CHAPTER 4

BER- ANALYSIS OF MIMO-OFDM SYSTEMS USING SPACE-TIME AND SPACE-FREQUENCY CODING SCHEMES

4.1 INTRODUCTION

The combination of STBC with OFDM, termed ‘STBC-OFDM’ was first proposed by Agrawal in [24]. Following this development, various researchers have focused on designs for scenarios where the channel is assumed to be known at the receiver, for example, the designs proposed in [21] [25][161]. The results from these works are consistent with the findings in [26][120]. Which indicate that the combination of MIMO techniques with OFDM improve the transmission rate, range and reliability. Moreover, frequency diversity can be achieved in addition to the space and time diversity exploited by STBC. Hence, the combination of MIMO with OFDM mix to another code known as SFBC or SFBC-OFDM, which exploit the maximum diversity available in MIMO channels [27]. In STBC-OFDM, the information symbols are coded across multiple antennas and time via the use of multiple consecutive OFDM symbols [28], whereas, SFBC symbols are coded across multiple antennas and multiple OFDM subcarriers [26].

In this chapter, we present the performance of STBC-OFDM and SFBC-OFDM. Simulation results are presented and analyzed for different number of transmit and receive antennas.

4.2 PERFORMANCE OF SPACE-TIME BLOCK CODES

The performance of space-time block codes depends on the type of modulation and the number of transmit and receive antennas used. Complex modulations give better bit error rate performance than real modulations and it is especially true when the number of
transmit antennas is more than 1. However, space-time block code with real modulation would have better bandwidth efficiency performance than complex modulation. This is because space-time block codes with real modulation require transmission of less data than space-time block codes with complex modulation. On the other hand, space-time block codes with two transmit antennas always give better performance because they transmit more data. This would give the receiver the ability to recover the transmitted data. Moreover, with larger number of receive antennas, the same transmitted data would be received by more than one receive antenna. This is an advantage because, if the one receive antenna did not recover the transmitted data correctly, the second receive antenna could do it. The chance that at least one out of two receive antennas would receive the transmitted data uncorrupted, is always higher than, if there is only one receive antenna.

A simple Space-time Code suggested by Mr. Siavash M Alamouti [21], presented a simple method for achieving spatial diversity with two transmit antennas. Here, we assume that the channel is a Rayleigh multipath channel and the modulation is BPSK/QPSK. The scheme is as follows [176] -

1. Consider that we have a transmission sequence, for example \( \{s_1, s_2, s_3, ..., s_n\} \)

2. In normal transmission, we will be sending \( s_1 \) in the first time slot, \( s_2 \) in the second time slot, \( s_3 \) in the third time slot and so on.

3. In this scheme, the symbols are to be grouped into groups of two. In the first time slot, send \( s_1 \) and \( s_2 \) from the first and second antenna. In second time slot send \(-s_2^*\) and \( s_1^* \) from the first and second antenna. In the third time slot send \( s_3 \) and \( s_4 \) from the first and second antenna. In fourth time slot, send \(-s_4^*\) and \( s_3^* \) from the first and second antenna, and so on.
4. Though we are grouping two symbols, we still need two time slots to send two symbols. Hence, there is no change in the data rate.

4.2.1 2-Transmit, 1-Receive Alamouti STBC coding

Assumptions

1. The channel experienced by each transmit antenna is independent from the channel experienced by other transmit antennas.

2. For the \( i^{th} \) transmit antenna, each transmitted symbol gets multiplied by a randomly varying complex number \( h_i \). As the channel under consideration is a Rayleigh channel, the real and imaginary parts of \( h_i \) are Gaussian distributed having mean

\[
\mu_{h_i} = 0 \quad \text{and variance} \quad \sigma^2_{h_i} = \frac{1}{2}.
\]

3. The channel between each transmit and receive antenna varies randomly with time. However, the channel is assumed to remain constant over two time slots.

4. On the receive antenna, the noise \( w \) has the Gaussian probability density function

\[
p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}} \quad \text{with} \quad \mu = 0 \quad \text{and} \quad \sigma^2 = \frac{N_0}{2}.
\]
5. The channel \( h_i \) is known at the receiver.

### 4.2.1.1 Receiver with Alamouti STBC

In the first time slot, the received signal is

\[
y_1 = h_1s_1 + h_2s_2 + w_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + w_1
\]  

(4.1)

In the second time slot, the received signal is,

\[
y_2 = -h_1s_2^* + h_2s_1^* + w_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix} + w_2
\]  

(4.2)

where \( y_1 \) and \( y_2 \) is the received symbol on the first and second time slot respectively, \( h_1 \) is the channel from 1\(^{st} \) transmit antenna to receive antenna, \( h_2 \) is the channel from 2\(^{nd} \) transmit antenna to receive antenna, \( s_1 \) and \( s_2 \) are the transmitted symbols and \( w_1 \) and \( w_2 \) is the noise on 1\(^{st} \) and 2\(^{nd} \) time slots.

Since the two noise terms are independent and identically distributed,

\[
E\left( \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} w_1^* \\ w_2^* \end{bmatrix} \right) = \begin{bmatrix} |w_1|^2 & 0 \\ 0 & |w_2|^2 \end{bmatrix}
\]  

(4.3)

For convenience, the above equation can be represented in matrix notation as follows:

\[
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}
\]  

(4.4)

Let us define \( H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \). To solve for \( \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \), we need to find the inverse of \( H \).

We know that, for a general m x n matrix, the pseudo inverse is defined as,

\[
H^+ = \left( H^H H \right)^{-1} H^H
\]  

(4.5)

The term,
(H^H H) = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2 & -h_1 \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \tag{4.6}

Since this is a diagonal matrix, the inverse is just the inverse of the diagonal elements, i.e.,

\[ (H^H H)^{-1} = \begin{bmatrix} 1/(|h_1|^2 + |h_2|^2) & 0 \\ 0 & 1/(|h_1|^2 + |h_2|^2) \end{bmatrix} \tag{4.7} \]

The estimate of the transmitted symbol is,

\[
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \left( H^H H \right)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \left( H^H H \right)^{-1} H^H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{4.8}
\]

4.2.1.2 BER with Alamouti STBC
Two transmitting antennas are used in Alamouti’s STBC. Hence the total transmit power in the Alamouti’s scheme is twice that of the power used in 1Tx, 2Rx system that uses MRC. We can see that BER performance of 2Tx, 1Rx Alamouti’s STBC case has a roughly 3dB poorer performance than that 1Tx, 2Rx MRC case [176].

The BER for BPSK modulation in Rayleigh channel with 1 transmit, 2 receive case is, on MRC

\[ P_{e,\text{MRC}} = P_{\text{MRC}}^2 \left[ 1 + 2(1 - P_{\text{MRC}}) \right] \tag{4.9} \]

\[ \text{where} \quad P_{\text{MRC}} = \frac{1}{2} - \frac{1}{2} \left( 1 + \frac{1}{E_b/ \mathcal{N}_0} \right)^{\frac{1}{2}} \tag{4.10} \]

With Alamouti’s 2 transmit antenna, 1 receive antenna STBC case,
\[ P_{STBC} = \frac{1}{2} - \frac{1}{2} \left( 1 + \frac{2}{E_b / N_0} \right)^{\frac{1}{2}} \]  

(4.11)

and BER is

\[ P_{e,STBC} = P_{STBC}^2 \left[ 1 + 2(1 - P_{STBC}) \right] \]  

(4.12)

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Table 4.1 Simulation parameters for MIMO-OFDM

Simulations have been performed under multipath frequency selective channels and results are obtained in terms of BER versus SNR. The simulation parameters used for MIMO-OFDM are shown in Table 4.1.
Compared to the BER plot for \( n_{Tx}=1, n_{Rx}=2 \) MRC shown in Fig. 4.2, we can see that, the Alamouti Space-time Block Coding has around 3dB poorer performance.

### 4.2.2 2-Transmit, 2-Receive Alamouti STBC coding

![Diagram of 2-Transmit, 2-Receive Alamouti’s STBC coding](image)
Extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is $[176][181]$.

$$y_1 = h_{1,1}s_1 + h_{1,2}s_2 + w_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + w_1$$  \hspace{1cm} (4.13)

The received signal on the second receive antenna is

$$y_2 = h_{2,1}s_1 + h_{2,2}s_2 + w_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + w_2$$  \hspace{1cm} (4.14)

where, $y_1$ and $y_2$ are the received symbol on the first and second antenna respectively, $h_{j,i}$ is the channel coefficient of the channel between $j^{th}$ receiving antenna and $i^{th}$ transmitting antenna, $s_1$ and $s_2$ are the transmitted symbols and $w_1$ and $w_2$ is the noise on $1^{st}$ and $2^{nd}$ receive antennas.

For convenience, the above eqn. (4.13) and (4.14) can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$  \hspace{1cm} (4.15)

Equivalently,

$$y = Hs + w$$  \hspace{1cm} (4.16)

### 4.2.2.1 Receiver Structures

1. **MIMO with ZF Equalization**

The ZF approach tries to find a matrix $U$ which satisfies $UH=I$. The ZF linear detector for meeting this constraint is given by,

$$U = \left(H^H H\right)^{-1} H^H$$  \hspace{1cm} (4.17)

2. **MIMO with MMSE Equalization**

The MMSE approach tries to find a coefficient $W$ which minimizes the criterion,
where,

\[ U = \left( H^H H + N_0 I \right)^{-1} H^H \] (4.19)

Using the MMSE equalization, the estimate of the two transmitted symbols \( s_1 \) and \( s_2 \) is given by

\[ \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \left( H^H H + N_0 I \right)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \] (4.20)

### 3. ZF Equalization with Successive Interference Cancellation

Using the ZF equalization approach described above, the estimate of the two transmitted symbols \( s_1 \) and \( s_2 \) is obtained as

\[ \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \left( H^H H \right)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \] (4.21)

Taking one of the estimated symbols (for example \( \hat{s}_2 \)) and subtracting its effect from the received vector \( y_1 \) and \( y_2 \), we get

\[ \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} \hat{s}_2 \\ y_2 - h_{2,2} \hat{s}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} s_1 + w_1 \\ h_{2,1} s_1 + w_2 \end{bmatrix} \] (4.22)

Expressing in matrix notation,

\[ \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} s_1 + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \] (4.23)

\[ r = h s_1 + w \] (4.24)

The equalized symbol is,

\[ \hat{s}_1 = \frac{h^H r}{h^H h} \] (4.25)

Following similar procedure the estimate of \( s_2 \) is also obtained.
4. ZF Equalization with Optimally ordered SIC

In classical SIC, the receiver arbitrarily takes one of the estimated symbols, and subtracts its effect from the received symbol \( y_1 \) and \( y_2 \). However, we can intelligently choose the order in which the symbol effect is subtracted. This is done based on the powers of the received symbols in which the symbol with higher power is subtracted first and then the other.

The received power at both the antennas corresponding to the transmitted symbol \( s_1 \) is,

\[
P_{s_1} = \left| h_{1,1} \right|^2 + \left| h_{2,1} \right|^2
\]  
(4.26)

The received power at both the antennas corresponding to the transmitted symbol \( s_2 \) is,

\[
P_{s_2} = \left| h_{1,2} \right|^2 + \left| h_{2,2} \right|^2
\]  
(4.27)

If \( P_{s_1} > P_{s_2} \) then the receiver decides to remove the effect of \( \hat{s}_1 \) from the received vectors \( y_1 \) and \( y_2 \) and then re-estimate \( \hat{s}_2 \). Else the receiver decides to subtract effect of \( \hat{s}_1 \) from the received vector \( y_1 \) and \( y_2 \), and then re-estimate \( \hat{s}_1 \).

5. MMSE equalization with optimally ordered SIC

Using the MMSE equalization, the receiver can obtain an estimate of the two transmitted symbols \( s_1, s_2 \), given by

\[
\begin{bmatrix}
\hat{s}_1 \\
\hat{s}_2
\end{bmatrix} = \left( H^H H + N_0 I \right)^{-1} \begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
\]  
(4.28)

If \( P_{s_1} > P_{s_2} \) then the receiver decides to remove the effect of \( \hat{s}_1 \) from the received vector \( y_1 \) and \( y_2 \) and then re-estimate \( \hat{s}_2 \). Otherwise the receiver decides to subtract effect of \( \hat{s}_2 \) from the received vector \( y_1 \) and \( y_2 \), and then re-estimate \( \hat{s}_1 \).
6. MIMO with ML equalization

The ML receiver tries to find $\hat{s}$ which minimizes, $J = \left| y - H\hat{s} \right|^2$

$$J = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix}^2$$

(4.29)

Since the modulation is BPSK, the possible values of $s_1$ and $s_2$ are $+1$ or $-1$. So, to find the ML solution, we need to find the minimum from all the four combinations of $s_1$ and $s_2$.

4.2.2.2 Simulation Results

Simulations have been performed under multipath frequency selective channels and results are obtained in terms of BER versus SNR. The plots simulated and given below from Fig.4.4 (a-f) show BER for 2-transmit 2-receive MIMO-OFDM transmission/reception for BPSK modulation in a Rayleigh channel, for different equalizer structures discussed above.
Observations

1. The BER performance of 2-transmit 2-receive Alamouti’s case is much better than 1-transmit 2-receive MRC case. This is because the effective channel concatenating the information from 2 receive antennas over two symbols results in a diversity order of 4. The improvement is brought in, because decoding of the information from the first spatial dimension ($s_1$) has a lower error probability than that of the symbol transmitted from the second dimension.
2. The BER curve with ZF equalization for 2×2 MIMO channel is identical to BER plot for 1 transmit 1 receive system. The ZF equalizer is not the best possible way to equalize the received symbol. The ZF equalizer helps us to achieve the data rate gain, but cannot take the advantage of diversity gain (as we have two receive antennas).

3. Compared to the case of ZF equalization alone, addition of SIC results in an improvement around 2.2dB for BER of $10^{-3}$.

4. Optimal ordering of SIC shows better performance than the simple SIC.

5. Compared to the case of ZF equalizer, at BER of $10^{-3}$, it can be seen that the MMSE equalizer results in improvement of around 3dB.

6. SIC with optimal ordering improves the performance with ZF equalization. Compared to the case of Minimum Mean Square Equalization with simple SIC, addition of optimal ordering results in around 5.0 dB of improvement for BER of $10^{-3}$.

7. MMSE equalization with ordered SIC provides performance is slightly poorer than ML.

8. With ML equalization, we come close to the performance of 1-transmit 2-receive MRC case. We gain both throughput gain and diversity gain.

Many simulations have been done on the performance of different space-time block codes using different types of modulation schemes and different numbers of transmit and receive antennas. In our simulation on the different implementations of space-time block codes, the channel coefficients are always assumed flat Rayleigh.
From the Fig.4.5, we can see the performance of space-time block codes using BPSK and QPSK modulation schemes. The bit-error-rate performance of the system using BPSK modulation is better than the performance of space-time block codes using QPSK modulation by approximately 3 to 4 dB. The performance of space-time block codes using two transmitters and two receivers shows better performance than that of space-time block codes using two transmitters and one receiver antenna by approximately 7 to 8 dB. From the plots, it can be seen that the BER reduces with the number of transmit and receive antennas and BER increases with the increase in modulation order.

4.2.3 Conclusion

In this section, Alamouti scheme and space-time block codes encoding, decoding and BER performance analysis were covered and explained in detail. Different modulation schemes were used with different implementations (2-transmit and 1 or 2-
receive antennas) of space-time block codes. All simulation results were shown and explained in detail.

4.3 PERFORMANCE OF SPACE-FREQUENCY BLOCK CODES

4.3.1 Introduction

In this section, SFBC transmit diversity technique is applied into the OFDM system. Simply, the 2Tx/1Rx and 2Tx/2Rx antenna configurations are considered to compare the system performance of the MIMO OFDM system. Here, we discuss the traditional MIMO SFBC-OFDM structure with 2Tx/1Rx and 2Tx/2Rx antennas. In this section, SFBC has been with different digital modulation schemes such as BPSK and QPSK modulation techniques.

Space Frequency Coding (SFC) is an efficient approach to exploit the enormous diversity offered by the MIMO. It is used to obtain gains due to spatial diversity via multiple transmit and receive antennas. Moreover, a diversity gain proportional to the number of antennas at both transmit and receive sides can be achieved. One popular representation of these codes is the Alamouti’s scheme [21] for two transmit antennas.

4.3.2 Alamouti’s scheme with 2Tx-1Rx antennas

Alamouti introduced a very simple scheme of SFBC allowing transmissions from two antennas with the same data rate as on a single antenna, but increasing the diversity at the receiver from one to two in a Rayleigh fading channel. As shown in Fig.4.6, the Alamouti’s algorithm uses the space and frequency domain to encode data with increase in the performance of the system by coding the signals over the different transmitter branches [26]. Thus, the Alamouti’s code achieves diversity two with full data rate as it transmits two symbols in two frequency slots. In the first frequency slot, transmit antennas Tx1 and Tx2 are sending symbols $s_0$ and $s_1$. 
respectively. In the next frequency slot, symbols \(-s_1^*\) and \(s_0^*\) are sent, where \((\cdot)^*\) denotes complex conjugation. Furthermore, it is supposed that the channel, which has transmission coefficients \(h_1\) and \(h_2\), remains constant and frequency is flat over the two consecutive time steps. The received vector, \(R\), is formed by stacking two consecutively received data samples in frequency, resulting in

\[
R = Sh + w
\]

(4.30)

where \(R = [r_0, r_1]^T\) represents the received vector[186].

Fig. 4.6 2 x 1 SFBC-OFDM Transceiver

\[
h = [h_1, h_2]^T\]

is the complex channel vector, \(w = [w_1, w_2]^T\) is the noise at the receiver and \(S\) defines the SFBC [27],

\[
S = \begin{pmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{pmatrix}
\]

(4.31)

The vector equation in (4.30) can be read explicitly as
\[ r_0 = s_0 h_1 + s_1 h_2 + w_0 \\
\ h_1 = s_0 h_1 + s_1 h_2 + w_1 \tag{4.32} \]

At the receiver, the vector \( y \) of the received signal is formed according to \( y = \begin{bmatrix} r_0, r_1^* \end{bmatrix}^T \).

The above eqn. (4.32) can be rewritten in a matrix system as

\[
\begin{pmatrix} r_0 \\ r_1^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + \begin{pmatrix} w_0 \\ w_1^* \end{pmatrix} \tag{4.33} \]

The short notation for this system is the following:

\[ y = H_v s + \tilde{w} \tag{4.34} \]

where \( \tilde{w} \) represents the new noise vector obtained after the conjugation of the second equation, \( \tilde{w} = \begin{bmatrix} w_0, w_1^* \end{bmatrix}^T \). The estimated transmitted signal is then calculated from the formula

\[ \tilde{s} = H_v^H y \quad \text{where} \quad y = \begin{bmatrix} r_0, r_1^* \end{bmatrix}^T \]

\[
\begin{align*}
  r_0 &= s_0 h_1 + s_1 h_2 + w_0 \\
  r_1^* &= s_0^* h_1^* - s_1^* h_2^* + w_1^* 
\end{align*} \tag{4.35} \]

where \( H_v^H = \begin{pmatrix} h_1^* & h_2^* \\ h_2^* & -h_1^* \end{pmatrix} \) is Hermitian of the virtual channel matrix \( H_v \).

\[
\tilde{s} = \begin{pmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{pmatrix} = H_v^H H_v \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + H_v^H \begin{pmatrix} w_0 \\ w_1^* \end{pmatrix} \tag{4.36} \]

The resulting virtual (2 \( \times \) 2) channel matrix \( H_v \) is orthogonal, i.e.

\[ H_v^H H_v = H_v H_v^H = h^2 I_2 \tag{4.37} \]

Due to this orthogonality, the Alamouti’s scheme decouples the MISO channel into two virtually independent channels with channel gain \( h^2 \) and diversity \( d = 2 \). The channel gain specifies that transmitted symbols can be estimated at the receiver as the result of multiplying the received signals by the Hermitian of the virtual channel
matrix. After performing the corresponding operations it results in a signal with a gain of $h^2$ plus some modified noise.

$$\hat{s} = h^2 I_2 s + \hat{w}$$  \hspace{1cm} (4.38)

Where, $s$ is the transmitted signal, $I_2$ is the 2×2 identity matrix, $h^2 = |h_1|^2 + |h_2|^2$

and $\hat{w} = \begin{pmatrix} h_1^* w_0 + h_2^* w_1 \\ h_2^* w_0 - h_1^* w_1 \end{pmatrix}$ is modified noise.

The plot shown in Fig.4.7 shows the BER comparative performance of Alamouti’s SFBC-OFDM in different channel situations. The 2 TX and 1 RX SFBC-OFDM with Rayleigh channel shows outperformance than standard OFDM system.

![BER performance of 2 x 1 Alamouti SFBC-OFDM](image)

**Fig. 4.7** BER performance of 2 x 1 Alamouti SFBC-OFDM

### 4.3.3 Alamouti’s with 2Tx - 2Rx Antennas

Systems with two transmit antennas and two receive antennas as the one shown in Fig. 4.8, is analyzed next. The already explained steps are applied to each of the
receive antennas, denoting the received signal in the first and second time slot as \( r_1 \) and \( r_2 \), respectively [182].

The received signal from a 2 × 2 Alamouti’s scheme, as depicted above, is

\[
y = \begin{pmatrix} r_0(1) \\ r_0(2) \\ r_1^*(1) \\ r_1^*(2) \end{pmatrix} = \begin{pmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \\ h_{11}^* & -h_{21}^* \\ h_{22}^* & -h_{12}^* \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + \begin{pmatrix} w_0(1) \\ w_0(2) \\ w_1^*(1) \\ w_1^*(2) \end{pmatrix}
\]  

(4.39)

The estimated transmitted signal can be calculated from \( \hat{s} = H_v^H y \)

\( y = \begin{pmatrix} r_0(1), r_0(2), r_1^*(1), r_1^*(2) \end{pmatrix}^T \) and \((.)^H \) represents the Hermitian operation.

Fig. 4.8 2 x 2 SFBC-OFDM Transceiver

The virtual channel matrix, is expressed as \( H_v \) is expressed as
Therefore \( H^H_v = \begin{pmatrix}
 h^*_{11} & h^*_{12} & -h^*_{21} & -h^*_{22} \\
 h^*_{21} & h^*_{22} & -h^*_{11} & -h^*_{12}
\end{pmatrix} \)  
(4.40)

The obtained result for the process of estimating the transmitted symbols is
\[
\hat{s} = h^2I_2s + \hat{w}
\]  
(4.41)

Where, \( I_2 \) is the 2×2 identity matrix, \( s \) is the transmitted Signal,
\[
h^2 = |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2
\] is the power gain of the channel, and
\[
\hat{w} = \begin{pmatrix}
 h^*_1w_0(1) + h^*_2w_0(2) + h_{21}w^*_1(1) + h_{22}w^*_1(2) \\
 h^*_1w_0(1) + h^*_2w_0(2) - h_{11}w^*_1(1) - h_{12}w^*_1(2)
\end{pmatrix}
\] represents some modified noise.

Fig. 4.9 BER performance of 2 × 2 Alamouti SFBC-OFDM

The plot shown in Fig. 4.9 shows the BERs comparative performance of Alamouti’s SFBC-OFDM in different channel situations. The 2 TX and 2 RX SFBC-OFDM with
Rayleigh channel shows outperformance than standard OFDM system for low SNR values.

4.3.4 Simulation Results

From the Fig. 4.10, we can see the performance of space-frequency block codes using QPSK, and BPSK modulation schemes. The BER performance of the system using BPSK modulation is better than the BER performance of space-time block codes using QPSK modulation by approximately 3 to 4 dB. The performance of space-time block codes using two transmitters and two receivers shows better performance than that of space-time block codes using two transmitters and one receiver antenna by approximately 7 to 8 dB. From the plots, it can be seen that the BER reduces with the number of transmit and receive antennas and BER increases with the increase in modulation order.

Fig. 4.10 BER performance of Alamouti’s SFBC-OFDM (BPSK vs QPSK)
4.4 SIMULATION RESULTS

A BER performance comparison between STBC-OFDM and SFBC-OFDM systems is shown in Fig. 4.11. From the performance results, it can be seen that STBC-OFDM and SFBC-OFDM systems achieve similar performance under similar condition when channel parameters are known at the receiver. Such results were expected because similar assumptions have been made and that the coding technique is similar with the exception that STBC-OFDM symbols are coded over OFDM symbols while SFBC-OFDM symbols are coded over subcarriers.

4.5 CONCLUSION

The combination of OFDM with STBC allowed the creation of codes known as STBC-OFDM and SFBC-OFDM offering low decoding complexity and bandwidth efficiency as realized in STBC for single carrier systems. STBC-OFDM and SFBC-
OFDM systems have been adopted by standard such as WiMax and 4G and have been under extensive investigation by researchers.

Both STBC/SFBC achieve satisfy performance with slight difference at low mobility with small delay spread; at high mobility, SFBC often outperforms STBC, while with large delay spread, SFBC has worse performance than STBC. The performance difference between STBC and SFBC mainly comes from the sensitivity of the used ML decoder to the channel selectivity in an Alamouti code block. MMSE decoder can improve the performance of Alamouti code in the selective channel at cost of complexity, but has still some performance degradation due to residual ISI of MMSE decoder. Therefore, for high spatial diversity gain and low decoding complexity recommends both STBC and SFBC should be supported to adapt to various application environments in MIMO OFDM system.

In this chapter, a comprehensive investigation of OFDM and in particular MIMO-OFDM systems was conducted. Two coding schemes, STBC-OFDM and SFBC-OFDM, have been described in detail and simulation results for both schemes have been presented for different number of transmit and receive antennas and under various modulation schemes. A comparison of the two schemes has also been made which led to the conclusion that STBC-OFDM and SFBC-OFDM achieve similar performances.

In the next chapters that is, in the chapter-5, we are concerned with interference cancellation for STBC-MIMO-OFDM Systems. In chapter-6, we are concerned with the detection of SFBC-OFDM signals on time and frequency-selective MIMO channels. Specifically in chapter-6, we proposed and evaluated the performance of an interference cancelling receiver for SFBC-OFDM which effectively alleviates the effects of ISI and ICI.