Chapter IV

THE GOLDEN RATIO IN TRADITIONAL ARCHITECTURE

4.1. INTRODUCTION

The Golden Ratio (or Golden section) which is known as 'Kanakamuri' in traditional architecture is an important concept in both ancient and modern artistic and architectural design. It is the geometrical proportion in which a line AB is divided into two parts by an interior point P in such a way that $\frac{AB}{AP} = \frac{AP}{PB}$ [cf Fig. 4(a)].

![Fig. 4(a)](image)

A rectangle whose length is in this ratio to its breadth is called a golden rectangle. In this chapter we illustrate the existence of golden ratio in traditional architecture, relation between golden ratio and arddhādhika, and its application in the constructions of Nātyamaṇḍapa (Kūthampalam) and idols of deities (bimbā).
4.2. THE CONCEPT 'GOLDEN RATIO'

Dividing a segment into two parts in mean and extreme proportion, so that the smaller part is to the larger part as the larger is to the entire segment, yields the so-called Golden section and the ratio \( \frac{\sqrt{5} + 1}{2} = 1.618 \) (approximately) designated as \( \phi \), is known as the golden number. The ratio \( \frac{\sqrt{5} - 1}{2} = 0.618 \), approximately, is the reciprocal of \( \phi \). This number has many fascinating qualities and the ancient Greeks considered the regular pentagon which includes a number of 'golden ratio' relationships, as a holy symbol. In a regular pentagon PQRST there is a golden ratio relationship between any diagonal and any side of it, namely,

\[
\frac{PR}{PU} = 1.618.....
\]

Further, all the diagonals intersect each other in golden ratio such that, [cf Fig.4(b)],

\[
\frac{PR}{LR} = \phi, \quad \frac{LR}{MR} = \phi, \quad \frac{MR}{LM} = \phi, \quad \text{when } \phi = 1.618, \text{ approximately.}
\]
In a regular decagon (10 sided polygon) the ratio of a side to the radius of the circumcircle is also $\phi$.

4.3. **GOLDEN RECTANGLE AND ITS PROPERTY**

A rectangle in which the ratio of the length to width is equal to $1.618$ approximately, is called a golden rectangle [cf Fig.4(c)]. This number $\phi$ produces a set of nesting rectangles.

$PQRS$ is a rectangle such that $\frac{PO}{QR} = \phi$, and $PTUS$ is a square such that $\frac{QR}{TQ} = \phi$.

![Fig. 4(c)](image-url)
This is a representation of the so called 'golden rectangle'. If the largest square in a golden rectangle is cut away, the figure remaining will also be a golden rectangle. Such rectangles are characterised by a length to width ratio of \((1 + \sqrt{5})/2\), the golden ratio\(^{(4)}\). It is believed that the ancient Egyptians may have used this ratio in the construction of Pyramids. This ratio recurs often in number theory; for example

\[
\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \frac{1 + \sqrt{5}}{2}
\]

\[
= 1.618 \text{ approximately},
\]

where \(F_n\) is the \(n^{th}\) Fibonacci number\(^{(4)}\). This is an irrational number which is the solution of the equation

\[
x^2 - x - 1 = 0.
\]

Solving the above equation we have,

\[
x = \frac{1 \pm \sqrt{5}}{2}
\]

The golden rectangle whose sides are in the ratio 1:\(\phi\) has the following property that the ratio of the length of the smaller side to the greater side is equal
to the ratio of the length of the greater side to the sum of the lengths of the two sides \(49\)

\[ \frac{1}{\phi} = \frac{\phi}{\phi + 1}, \]

or \(\phi\) is the mean proportion between 1 and \(\phi + 1\).

\[ \text{ie}, \quad 1 + \phi = \phi^2 \]

\[ \text{ie}, \quad \phi^2 - \phi - 1 = 0 \]

If we divide the golden rectangle into two parts such that one of the smaller resulting rectangles is a square then it follows that the proportion of the second rectangle is 1:\(\phi\) itself [cf Fig. 4(d)].

![Diagram of golden rectangle](image)

The proportion is \(\frac{\phi - 1}{1}\) and multiplying by \(\phi\) we get \(\frac{\phi^2 - \phi}{\phi}\). But \(\phi^2 - \phi = 1\).

\[ \therefore \frac{\phi^2 - \phi}{\phi} = \frac{1}{\phi} \]

which is the same as the original proportion.
4.4. GOLDEN RECTANGLES AND FIBONACCI NUMBERS

A sequence of numbers each of which, after the second, is the sum of the two preceding numbers is known as Fibonacci numbers. The sequence 1, 1, 2, 3, 5, 8, 13, ........ is a Fibonacci sequence of numbers. This sequence was discovered by Leonardo Fibonacci, also known as Leonardo of Pisa (1170-1250). The formula for generating the sequence is

\[ x_n = x_{n-1} + x_{n-2}, \text{ where } x_n \text{ is the } n^{th} \text{ term of the sequence, } n > 2. \]

Another formula for generating the Fibonacci numbers is attributed to Lucas\(^4\). It is given as

\[ x_n = \frac{1}{\sqrt{5}} \left[ (\frac{1 + \sqrt{5}}{2})^n - (\frac{1 - \sqrt{5}}{2})^n \right] \]

Further, the sequence of numbers\(^{49}\),

1, 1.618, 2.618, 4.236, 6.854, 11.090, ........

(correct to three places of decimals only) have the property that if we add the first two terms together (1 + 1.618) we get the third, 2.618. In the same way the sum of the second and third 1.618 + 2.618 gives the
fourth, 4.236; and so on. Thus each successive term is the sum of the preceding two. Therefore it is a Fibonacci sequence of numbers. In the above sequence the ratio of any term to its preceding term is 1.618 which is the golden ratio.

The Fibonacci numbers can be used to make a golden rectangle. Consider a unit square representing the first term of Fibonacci sequence. Then add a second square. Add a third square to fit the longest side. Again add a fourth square with its side as the longest side of the above square. If we continue the process we will eventually get a golden rectangle [cf Fig.4(e)].

![Fig.4(e)](image)

Thus the Fibonacci numbers and the golden rectangle are closely related.
4.5. GOLDEN RATIO AND ARDDHĀDHĪKA

In Vāstusāstras, four types of width to length ratios are defined in determining the width of the house in guṇāmsa method. They are Samatata, Pādādhika, Arddhādhika, and Pādōna (explained in Chapter I). They define four categories of ratios of width to length in rectangular buildings. The arddhādhika ratios are $1:1.5, 1:2.5, 1:3.5, 1:4.5, 1:5.5, 1:6.5$. It is to be noted that the ratios from $1:1.5$ upto $1:1.75$ are also considered as arddhādhika. According to Manuṣyālayacandrika, pādādhika ratios are most suitable for domestic buildings and arddhādhika ratios are accepted for buildings, idols, nātyamaṇḍapa etc, which have aesthetic values. Thus the golden ratio belongs to the arddhādhika ratios in traditional architecture. The existence of golden ratio in traditional architecture may be understood from the construction of Kūthampalam of Kerala.

4.6. GOLDEN RATIO AND NĀTYAMAṆḌAPAM (KUTHAMPALAM)

'Tantrasamuccaya' (Śilpabhatam) of Kanippayyur Damodaran Namboothirippad, gives a special method of constructing a Nātyamaṇḍapam (Kuthampalam) in temple
complexes of Kerala. The method is explained in the verse given below:

paryantē pratiyōnibhājī bāhirutthē vōttarasyā thavā
maddhyastē dalītē, tatō vibhajitē samyak caturväggakai:
syādamśa: pada, māyatistu vitatērdwābhyām padēbhyām yutam,
tacchiśtā tati, ruttaram natanadhāmnām dwitisamkhyam matam
(Tantrasamuccaya (Silpabhamgam), Chapter 10, slōka 1)

This states that the perimeter along the uttara (beam) of the nātyamaṇḍapa must be of the 'pratiyōni' (opposite yōni) of that of the prāsāda (Srīkovil). [This is due to the fact that the nātyamandapam and the Srīkovil are facing each other]. This perimeter may be measured either along the central line or along the boundary line of the uttara (beam). Divide the semiperimeter into 16 equal parts and each part is called a 'pada'. The length of the maṇḍapa (along uttara) is obtained by adding 2 pada to the half of the semiperimeter (to ¼ of the perimeter) and the width is determined by subtracting this length from the semiperimeter.

Let us examine the ratio of the length to width in the above construction of rectangular Nātyamaṇḍapa.
Let the perimeter of a Nātyamandapa be $P$ and $a$ be the side of a square of the same perimeter.

Then $P = 4a$

\[ \text{Semiperimeter} = \frac{P}{2} \]
\[ = 2a \]

Dividing by 16, we get, $\frac{P}{32} = \frac{a}{8}$. This is the unit 'Pada' defined in the text. "." by the above verse we have,

Length of the Nātyamandapam $= a + \frac{a}{8} \times 2$

\[ = a + \frac{a}{4} \]
\[ = \frac{5a}{4} \]

Width of the Mandapa $= a - \frac{a}{8} \times 2$

\[ = a - \frac{a}{4} \]
\[ = \frac{3a}{4} \]

\[ \therefore \frac{\text{Width}}{\text{Length}} = \frac{\frac{3a}{4}}{\frac{5a}{4}} \]
\[ = \frac{3}{5} \]

ie, Width:Length $= 3:5$
or \( L:W = 5:3 \)

\[
= \frac{5}{3}
\]

\[
= 1.66.......
\]

But the golden ratio is 1.618 (approximately). Hence the above ratio is very close to the golden ratio.

4.7. **CONSTRUCTION OF BIMBA (IDOL) AND GOLDEN RATIO**

It is significant to note that the bodies of many living beings (natural organisms) including man (the human body), are really based on golden ratio relationship. For example, the ratio of the height of the navel from the feet to the height of the head from the navel (of a man of standard height) is \( \phi \). In the construction of Navatāla or Daśatāla bimba (idol), the ratio accepted in Tantrasamuccaya\(^{15}\) is 1.6, approximately, which is very close to \( \phi \).

In the case of Navatāla bimba, the total height of the bimba (idol) is divided into 108 equal parts and each part is called an angula. Two angula form a 'Kala' or 'Gōlaka' and 12 angula constitute a 'Tāla'. Since
the height of the bimba is 9 Tāla, it is known as Nāvatāla bimba. The height of the idol is divided into two parts at the navel point in such a way that 66 angulas are below the navel point and 42 angulas are above it. Then the ratio of these lengths is equal to $\frac{66}{42} = 1.6$, approximately, which is very close to the golden ratio. Further, the ratio of the total length to the height of the bimba upto the navel is

$$\frac{108}{66} = 1.6363, \text{ approximately}.$$  

This is also very close to the golden ratio and belongs to the arddhādhika.

Similarly, in the case of Dasatāla bimba, the total number of divisions is 120. The ratio of the height of the navel from the feet to the height of the head from the navel is $73/47$. The value of the ratio is approximately equal to 1.6 and the ratio of the total height to the height of the navel from the feet is 1.64, approximately. Thus these two ratios are very close to the golden ratio and they belong to the arddhādhika.
Thus the golden ratio which is a special case of arddhādhika ratios is used for constructing artefacts having aesthetic values, in the traditional architecture of Kerala.