuning

The controller tuning process is used to find the optimal values for controller parameters from the search space that minimizes the objective
function. The performance of the heuristic algorithm based controller tuning depends mainly on the objective function which guides the optimization search to obtain appropriate values for the controller parameters. Figure 3.12 illustrates the basic block diagram of the IPSO based PID controller tuning. In this, the dimension of the optimization search is three ($K_p$, $K_i$ and $K_d$).

![Figure 3.12 IPSO based PID controller design](image)

The IPSO based controller design is attempted to study and improve the performance of the class of unstable systems using a parallel form of the PID with a derivative filter (with a filter value of 10). The optimal tuning procedure is repeated 10 times independently, and the best value among the trials is considered for stabilizing the system.

In this research work, the Improved Particle Swarm Optimization (IPSO) technique is attempted to design the controller for the Kharitonov polynomials. The IPSO algorithm attempts to mimic the natural process of group communication of individual knowledge, to achieve some optimum property. The swarm is initialized with a population of random solutions. Each particle in the swarm is a different possible set of the unknown parameters to be optimized. Representing a point in the solution space, each
particle adjusts its flying toward a potential area, according to its own flying experience, and shares the social information among particles. The goal is to efficiently search the solution space by swarming the particles toward the best fitting solution encountered in previous iterations, with the intent of encountering better solutions through the course of the process, and eventually converging on a single minimum error. At the beginning, each particle of the population is scattered randomly throughout the entire search space. Under the guidance of the performance criterion, particles in their flight dynamically adjust their velocities, according to their own flying experience and their companions flying experience. Each particle remembers its best position obtained so far, which is denoted $p_{\text{best}}$. It also receives the globally best position achieved by any particle in the population, which is denoted as $g_{\text{best}}$. The updated velocity of each particle is calculated using the present velocity and the distances from the $p_{\text{best}}$ and $g_{\text{best}}$.

The mathematical expression of the particle’s velocity and position for the improved PSO is shown in Equations (3.34) and (3.35).

$$V_{i,D}^{t+1} = W \times V_{i,D}^{t} + \sum_{j=1}^{n} \left[ \sum_{k=1}^{n} C_1 \times R_1 \times (p_{j,D}^{t} - X_{i,D}^{t}) + \sum_{k=0}^{n} C_2 \times R_2 \times (g_{D}^{t} - X_{i,D}^{t}) + C_3 \times R_3 \times (p_{D}^{t} - X_{i,D}^{t}) \right]$$

(3.34)

$$X_{i,D}^{t+1} = X_{i,D}^{t} + V_{i,D}^{t+1}$$

(3.35)

where

- $V_{i,D}^{t}$ - current velocity of particle i at iteration k
- $V_{i,D}^{t+1}$ - updated velocity of particle i
- $W$ - Inertia weight (0.75)
- $C_1, C_2, C_3$ - positive constants
- $X_{i,D}^{t}$ - current position of particle i at inertia k
- $R_1, R_2, R_3$ - random number between 0 and 1
The overall performance (speed of convergence, efficiency and optimization accuracy) of the heuristic algorithm depends on the objective function, which monitors the optimization search. The objective function is chosen to maximize the domain constraints or to minimize the preference constraints. During the search, without loss of generality, the constrained optimization problem minimizes a scalar function ‘\( J \)’, of some decision variable vector ‘\( D \)’ in a universe ‘\( U \)’. The objective function is to be framed by assuming, that there exist at least one set of optimal parameters in ‘\( U \)’ which satisfies all the constraints. In many optimization cases, it is very difficult to satisfy all the considered constraints. Hence, there should be some negotiation between the preference constraint parameters without compromising on the domain constraint (Liu et al 2008).

In heuristic algorithm based optimization, the minimization of a single objective function (error minimization) is very popular. The algorithm explores the D dimensional search space until the objective function is minimised. Equations (3.36) to (3.39) represent the general objective function in process control.

\[
IAE = \int_{0}^{\infty} |e(t)| \ dt = \int_{0}^{\infty} |r(t) - y(t)| \ dt \\
(3.36)
\]

\[
ISE = \int_{0}^{\infty} e^2(t) \ dt = \int_{0}^{\infty} [r(t) - y(t)]^2 \ dt \\
(3.37)
\]

\[
ITAE = \int_{0}^{\infty} t |e(t)| \ dt = \int_{0}^{\infty} t |r(t) - y(t)| \ dt \\
(3.38)
\]

\[
ITSE = \int_{0}^{\infty} t e^2(t) \ dt = \int_{0}^{\infty} t [r(t) - y(t)]^2 \ dt \\
(3.39)
\]

where \( e(t) = \) error, \( r(t) = \) reference input (set point), \( y(t) = \) process output, \( IAE = \) integral of absolute error, \( ISE = \) integral square error, \( ITAE = \) integral
of time multiplied by absolute value of error, and \( ITSE \) = integral of time multiplied by the square error criterion.

Controller Settings for the reduced order interval model are obtained using IPSO. Figure 3.13 is the convergence chart for the proportional controller parameter. It is observed that the proportional constant (\( K_p \)) is converged to 0.273 for the Kharitonov polynomial \( K_{11} \). The optimization algorithm searches the proportional constant in a wide span, and after the 66\(^{th} \) iteration it has converged to the constant value.

![Convergence chart for controller setting (Kp)](image)

**Figure 3.13 Convergence chart for controller setting (\( K_p \))**

From the Figure 3.14, it is observed that the integral constant (\( K_i \)) is converged to 0.009 for the Kharitonov polynomial \( K_{11} \). The optimization algorithm searches the proportional constant in a wide span, and after the 66\(^{th} \) iteration it has converged to the constant value.
Figure 3.14 Convergence chart for controller setting \((K_i)\)

From Figure 3.15, it is observed that the derivative Constant \((K_d)\) is converged to \(K_d=0.0049\) for the Kharitonov polynomial \(K_{11}\). The optimization algorithm searches the proportional constant in a wide span, and after the 66\(^{th}\) iteration, it has converged to the constant value.

Figure 3.15 Convergence chart for controller setting \((K_d)\)

Figures 3.13, 3.14 and 3.15 show the convergence flow of the controller setting. Figure 3.16 gives the comparison of the step response of the
higher and reduced order models. $K_p=1.2865$, $K_i=0.0629$, $K_d=0.0049$ for $K_{11}$, and similarly, the controller parameter is calculated for all the other sets of Kharitonov polynomials $K_{12}$, $K_{13}$ and $K_{14}$. From the set of controller parameter identified for all 16 Kharitonov polynomials, the controller parameters in the interval form are computed by using the hull set. From the hull set it is observed, that the proportional controller gain varied from 1.249 to 1.368. Similarly, the integral time constant varies from 0.0184 and 0.125. The derivative constant varies from 0.0126 to 0.5821. From the different set of controller parameters, using interval algebra controller parameter variations are obtained and shown in Equation (3.40)

$$K_p = [1.249, 1.368]$$

$$K_i = [0.0184, 0.12528]$$

$$K_d = [0.0126, 0.5821].$$

![Figure 3.16 Step responses of the higher and lower order models with controller](image)

Figure 3.16 Step responses of the higher and lower order models with controller
3.7 CONCLUSION

The proposed method simplifies the mathematical complexities of model order reduction technique. It is based on the convergence technique, generates a stable reduced order model for stable higher order model by retaining both the initial time moments and the Markov parameters. The reduced order model performance matches the higher order model performance, for both the steady and transient states of the time response as well as frequency response. A controller parameter is identified for the reduced order model, using the IPSO, which controls the higher order model effectively. The time and frequency response curve shows the closeness of higher and lower order model response. When the interval model of a physical system is designed, the sensitive parameter has to be identified and the range of permitted variation of each parameter should be determined. A multiple experiment based method is proposed in the chapter 4, by which the width of the interval coefficients can be estimated and validated using a statistical method.