CHAPTER 5

SENSITIVITY ANALYSIS FOR PAIR OF UNEQUAL UNIT STANDBY RELIABILITY MODEL

5.1 Introduction:

The standby system is often applicable in various industrial and other manufacturing establishments for its smooth functioning. The observation has been realized in order to predict the reliability and availability due to occurrence of common cause failures and availability of repairman at that particular time. Most of the research work has been finished for a pair of identical or non-identical units repairable redundant systems models with certain assumption like as soon as an unit fails then repair started instantaneously by the repairman. Here question arises that repairman may be available or not at that time due to high competition and busy of repairman in the supermarket. The maximum engagement of human resources should be done with the objective of maximum earning.

It may be seen that the repairmen may not always available with the system in general practice. In such situations, failed unit waits for repair until the repairman available in the system. Hence it is assumed that failure and repair time of units is not correlated if this is treated as random variables. But in practical situations it seems to be unrealistic because in many cases, there may be sort of correlation between failure and repair time. For increasing the reliability of the system/component the essential requirement to give the facility of repair and post repair after inspection for maintenance of system. When component of the system is not cent percent perfect and already worked till sufficient time as per capacity or as per cost incurred at the time of purchased equipment then it is necessary to inspect the component for maintenance point of view to check whether the repair is required or not by repairman. If necessity of repair has been decided by repairman then it is needful for better maintenance. Most important, it must be inspected whether repair has been done perfectly or not by repairman of corresponding component If repair is found unsatisfactory during inspection, the component may be sent again for post-repair.
5.2 System Description

This research work has been carried on extension of previous work for two non-
identical unit standby units when repairman is not always available with the system for
sensitivity point of view and involvement of various cost for maintaining the system
reliability due to failure of any unit. The repair of failure unit is one of the major
exercised for optimized the reliability of system. The sensitivity of profit is fully
depends on cost incurred during repair. The availability or non-availability of repair
man is one of them with number of visit by the repairman. Due to this, the concept of
availability and non-availability of repairman has been used for better repair policy. It is
clear that repair is required when the system become failure in two ways i.e. normal
failure and complete failure. If these units may be named as unit-1 and unit-2. Initially
system starts with operation state $S_0$ in which unit-1 is operative and unit-2 is kept in
cold standby and repairman is not available. Upon failure of an operative unit the cold
standby unit becomes operative instantaneously. After the repair of unit-1 it goes for
inspection to decide whether the repair is perfect or not. If the repair of a unit is found
to be perfect then the repaired unit becomes operational otherwise it is sent for post
repair. The probability of perfect repair is fixed. Unit-2 becomes as good as new after
repair. The overall process can be understood in this way:

i. The first unit goes for repair, inspection and post repair whereas the second unit
becomes as good as new after repair. Here the priority in operation is given to
the first unit as it is highly sophisticated, costly unit which provides high quality
product at low running cost.

ii. The second unit is ordinary unit which has high running cost. Priority in repair
is given to second unit as repair of second unit is less time consuming and
cheaper as compared to the first unit.

Our aim is to analyzed a pair of unequal unit standby system model with repair,
inspection and post repair by using joint distribution especially bivariate exponential
for two different random variables of failure and repair time. The distributions of time
for available and non-available of repairman are taken as exponential with different
parameters. The result may be varies as per repair time by regular repairman which may
follows the different distribution. It may be either exponential, negative exponential, inverse Gaussian distribution, e.t.c. The regenerative point technique has been used for measuring the reliability with its sensitivity analysis. Attempted has been covered to analysis the behaviour of the following measures.

1) Possibility of Up and Down states for Transition Diagram.
2) Development process Transition probability.
3) Possible no of Up and Down states for Transition Diagram
4) Reliability calculation for each component.
5) Mean Time to System Failure (MTSF) in every case.
6) Point wise and steady state availability of the system.
7) Busy period Analysis And Number of visit by the repairman
8) Cost and profit Analysis with its sensitivity.
9) Concise way presentation of Sensitivity Analysis.

5.3 Assumptions:

The major used assumption for development of this model is as follows.

Type of Unit:- The pair of unequal unit involved with the behaviour, one unit is operative and other is standby with repair if required.

Type of Repair: A single repair facility is used to repair of both units and inspection and post repair of unit-1.

Behaviour of Repairman: The repairman is not always available due competitive nature of super market. Hence it may be available or non-available with random nature.

Decision on Repairman: If Unit-2 gets preference in repair over unit-1 while unit-1 gets preference in operation.

Discipline of Repairman: Once the repairman enters the system, he will not leave the system till the repair of all the failed units is completed.

Distribution used: The distribution of time to available and non-available of service provided by repairman is taken as exponential with different parameters however the failure time of a unit must be follow (negative) exponential. Hence the joint distribution of failure and repair times of each unit are bivariate.
exponential i.e. The inspection and post repair time distribution of unit-1 are taken to be independent having the exponential function as follows:

\[ J(t) = \lambda \exp(-\lambda t) t \geq 0 \text{ and } \lambda > 0 \]

\[ h(t) = \mu \exp(-\mu t) t \geq 0 \text{ and } \mu > 0 \]

### 5.4 Symbol and Notations:

- \( X_i/Y_i \): Random variable for the failure/repair time of respective one.
- \( \theta \): Constant available rate of the repairman
- \( \varnothing \): Constant non-available rate of the repairman
- \( g_i(x) \): Marginal p.d.f. of \( X_i = x \), \( = \alpha_i(1 - r_i)e^{-\alpha_i(1-r_i)x} \)
- \( G_i(x) \): c.d.f. of \( X_i = x \)
- \( m(t) \): Inspection rate of first unit having the form \( \mu \exp(-\mu t) \), \( \mu > 0 \)
- \( e(t) \): Post repair rate of first unit having the form \( \lambda \exp(-\lambda t) \), \( \lambda > 0 \)
- \( a/b \): Probabilities that the repair of unit-1 is perfect or imperfect (\( a+b=1 \))
- \( Y \): Random variable representing the residual repair time
- \( E \): Set of regenerative states i.e., \( E = \{0,1,2,3,4,5,6,7,8,9,10,11\} \)
- \( h(y) \): p.d.f. of \( Y = \beta e^{-\beta y} \), \( y > 0, \beta > 0 \)
- \( N_{IO}/N_{IS} \): Unit in normal mode (N-mode) and operative/standby.
- \( F_{IR}/F_{IWR} \): Unit in failure mode (F-mode) and under repair/waiting for repair
- \( F_{IPR}/F_{IWPR} \): Post repair/waiting for post repair/inspection
- \( F_{II} \): Unit in failure mode and under inspection
- \( NA/A \): Repairman not available/available
5.5 Transition Diagram:

As per description of model here we have considered eleven states for analysis the stochastic behaviour of the system. On the basis of this we are able to complete the transition diagram and able to write the transition probability equation or matrices. The uptime and downtime are as follows.

The transition diagram has been drawn on the basis of inspection, repair and post repair involved in two unit unequal standby system for better understanding of diagram and with the help of this the transition probability equation has been form.

<table>
<thead>
<tr>
<th>Up States</th>
<th>Down States</th>
</tr>
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<tbody>
<tr>
<td>(0,1,2,5,7,9)</td>
<td>(3, 4, 6,8,10)</td>
</tr>
<tr>
<td>$S_0 = \left(\begin{array}{c} N_{10}, N_{2S} \ NA \end{array}\right)$</td>
<td>$S_3 = \left(\begin{array}{c} F_{1WR}, F_{2WR} \ NA \end{array}\right)$</td>
</tr>
<tr>
<td>$S_1 = \left(\begin{array}{c} N_{10}, N_{2S} \ A \end{array}\right)$</td>
<td>$S_4 = \left(\begin{array}{c} F_{1WR}, F_{2R} \ A \end{array}\right)$</td>
</tr>
<tr>
<td>$S_2 = \left(\begin{array}{c} F_{1WR}, N_{20} \ NA \end{array}\right)$</td>
<td>$S_6 = \left(\begin{array}{c} F_{1WR}, F_{2R} \ A \end{array}\right)$</td>
</tr>
<tr>
<td>$S_5 = \left(\begin{array}{c} F_{1R}, N_{20} \ A \end{array}\right)$</td>
<td>$S_8 = \left(\begin{array}{c} F_{1WR}, F_{2R} \ A \end{array}\right)$</td>
</tr>
<tr>
<td>$S_7 = \left(\begin{array}{c} F_{1PR}, N_{20} \ A \end{array}\right)$</td>
<td>$S_9 = \left(\begin{array}{c} F_{1L}, N_{20} \ A \end{array}\right)$</td>
</tr>
<tr>
<td>$S_9 = \left(\begin{array}{c} F_{1L}, N_{20} \ A \end{array}\right)$</td>
<td>$S_{10} = \left(\begin{array}{c} F_{1WI}, F_{2R} \ A \end{array}\right)$</td>
</tr>
</tbody>
</table>
Fig.5.1 Transition Diagram
The transition diagram has been drawn on the basis of inspection, repair and post repair [1] involved in two unit unequal standby system for better understanding of diagram and with the help of this the transition probability equation has been form.

5.6 Calculation of Transition Probabilities:

Let $T_0(=0)$, $T_1, T_2, \ldots \ldots$ denote the regenerative epochs and $X_n$ denotes the state visited at epoch $T_n$ i.e., just after the transition at $T_n$. Then $\{X_n, T_n\}$ constitute a Markov-Renewal process.

Let $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t; X_n = i]$

Then the transition probability matrix (TPM) of the embedded Markov Chain is

$$P = p_{ij} = Q_{ij}(\infty) = Q(\infty)$$

The time dependent transition probabilities are

$Q_{01}(t) = P[\text{System transits from state } S_0 \text{ to } S_1 \text{ during } (u, u+du), u<t$

$$= \theta \int_0^t e^{-[\theta + \alpha_1(1-r_1)]u} du$$

$$= \frac{\theta}{[\theta + \alpha_1(1-r_1)]} \left[1 - e^{-[\theta + \alpha_1(1-r_1)]t}\right]$$

Similarly

$Q_{02}(t) = \alpha_1(1-r_1) \int_0^t e^{-[\theta + \alpha_1(1-r_1)]u} du$

$$= \frac{\alpha_1(1-r_1)}{[\theta + \alpha_1(1-r_1)]} \left[1 - e^{-[\theta + \alpha_1(1-r_1)]t}\right]$$

$Q_{10}(t) = \emptyset \int_0^t e^{-[\theta + \alpha_1(1-r_1)]u} du$

$$= \frac{\emptyset}{[\theta + \alpha_1(1-r_1)]} \left[1 - e^{-[\theta + \alpha_1(1-r_1)]t}\right]$$

$Q_{15}(t) = \alpha_1(1-r_1) \int_0^t e^{-[\theta + \alpha_1(1-r_1)]u} du$
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

\[ Q_{23}(t) = \alpha_2 (1 - r_2) \int_0^t e^{-[\theta + \alpha_2(1-r_2)]u} du \]

\[ = \frac{\alpha_2(1-r_2)}{[\theta + \alpha_2(1-r_2)]} [1 - e^{-[\theta + \alpha_2(1-r_2)]t}] \]

\[ Q_{25}(t) = \theta \int_0^t e^{-[\theta + \alpha_2(1-r_2)]u} du \]

\[ = \frac{\theta}{[\theta + \alpha_2(1-r_2)]} [1 - e^{-[\theta + \alpha_2(1-r_2)]t}] \]

\[ Q_{34}(t) = \theta \int_0^t e^{-\theta u} du \]

\[ = [1 - e^{-\theta t}] \]

\[ Q_{45}(t) = \beta \int_0^t e^{-\beta u} du \]

\[ = [1 - e^{-\beta t}] \]

\[ Q_{56}(t) = \alpha_2 (1 - r_2) \int_0^t e^{-[\beta + \alpha_2(1-r_2)]u} du \]

\[ = \frac{\alpha_2(1-r_2)}{[\beta + \alpha_2(1-r_2)]} [1 - e^{-[\beta + \alpha_2(1-r_2)]t}] \]

\[ Q_{59}(t) = \beta \int_0^t e^{-[\beta + \alpha_2(1-r_2)]u} du \]

\[ = \frac{\beta}{[\beta + \alpha_2(1-r_2)]} [1 - e^{-[\beta + \alpha_2(1-r_2)]t}] \]

\[ Q_{65}(t) = \beta \int_0^t e^{-\beta u} du \]

\[ = [1 - e^{-\beta t}] \]

\[ Q_{78}(t) = \alpha_2 (1 - r_2) \int_0^t e^{-[\lambda + \alpha_2(1-r_2)]u} du \]

\[ = \frac{\alpha_2(1-r_2)}{[\lambda + \alpha_2(1-r_2)]} [1 - e^{-[\lambda + \alpha_2(1-r_2)]t}] \]

\[ Q_{71}(t) = \lambda \int_0^t e^{-[\lambda + \alpha_2(1-r_2)]u} du \]
\[
\frac{\lambda}{[\lambda + \alpha_2(1-r_2)]} \left[ 1 - e^{-[\lambda + \alpha_2(1-r_2)]t} \right]
\]

\[
Q_{87}(t) = \beta \int_0^t e^{-\beta u} du = \left[ 1 - e^{-\beta t} \right]
\]

\[
Q_{9,10}(t) = \alpha_2(1-r_2) \int_0^t e^{-[\mu + \alpha_2(1-r_2)]u} du = \frac{\alpha_2(1-r_2)}{[\mu + \alpha_2(1-r_2)]} \left[ 1 - e^{-[\mu + \alpha_2(1-r_2)]t} \right]
\]

\[
Q_{91}(t) = a\mu \int_0^t e^{-[\mu + \alpha_2(1-r_2)]u} du = \frac{a\mu}{[\mu + \alpha_2(1-r_2)]} \left[ 1 - e^{-[\mu + \alpha_2(1-r_2)]t} \right]
\]

\[
Q_{97}(t) = b\mu \int_0^t e^{-[\mu + \alpha_2(1-r_2)]u} du = \frac{b\mu}{[\mu + \alpha_2(1-r_2)]} \left[ 1 - e^{-[\mu + \alpha_2(1-r_2)]t} \right]
\]

\[
Q_{10,9}(t) = \beta \int_0^t e^{-\beta u} du = \left[ 1 - e^{-\beta t} \right]
\]

(5.1-5.18)

Steady state probabilities of transition are \( p_{ij} = \lim_{t \to \infty} Q_{ij} \)

\[
p_{01} = \frac{\theta}{[\theta + \alpha_1(1-r_1)]}
\]

\[
p_{02} = \frac{\alpha_1(1-r_1)}{[\theta + \alpha_1(1-r_1)]}
\]

\[
p_{10} = \frac{\varnothing}{[\varnothing + \alpha_1(1-r_1)]}
\]

\[
p_{15} = \frac{\alpha_1(1-r_1)}{[\varnothing + \alpha_1(1-r_1)]}
\]
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

\[ p_{23} = \frac{\alpha_2(1 - r_2)}{[\theta + \alpha_2(1 - r_2)]} \]

\[ p_{25} = \frac{\theta}{[\theta + \alpha_2(1 - r_2)]} \]

\[ p_{34} = 1 \]

\[ p_{45} = 1 \]

\[ p_{56} = \frac{\alpha_2(1 - r_2)}{[\beta + \alpha_2(1 - r_2)]} \]

\[ p_{59} = \frac{\beta}{[\beta + \alpha_2(1 - r_2)]} \]

\[ p_{65} = 1 \]

\[ p_{78} = \frac{\alpha_2(1 - r_2)}{[\lambda + \alpha_2(1 - r_2)]} \]

\[ p_{71} = \frac{\lambda}{[\lambda + \alpha_2(1 - r_2)]} \]

\[ p_{87} = 1 \]

\[ p_{9,10} = \frac{\alpha_2(1 - r_2)}{[\mu + \alpha_2(1 - r_2)]} \]

\[ p_{91} = \frac{a \mu}{[\mu + \alpha_2(1 - r_2)]} \]

\[ p_{97} = \frac{b \mu}{[\mu + \alpha_2(1 - r_2)]} \]

\[ p_{10,9} = 1 \] \hspace{1cm} (5.19-5.36)

Ultimately the relationship of transition probabilities may looks like :-

\[ p_{01} + p_{02} = 1, \quad p_{10} + p_{15} = 1, \quad p_{23} + p_{25} = 1. \]
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

\[ p_{34} = 1, \quad p_{45} = 1, \quad p_{56} + p_{59} = 1, \]
\[ p_{65} = 1, \quad p_{78} + p_{71} = 1, \quad p_{97} = 1, \]
\[ p_{9,10} + p_{91} + p_{97} = 1, \quad p_{10,9} = 1 \]

5.7 Calculating Sojourn Times:

We observe that as long as the system is in state \( S_i \), there is no transition to any other state. If \( T_i \) denotes the sojourn time in state \( S_i \), then the mean sojourn time in \( S_i \) is given by \( \psi_i = \int P(T_i > t) dt \); Now, we find out the mean sojourn times in various states

\[ \psi_0 = \int_0^\infty e^{-[\theta + \alpha_1(1-r_1)]u} du = \frac{1}{[\theta + \alpha_1(1-r_1)]} \]
\[ \psi_1 = \int_0^\infty e^{-[\theta + \alpha_1(1-r_1)]u} du = \frac{1}{[\theta + \alpha_1(1-r_1)]} \]
\[ \psi_2 = \int_0^\infty e^{-[\theta + \alpha_2(1-r_2)]u} du = \frac{1}{[\theta + \alpha_2(1-r_2)]} \]
\[ \psi_3 = \int_0^\infty e^{-\theta u} du = \frac{1}{\theta} \]
\[ \psi_4 = \int_0^\infty e^{-\beta u} du = \frac{1}{\beta} \]
\[ \psi_5 = \int_0^\infty e^{-[\beta + \alpha_2(1-r_2)]u} du = \frac{1}{[\beta + \alpha_2(1-r_2)]} \]
\[ \psi_6 = \int_0^\infty e^{-\beta u} du = \frac{1}{\beta} \]
\[ \psi_7 = \int_0^\infty e^{-[\lambda + \alpha_2(1-r_2)]u} du = \frac{1}{[\lambda + \alpha_2(1-r_2)]} \]
\[ \psi_8 = \int_0^\infty e^{-\beta u} du = \frac{1}{\beta} \]
\[ \psi_9 = \int_0^\infty e^{-[\mu + \alpha_2(1-r_2)]u} du = \frac{1}{[\mu + \alpha_2(1-r_2)]} \]
\[ \psi_{10} = \int_0^\infty e^{-\beta u} du = \frac{1}{\beta} \] (5.37-5.47)
5.8 Reliability Calculation:

Let the random variable $T_i$ denotes the time to system failure when the system starts its operation from state $S_i \in E$. Then the reliability of the system is given by $R_i = P(T_i > t)$. To determine the reliability of the system, we regard the failed states $S_3$, $S_4$, $S_6$, $S_8$, and $S_{10}$ as absorbing. By simple probabilities argument one can easily develop the recurrence relations among $R_i(t)$ ($i = 0, 1, 2, 5, 7, 9$).

$$
R_0(t) = Z_0(t) + q_{01}(t)R_1(t) + q_{02}(t)R_2(t)
$$

$$
R_1(t) = Z_1(t) + q_{10}(t)R_0(t) + q_{15}(t)R_5(t)
$$

$$
R_2(t) = Z_2(t) + q_{25}(t)R_5(t)
$$

$$
R_5(t) = Z_5(t) + q_{59}(t)R_9(t)
$$

$$
R_7(t) = Z_7(t) + q_{71}(t)R_1(t)
$$

$$
R_9(t) = Z_9(t) + q_{91}(t)R_1(t) + q_{97}(t)R_7(t) \quad (5.48-5.53)
$$

Where

$$
Z_0(t) = e^{-[\theta + \alpha_1(1-r_1)]t}
$$

$$
Z_1(t) = e^{-[\theta + \alpha_1(1-r_1)]t}
$$

$$
Z_2(t) = e^{-[\theta + \alpha_2(1-r_2)]t}
$$

$$
Z_5(t) = e^{-[\beta + \alpha_2(1-r_2)]t}
$$

$$
Z_7(t) = e^{-[\lambda + \alpha_2(1-r_2)]t}
$$

$$
Z_9(t) = e^{-[\mu + \alpha_2(1-r_2)]t}
$$

As an illustration, $R_0(t)$ is the sum of the following contingencies:

(i) The system remains up in the state $S_0$ without making any transition to any other state up to time ‘t’. The probability of this event is

$$
Z_0(t) = e^{-[\theta + \alpha_1(1-r_1)]t}
$$
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

(ii) The system transits from state $S_0$ to state $S_1$ during $(u, u+du)$, $u < t$ and then starting from $S_1$, it remains up continuously during the remaining time $(t-u)$. The probability of this contingency is

$$
\int_0^1 q_{01}(u) du R_1(t-u) = q_{01}(t) \otimes R_1(t)
$$

(iii) The system transits from state $S_0$ to state $S_2$ during $(u, u+du)$, $u < t$ and then starting from $S_2$, it remains up continuously during the remaining time $(t-u)$. The probability of this contingency is

$$
\int_0^1 q_{02}(u) du R_2(t-u) = q_{02}(t) \otimes R_2(t)
$$

Taking Laplace transform of the above relation for $R_i(s)$ can be written in the matrix form as

$$
\begin{bmatrix}
R_0^* \\
R_1^* \\
R_2^* \\
R_5^* \\
R_7^* \\
R_9^*
\end{bmatrix} =
\begin{bmatrix}
1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\
-q_{10}^* & 1 & 0 & -q_{15}^* & 0 & 0 \\
0 & 0 & 1 & -q_{25}^* & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -q_{59}^* \\
0 & -q_{71}^* & 0 & 0 & 1 & 0 \\
0 & 0 & -q_{91}^* & 0 & 0 & -q_{97}^* & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
Z_0^* \\
Z_1^* \\
Z_2^* \\
Z_5^* \\
Z_7^* \\
Z_9^*
\end{bmatrix}
$$

For brevity, we have omitted the argument ‘$s$’ from $q_{ij}^*(s), R_i^*(s)$ and $Z_i^*(s)$. On solving for $R_0^*(s)$, we get

$$
R_0^*(s) = \frac{N_1(s)}{D_1(s)}
$$

Where

$$
N_1(s) = (1 - q_{51}^* q_{15}^* q_{59} - q_{71}^* q_{15}^* q_{57} q_{59}^*) (Z_0^* + q_{02} Z_2^*)
+ (q_{01}^* q_{15}^* + q_{02} q_{25}^*) (q_{59}^* (Z_5^* + q_{07} Z_7^*) + Z_5^*)
+ (q_{01}^* + q_{91} q_{02} q_{25} q_{59} + q_{02} q_{25} q_{37} q_{59}) (Z_1^*) \\
$$

$$
D_1(s) = 1 - q_{10}^* q_{01} - q_{51}^* q_{15}^* q_{59} - q_{10}^* q_{51} q_{02} q_{25} q_{59} - q_{71}^* q_{15} q_{57} q_{59}^* - q_{10}^* q_{71} q_{02} q_{25} q_{57} q_{59}^* \\
$$

106
Taking inverse Laplace transform of above expression, we get the reliability of the system when it starts from state $S_0$.

### 5.9 Measuring Mean Time to System Failure

On the basis of above discussed transition probability and reliability analysis the MTSF can be obtained using the following well known formula:

$$E(T_0) = \int_0^\infty R_0(t)dt = \lim_{s \to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)}$$

---(5.57)

To determine $N_1(0)$ and $D_1(0)$, we first obtain $Z_i^*(0)$.

$$\lim_{s \to 0} Z_i^*(s) = \int Z_i(t)dt$$

$$Z_0^*(0) = \int Z_0(t)dt$$

$$= \int e^{-[\theta + \alpha_1(1-r_1)]t} dt = \psi_0$$

$$Z_1^*(0) = \int Z_1(t)dt$$

$$= \int e^{-[\theta + \alpha_1(1-r_1)]t} dt = \psi_1$$

$$Z_2^*(0) = \int Z_2(t)dt$$

$$= \int e^{-[\theta + \alpha_2(1-r_2)]t} dt = \psi_2$$

$$Z_5^*(0) = \int Z_5(t)dt$$

$$= \int e^{-[\beta + \alpha_2(1-r_2)]t} dt = \psi_5$$
\[ Z_7^*(0) = \int Z_7(t)dt \]
\[ = \int e^{-[\lambda + a_2(1-r_2)]t} dt = \psi_7 \]
\[ Z_9^*(0) = \int Z_9(t)dt \]
\[ = \int e^{-[\mu + a_2(1-r_2)]t} dt = \psi_9 \]

Thus using \( q_{ij}^* (0) = p_{ij} \), we get

\[ N_1(0) = \]
\[ (1 - p_{91}p_{15}p_{59} - p_{71}p_{15}p_{97}p_{59})(\psi_0 + p_{02}\psi_2) + \]
\[ (p_{01}p_{15} + p_{02}p_{25})(p_{59}(\psi_9 + p_{97}\psi_7) + \psi_5) + \]
\[ (p_{01} + p_{91}p_{02}p_{25}p_{59} + p_{02}p_{25}p_{97}p_{59})(\psi_1) \]

\[ D_1(0) = \]
\[ 1 - p_{10}p_{01} - p_{91}p_{15}p_{59} - p_{10}p_{91}p_{02}p_{25}p_{59} - p_{71}p_{15}p_{97}p_{59} - p_{10}p_{71}p_{02}p_{25}p_{97}p_{59} \]

\( N_2(0), N_3(0), N_4(0) \) and \( D_2(0), D_3(0) \) may be calculated with the help of above discussed or generated recurrence ides. Using (5.64) and (5.65) in (5.57), we get \( E(T_0) \), the expected life time of the system starting from state \( S_0 \) which is also known as MTSF.

### 5.10 Availability Analysis:

By definition, \( A_i(t) \) is defined as the probability that the system is up at epoch ‘t’ when it initially starts from the state \( S_i \in E \).

By simple probability arguments, we have the following recursive relations among \( A_i(t) \)’s:

\[ A_0(t) = Z_0(t) + q_{01}(t)A_1(t) + q_{02}(t)A_2(t) \]
\[ A_1(t) = Z_1(t) + q_{10}(t)A_0(t) + q_{15}(t)A_5(t) \]
\[ A_2(t) = Z_2(t) + q_{25}(t)A_5(t) + q_{23}(t)A_3(t) \]
\[ A_3(t) = q_{34}(t)A_4(t) \]
\[ A_4(t) = q_{45}(t)A_5(t) \]
\[ A_5(t) = Z_5(t) + q_{56}(t)A_6(t) + q_{59}(t)A_9(t) \]
\[ A_6(t) = q_{65}(t)A_5(t) \]
\[ A_7(t) = Z_7(t) + q_{78}(t)A_8(t) + q_{71}(t)A_1(t) \]
\[ A_8(t) = q_{87}(t)A_7(t) \]
\[ A_9(t) = Z_9(t) + q_{9,10}(t)A_{10}(t) + q_{91}(t)A_1(t) + q_{97}(t)A_7(t) \]
\[ A_{10} = q_{10,9}(t)A_9(t) \]

where

\[ Z_0(t) = e^{-[\theta + \alpha_1(1-r_1)]t} \]
\[ Z_1(t) = e^{-[\theta + \alpha_1(1-r_1)]t} \]
\[ Z_2(t) = e^{-[\theta + \alpha_2(1-r_2)]t} \]
\[ Z_5(t) = e^{-[\beta + \alpha_2(1-r_2)]t} \]
\[ Z_7(t) = e^{-[\lambda + \alpha_2(1-r_2)]t} \]
\[ Z_9(t) = e^{-[\mu + \alpha_2(1-r_2)]t} \]

As an illustration \( A_0(t) \) is the sum of the following mutually exclusive contingencies:

(i). System continues to be up state \( S_0 \) till epoch ‘t’. The probability of this event is \( Z_0(t) \).
(ii). The system transits to state $S_1$ from state $S_0$ during $(u,u+du)$, $u<t$ and then starting from state $S_1$ at epoch $u$, it is found to be up at epoch $(t-u)$. The probability of this event is

$$\int_0^1 q_{01}(u)du A_1(t-u) = q_{01}(t) A_1(t)$$ \hspace{1cm} \text{(5.77)}$$

(iii). The system transits from state $S_0$ to state $S_2$ during $(u,u+du)$, $u<t$ and then starting from $S_2$, it remains up continuously during the remaining time $(t-u)$. The probability of this contingency is

$$\int_0^1 q_{02}(u)du A_2(t-u) = q_{02}(t) A_2(t)$$ \hspace{1cm} \text{(5.78)}$$

Taking Laplace transform of the above relation for $A_i^*(s)$ can be written in the matrix form as

$$\begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_3^* \\ A_4^* \\ A_5^* \\ A_6^* \\ A_7^* \\ A_8^* \\ A_{10}^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{10}^* & 1 & 0 & 0 & 0 & -q_{15}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^* & 0 & -q_{25}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -q_{34}^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{45}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^* & 0 & 0 & -q_{59}^* & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{65}^* & 1 & 0 & 0 & 0 \\ -q_{71}^* & 0 & 0 & 0 & 0 & 0 & 1 & -q_{78}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^* & 1 & 0 & 0 \\ -q_{91}^* & 0 & 0 & 0 & 0 & 0 & -q_{97}^* & 0 & 1 & -q_{9,10}^* \end{bmatrix}^{-1} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \\ Z_4^* \\ Z_5^* \\ Z_6^* \\ Z_7^* \\ Z_8^* \\ Z_{10}^* \end{bmatrix}$$

For brevity, we have omitted the argument ‘s’ from $q_{ij}^*(s)$, $A_i^*(s)$ and $Z_i^*(s)$.

On solving for $A_{10}^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$ \hspace{1cm} \text{(5.79)}$$

where
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

\[ N_2(s) = (1 - q_{65}q_{56} - q_{67}q_{78} + q_{56}q_{65}q_{87}q_{78} - q_{910}q_{109} + q_{65}q_{56}q_{910}q_{109} + q_{87}q_{78}q_{910}q_{109} - q_{65}q_{56}q_{87}q_{78}q_{910}q_{109}) (Z_0^*) \\
+ q_{01}(Z_1^* + Z_6^*) + q_{02}(Z_2^* + Z_7^*) + q_{02}q_{23}(Z_5^* + Z_8^*) \\
+ q_{02}q_{23}q_{34}(Z_4^* + Z_5^*) + (Z_5^* + Z_10^* + q_{56}Z_{11}^*) \\
(\frac{q_{01}q_{15} + q_{02}q_{25} + q_{02}q_{23}q_{34}q_{45} - q_{01}q_{15}q_{87}q_{78} - q_{02}q_{25}q_{87}q_{78} \\
- q_{02}q_{23}q_{34}q_{45}q_{87}q_{78} - q_{01}q_{15}q_{109} - q_{02}q_{25}q_{910}q_{109} + q_{02}q_{23}q_{87}q_{78}q_{910}q_{109} \\
+ q_{02}q_{25}q_{87}q_{78}q_{910}q_{109} + q_{02}q_{23}q_{34}q_{45}q_{87}q_{78}q_{109}q_{910}) \\
D_2(s) = 1 - q_{10}q_{01} - q_{65}q_{56} + q_{01}q_{10}q_{65}q_{56} - q_{87}q_{78} + q_{01}q_{10}q_{87}q_{78} \\
+ q_{65}q_{56}q_{87}q_{78} - q_{01}q_{10}q_{65}q_{56}q_{87}q_{78} - q_{91}q_{01}q_{15}q_{59} - q_{01}q_{10}q_{25}q_{59} \\
- q_{01}q_{02}q_{23}q_{34}q_{45}q_{59} - q_{71}q_{01}q_{15}q_{37}q_{59} \\
- q_{71}q_{02}q_{25}q_{97}q_{59} - q_{71}q_{02}q_{23}q_{34}q_{45}q_{97}q_{59} + q_{91}q_{01}q_{23}q_{87}q_{78}q_{59} \\
+ q_{91}q_{02}q_{25}q_{87}q_{78}q_{59} + q_{91}q_{02}q_{23}q_{34}q_{45}q_{87}q_{78}q_{59} - q_{109}q_{910} \\
+ q_{01}q_{10}q_{109}q_{910} + q_{65}q_{56}q_{109}q_{910} - q_{91}q_{01}q_{87}q_{78}q_{109}q_{910} + q_{10}q_{01}q_{65}q_{56}q_{87}q_{78}q_{109}q_{910} \\
q_{10}q_{01}q_{65}q_{56}q_{87}q_{78}q_{109}q_{910} \\
\text{---(5.80)} \]

Therefore, the steady state availability of the system is given by

\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to \infty} A_0(s) = \frac{N_2(0)}{D_2(0)} \text{ ---(5.81)} \]

Thus using \( q_{ij}^*(0) = p_{ij} \), we get

\[ N_2(0) = (1 - p_{65}p_{56} - p_{87}p_{78} + p_{56}p_{65}p_{87}p_{78} - p_{910}p_{109} + p_{65}p_{56}p_{910}p_{109} + p_{87}p_{78}p_{910}p_{109} - p_{65}p_{56}p_{87}p_{78}p_{910}p_{109}) (\psi_1 + \psi_6) \\
+ p_{01}(\psi_2 + \psi_7) + p_{02}p_{23}(\psi_3 + \psi_8) + p_{02}p_{23}p_{34}(\psi_4 + \psi_9) \\
+ (\psi_5 + \psi_{10} + p_{56}\psi_{11}) \]
And
\[ D_2(0) = 1 - p_{10}p_{01} - p_{65}p_{56} + p_{01}p_{10}p_{65}p_{56} - p_{97}p_{78} + p_{01}p_{10}p_{87}p_{78} \]
\[ + p_{65}p_{56}p_{07}p_{78} - p_{01}p_{10}p_{65}p_{56}p_{87}p_{78} - p_{91}p_{01}p_{15}p_{59} - p_{91}p_{02}p_{25}p_{59} \]
\[ - p_{91}p_{02}p_{23}p_{34}p_{45}p_{56} - p_{71}p_{01}p_{15}p_{97}p_{59} - p_{71}p_{02}p_{25}p_{97}p_{59} \]
\[ - p_{71}p_{02}p_{23}p_{34}p_{45}p_{97}p_{59} + p_{91}p_{01}p_{23}p_{87}p_{78}p_{59} + p_{91}p_{02}p_{25}p_{87}p_{78}p_{59} \]
\[ + p_{91}p_{02}p_{23}p_{34}p_{45}p_{87}p_{78}p_{59} - p_{10,9}p_{9,10} + p_{01}p_{10}p_{10,9}p_{9,10} \]
\[ + p_{65}p_{56}p_{10,9}p_{9,10} - p_{01}p_{10}p_{65}p_{56}p_{10,9}p_{9,10} + p_{87}p_{78}p_{10,9}p_{9,10} \]
\[ - p_{10}p_{01}p_{87}p_{78}p_{10,9}p_{9,10} - p_{65}p_{56}p_{87}p_{78}p_{10,9}p_{9,10} \]
\[ + p_{10}p_{01}p_{65}p_{56}p_{87}p_{78}p_{10,9}p_{9,10} \]
\[ = 0 \quad \text{(on simplification)} \quad \text{----(5.82)} \]

Hence by L’Hospital’s rule
\[ A_0 = \frac{N_2(0)}{D_2(0)} \]

To obtain \( D_2'(0) \), we collect the coefficients of \( q_{ij}^*(0) = -m_{ij} \).

On getting the value of \( D_2'(0) \), we get the expression for the steady availability of the system starting from \( S_0 \).

The expected uptime of the system during \((0,t)\) is given by
\[ \mu_{up}(t) = \int_0^t A_0(u)du \quad \text{----(5.83)} \]
So, that \( \mu_{up}^*(S) = A_0^*(S)/S \quad \text{----(5.84)} \)

### 5.11 Busy Period Analysis:

This reliability measure explains busy of repairman when failure a unit occurred. If repairman is not available at instant then unit wait for repair. The analysis of busy period in every states is:
\[ B_0(t) = q_{01}(t)B_1(t) + q_{02}(t)B_2(t) \]
\[ B_1(t) = q_{10}(t)B_0(t) + q_{15}(t)B_5(t) \]
\[ B_2(t) = q_{25}(t)B_5(t) + q_{23}(t)B_3(t) \]
\[ B_3(t) = Z_3(t) + q_{34}(t)B_4(t) \]
\[ B_4(t) = Z_4(t) + q_{45}(t)B_5(t) \]
\[ R_5(t) = Z_5(t) + q_{56}(t)B_6(t) + q_{59}(t)B_9(t) \]
\[ B_6(t) = Z_6(t) + q_{65}(t)B_5(t) \]
\[ B_7(t) = q_{78}(t)B_8(t) + q_{71}(t)B_1(t) \]
\[ B_8(t) = Z_8(t) + q_{87}(t)B_7(t) \]
\[ B_9(t) = q_{9,10}(t)B_{10}(t) + q_{91}(t)B_1(t) + q_{97}(t)B_7(t) \]
\[ B_{10}(t) = Z_{10}(t) + q_{10,9}(t)B_9(t) \]

Where
\[ Z_3(t) = G_1e^{-\theta t} \quad Z_4(t) = G_2e^{-\theta t} \quad Z_6(t) = e^{-\beta t}G_2(t) \quad Z_8(t) = e^{-\lambda t}G_2(t) \quad Z_{10}(t) = e^{-\mu t}G_2(t) \]

With the help of contingencies analysis of three condition discussed in above section and adopting the same process for solving for \( B_0^*(s) \). Initially busy period may be evaluated as
\[ B_0^*(s) = \frac{N_3(s)}{D_2(s)} \] In the long run the function of time for which the system is under repair is given by
\[ B_0 = \lim_{s \to 0} B_0(t) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3}{D_2} \quad ----(5.95) \]

In steady state, the total fraction of the discussion time of the expert repairman, is given by
\[ B_0 = \lim_{s \to \infty} sB_0^*(s) = \frac{N_3}{D_2} \quad ----(5.96) \]
and $D_2$ is already specified.

Where $N_3(s)$ can be found on the basis of above discussed procedure and $D_2(s)$ is already specified.

$$N_3 = P_{01} P_{12} P_{21} (P_{45} P_{54} P_{65} P_{56})$$

5.12 Expected Number of Visits by the Repairman:

On basis of number of visit we can Judge the potential of repairman either they are skilled or less skilled.

By probabilistic arguments, we have the following recursive relations:

$$V_0(t) = Q_{01}(t) S [1 + V_1(t)] + Q_{02}(t) S V_2(t)$$

$$V_1(t) = Q_{10}(t) S V_0(t) + Q_{15}(t) S V_5(t)$$

$$V_2(t) = Q_{25}(t) S V_5(t) + Q_{23}(t) S [1 + V_3(t)]$$

$$V_3(t) = Q_{34}(t) S [1 + V_4(t)]$$

$$V_4(t) = Q_{45}(t) S [1 + V_5(t)]$$

$$V_5(t) = Q_{56}(t) S [1 + V_6(t)] + Q_{59}(t) S V_9(t)$$

$$V_6(t) = Q_{65}(t) S V_5(t)$$

$$V_7(t) = Q_{78}(t) S [1 + V_8(t)] + Q_{71}(t) S V_1(t)$$

$$V_8(t) = Q_{87}(t) S V_7(t)$$

$$V_9(t) = Q_{10,9}(t) S V_9(t)$$

Taking Laplace Stieltjes Transforms (L.S.T.) of the above equations and solving them for $V_0^*(s)$, we get
$V_0^*(s) = \frac{N_4(s)}{D_3(s)}$  \hspace{1cm} ...(5.108)

In steady-state, the total number of visits by the ordinary repairman per unit time is given by

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} [sV_0^{**}(s)] = N_4/D_3 \hspace{1cm} ...(5.109)$$

Where $N_4(s)/N_4$ must be calculated as per above described process and $D_3$ is also specified

### 5.13 Cost and Profit Analysis with its sensitivity

Total cost incurred Expected up time and down time of the system and busy period of repair man in time $(0,t)$

$$\mu_{up}(t) = \int_0^t A_0(u)du = \mu_{up}^*(s) = \frac{A_1^*(s)}{s}$$

$$\mu_{dn}(t) = \mu_{dn}^*(t) = \frac{1}{s}\mu_{up}^*(s)$$

$$\mu_{sb}(t) = \int_0^t B_0(u)du = \mu_{up}^*(s) = \frac{B_1^*(s)}{s}$$

$$\mu_{V_0}(t) = \int_0^t V_0(u)du = \mu_{V_0}^*(s) = \frac{V_1^*(s)}{s} \hspace{1cm} ...(5.110-5.113)$$

Here cost is fully depends on the behavior of up time and downtime of system and available and non-available repairman. If repairman is busy for less time and minimum number of visit by repairman for repair the failure unit then cost incurred for repair is minimum otherwise its to high.
5.14 Profit Function Analysis:

We are now in a position to obtain the two profit functions of the system by considering the characteristics obtained in the preceding sections. Hence total profit may be calculated as per uptime or downtime of stochastic process of transition diagram.

Net expected Profit incurred in (0,t) = Expected total revenue in (0,t) - Expected total expenditure in (0,t)

Thus

\[ P_1(t) = C_0 \mu_{up}(t) - C_2 \mu_b(t) - C_3 V_0(t) \]
\[ P_2(t) = C_1 \mu_{dn}(t) - C_2 \mu_b(t) - C_3 V_0(t) \] (5.114-5.115)

Where \( C_0 \) & \( C_1 \) is the revenue per unit uptime & downtime, \( C_2 \) is the cost per unit time for which the repairman is busy and \( C_3 \) is the cost per unit visits by the repairman.

Therefore, the expected profit per unit time in steady state are given by

\[ P_1 = \lim_{t \to \infty} \frac{P_1(t)}{t} \]
\[ = \lim_{s \to 0} s^2 P_1^*(s) \]
\[ = C_0 A_0 - C_2 B_0 - C_3 V_0 \] (5.116)

And

\[ P_2 = \lim_{t \to \infty} \frac{P_2(t)}{t} \]
\[ = \lim_{s \to 0} s^2 P_2^*(s) \]
\[ = C_1 A_0 - C_2 B_0 - C_3 V_0 \] (5.117)

The overall profit varies as per sensitivity of above discussed two type of cost during repair by the repairman as per his availability and non-availability.
5.15 Sensitivity due to Repairman cost:

Sensitivity of profit is fully depends cost of revenue, cost incurred in repair when repairman is busy and cost incurred in number of visit by repairman if repairman is not completely skilled. Its variation is either $C_2 > C_3$ or $C_3 > C_2$ with either it may be uptime or down time. The another case arises as per behaviour of $C_0 > C_1$ or $C_1 > C_0$.

5.16 Sensitivity due to Distribution:

The wide applicability of model can be analysis by the various sensitivity analysis involved in type of distribution applied in the form of p.d.f. of repair time of availability or non availability of repairman. The repair facility may be any of the distribution i.e. general distribution. Here the particular case has been imposed on Inverse Gaussian distribution.

When we used the repair time of available repairman follow Inverse Gaussian Distribution having p.d.f.

$$G(t)=\frac{1}{\sqrt{2\pi} nt^{3/2}} e^{-(t-\theta)^2/2\theta^2 t}; \quad p\leq t \quad (5.118)$$

This can be solved by Laplace Transformation for above density function and using Laplace Stieltjes Transform used as above calculation.

Hence c.d.f. become

$$g^*(s)=G^*(s)=e^{\left\{1-\sqrt{(1+2s\theta^2)}\right\}/\theta} \quad (5.119)$$

The remaining result can be obtained for the above discussed equation for Reliability, Availability, Busy Period and Number of visit by repairman and then profit will be obtained.

5.17 Sensitivity due to time interval

The failure time and repair time interval is the one of the factor for the cost of repair as per uptime and downtime.
They are mutually exclusive and exhaustive i.e.

1) The interval \((0,t+\Delta t)\) is not intercepted by a system down state

2) The interval\((0,t+\Delta t)\) is intercepted by a system down state

Hence Reliability varies \(R(t, \Delta t)= R(t, t+\Delta t)\)

Finally due to maximum interval between failure and repair revels the maximize the cost during repair.

5.18 Application of Model

Most commonly the \textit{pair of unequal unit standby} system model has been used in Generation and Maintenance in power plants with optimum distribution of electricity, manufacturing industries and elsewhere. Due to maintaining a high or required level of reliability, It is an essential requirement by the manager. Hence reliability of the system or its component may be increased by the repair of failed component/systems is an important part of the analysis. In this case repairman is one of the essential parts of repairable systems for economical point of view. In order to improve the reliability or raise the availability it is essential requirement to reduce the loss of entire component for optimum utilization of resources.

5.19 Sensitivity Analysis in Graphical View

For a more concrete study of the system behavior we plot the MTSF and Steady state functions w.r.t \(\alpha_1\) (failure rate) and \(\alpha_2\) for different values of \(r_1(=0.15,0.30,0.45)\) while the other parameters are kept fixed as \(\beta_1=0.03, \beta_2=0.06, \theta=0.09, \vartheta=2, \omega=2, r_2=0.40\). The curves so obtained are shown in Fig.5.2, Fig.5.3 and Fig.5.4 respectively who gives the sensitivity behaviour of MTSF and the Profit \(P_1\) for uptime & \(P_2\) for downtime. The another part of cost and profit sensitivity analysis may be seen in Fig.5.5 and Fig.5.6 which is discussed for fixed revenue (for uptime and downtime) and the corresponding value of repairman cost involved as per Number of visit. Here we have assumed the single visit is sufficient for complete the repair work by the repairman.
From Fig. 5.2 it is observed that the MTSF of the system decreases w.r.t $\alpha_1$ irrespective of other parameters.

**Fig. 5.2 Sensitivity on MTSF & Failure rate ($\alpha_1$)**

**Fig. 5.3 Sensitivity on Profit($P_1$) & Failure Rate($\alpha_1$)**
For different value of $\alpha_1$ and $r_1$, MTSF varies with indicating higher MTSF for higher values of failure rate as well as repair rate. Hence from Fig.5.2, we conclude that the high correlation ($r_1$) between failure and repair times tends to increase expected life of the system.

Fig.5.4 Sensitivity of Profit ($P_2$) VS Failure Rate($\alpha_2$) for different post repair rate($r_1, r_2, r_3$)
Sensitivity Analysis for pair of unequal unit standby Reliability Mode

**Fig. 5.5** Sensitivity of Profit ($P_1$) VS Revenue ($C_0$) at various repair cost ($C_2$) for single visit

**Fig. 5.6** Sensitivity of Profit ($P_2$) VS Revenue ($C_1$) at various repair cost ($C_2$) for single visit
In Fig.5.5 and Fig.5.6, curves respectively show the variations in steady state profits $P_1$ and $P_2$ w.r.t $\alpha_1$ for different values of $r_1$ while we fix $C_0=1000$, $C_1=400$, $C_2=150$ and $C_3=50$ with fixed value of in addition to the parameters taken in MTSF seen in Fig5.2. From Fig.5.3 and Fig.5.4 it is observed that values of $P_1$ and $P_2$ deceases as value of $\alpha_1$ increases.