CHAPTER 4
CFD STUDIES ON NATURAL CONVECTION IN SODIUM FILLED CYLINDRICAL ENCLOSURES

4.1 INTRODUCTION

The main objective of this numerical study is to develop heat transfer correlations for natural convection in sodium filled cylindrical cavities heated on the top side and cooled at the cylindrical wall. Both thin plate and thick plate conditions of the top wall are considered as shown in Figs. 4.1(a) and Fig. 4.1(b). Conjugate heat transfer analysis is carried out for the case of thick top plate configuration. The computational model is built and solved using a commercial CFD code PHOENICS. Grid optimization has been carried out. The details of model validation for two case studies viz., differentially heated cavity and bottom heated cavity are also described.

4.2 GRID OPTIMISATION

In any numerical study grid optimization is carried out with a view to minimize the errors. The spatial grid size is decreased in steps until no appreciable change in the output parameter, say, temperature at a selected point is reached. In the present study, turbulence is modeled by a low Reynolds number k-ε model. Therefore, the grid size is so chosen that mesh points extend upto the conduction dominated laminar sub-layer. This implies that computations are extended through the viscous sub-layer close enough to the wall and excludes the use of wall functions. The number of nodes in the axial direction is optimized such that the typical value of y⁺ is close to one, as required by the turbulence model. For the reactor sodium plenum, 100 nodes along axial direction and 50 nodes along radial direction are taken. Increasing the number of nodes to 100 in radial direction did not alter the solution appreciably. Therefore the domain was discretised by 50×100
nodes. Global convergence criterion is fixed at $10^{-3}$. Reducing this value to $10^{-4}$ did not produce perceptible change in heat transfer rates.

![Diagram](image)

**Fig. 4.1** Schematic of computational domain (a) Thin top plate and (b) Reactor sodium plenum with thick top plate

Computations are carried out upto 500 s, adopting a time marching approach for steady state solutions. Before this time, steady state has been attained in all the cases.
Typical values of y+ along the lateral wall for the reactor cavity were found to lie between 0.2 and 3 as discussed later in Section 4.5.

4.3 VALIDATION

Natural convection results for top wall heated and side wall cooled sodium filled enclosures are not available in open literature. Hence, for the validation of computational procedure used, two rectangular enclosures with well established benchmark results are considered. In one of the enclosures, the vertical walls are differentially heated, while the horizontal walls are maintained adiabatic. Mohamad and Viskanta (1993) have studied natural circulation in such differentially heated enclosures in detail and proposed a correlation for Nusselt number in terms of Boussinesq number given by Eqn. (2.7). Considering the test fluid to be sodium with Boussinesq number $3.7 \times 10^8$, results obtained in the present study are compared with their correlation. The comparison is given in Table 4.1. The steady state velocity vectors and temperature field obtained for this case from the present study are illustrated in Fig. 4.2 and Fig. 4.3 respectively. Next, the present model is run for Viskanta’s case study of $Ra=10^6$ and $Pr=0.02$ to compare flow and temperature patterns (Viskanta et al., 1986). The comparison of results is illustrated in Fig. 4.4 and Fig. 4.5. The flow and temperature patterns obtained from the present study compare very well with the benchmark patterns.

In the next case, the bottom wall of the enclosure is heated and top wall is cooled isothermally. Rossby (Kek and Muller, 1993) has studied natural circulation driven by vertical temperature gradient in these enclosures and proposed a correlation given by Eqn. (2.8). The same problem was analysed by the present method and the predicted Nusselt number is given in Table 4.1.
Table 4.1: Comparison of present results with benchmark data for liquid metals

<table>
<thead>
<tr>
<th>Square cavity</th>
<th>Configuration</th>
<th>Bo</th>
<th>Empirical Nu</th>
<th>Computed Nu from present study</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Heated</td>
<td>(Rossby et al. referred by Kek and Muller, 1993)</td>
<td>$1.48 \times 10^6$</td>
<td>23.4</td>
<td>24.3</td>
<td>3.5</td>
</tr>
<tr>
<td>Differentially Heated</td>
<td>(Mohamad and Viskanta, 1993)</td>
<td>$3.7 \times 10^8$</td>
<td>19.7</td>
<td>18.7</td>
<td>5</td>
</tr>
</tbody>
</table>

Fig. 4.2 Velocity vectors (m/s) in a differentially heated cavity (Bo=3.7×10^8)
Fig. 4.3  Temperature Field (K) in a differentially heated cavity (Bo=3.7×10^8)

Fig. 4.4  Comparison of velocity field of present work with that of Viskanta’s work
(Ra=10^6, Pr=0.02)
Fig. 4.5 Comparison of non dimensional temperature field of present work with that of Viskanta’s work ($Ra=10^6$, $Pr=0.02$)

For the bottom heated cavity, velocity and temperature fields for this problem are depicted in Fig. 4.6 and Fig. 4.7. Two counter rotating vortices are predicted in the enclosure with the temperature field symmetric about $x/H=0.5$, as expected for the boundary conditions employed. From the validation cases, it is found that present results differ from the published data only by 5%.

Fig. 4.6 Velocity vectors (m/s) for a bottom heated cavity ($Bo=1.48\times10^6$)
4.4 RESULTS AND DISCUSSION FOR THIN PLATE ENCLOSEMENT

After satisfactory validation of the present low Reynolds number k-ε model for both laminar and turbulent regimes, detailed parametric studies have been carried out for axisymmetric cylindrical enclosures for Pr=0.004.

4.4.1 Calculation of Nusselt Number

Steady state analysis is carried out in cylindrical enclosures depicted in Fig. 4.1(a). The top surface is the heat source maintained either at constant temperature or at constant heating rate, and the cylindrical side wall is the isothermal heat sink. The thickness of the top wall is considered to be negligible. Heat transfer coefficient ($h$) in liquid sodium below this plate is estimated by equating the conductive heat transfer rate to convective heat transfer rate as given in the following equation.

$$ h_i = \frac{k_{liq} \frac{\partial T}{\partial z}}{T_i - T_{wall}} $$  \hspace{1cm} (4.1)

$$ h_{ave} = \frac{\sum h_i dA_i}{\sum dA_i} $$  \hspace{1cm} (4.2)
where, $dA_i$ is the area associated with $i^{th}$ node, $N$ is the number of radial nodes, $T_p$ is the hot plate temperature, $h_i$ is local heat transfer coefficient and $h_{ave}$ is the average heat transfer coefficient.

Using this heat transfer coefficient, Nusselt number is estimated as

$$Nu_{ave} = \frac{h_{ave}L}{\kappa_{Na}}$$

(4.3)

where the characteristic length $L$ is taken as the diameter of the cylindrical enclosure. In liquid metals since $Pr \ll 1$, Nusselt number is correlated with Boussinesq number (Bejan, 2004) where Boussinesq number is defined as

$$Bo = \frac{g \beta \Delta T L^3}{\alpha^2}$$

(4.4)

Hence, the analysis is carried out with the view to arrive at such a correlation between Nusselt number and Boussinesq number.

**4.4.2 Isothermal Top Surface**

In the first case, natural convective heat transfer in liquid sodium is studied by prescribing constant temperature boundary condition on the top surface of the cylindrical cavity. The side cylindrical wall is kept at 673 K which is the typical temperature prevailing in the lower plenum of a fast breeder reactor. The aspect ratio of the cylindrical enclosure is 0.4. Based on systematic parametric studies, the mean Nusselt number is computed for a large range of Boussinesq number which is of interest in the accident analysis of a fast breeder reactor.

For a typical case with hot wall at 800 K, the predicted results of velocity field and isotherms are presented in Fig. 4.8 and Fig. 4.9 respectively. Liquid sodium upon getting cooled near the side wall descends due to gravity. This descending flow induces a weak circulation in the cavity which is otherwise stably stratified with hot sodium at the top. The maximum value of sodium velocity at $Bo=1.38\times10^{10}$ is 8 cm/s as can be seen in
From the corresponding isotherms depicted in Fig. 4.9, it is observed that the region close to the axis of symmetry is unaffected by the weak natural convection.

The temperature difference between the hot and cold walls which is the driving force for convection is not strong enough to encompass the entire enclosure. The bulk temperature of the enclosure is close to 673 K. The dependence of mean Nusselt number on Boussinesq number is seen to vary as

\[ Nu = 3.6811(Bo)^{0.1022} \quad \text{for } 2 \times 10^3 < Bo < 2 \times 10^6 \]  

(4.5)
as shown in Fig. 4.10. Turbulence is found to be dominant for higher Boussinesq numbers and Nusselt number is seen to exhibit a stronger dependence on Boussinesq number as given by

\[ Nu = 0.8715(Bo)^{0.2029} \quad \text{for} \quad 2 \times 10^6 < Bo < 2 \times 10^{11} \]  

(4.6)

The regression coefficient for both the equations is 0.99. The graphical representation of these results is given in Fig. 4.10.

![Graph showing Nusselt number vs Boussinesq number](image)

**Fig. 4.10 Nusselt number vs Boussinesq number for isothermal top wall**

### 4.4.3 Comparison of Present Results with Published Data

The works of Sheriff and Davies (1979) and that of Rigoleur (1982) which are focused towards nuclear reactor accident analysis are taken for comparison with the present work. Both of them have presented their results for downward facing hot plate in a pool of liquid metal. The exact ranges of applicability of these empirical correlations are not explicitly reported in the published literature. However, the Nusselt number predicted
by the present study is compared with that reported by Sheriff and Davies and Rigoleur for Boussinesq number in the range of $10^6$ to $10^{11}$, in Fig. 4.11.

The Nusselt number predicted by the present study for a vertical cylindrical enclosure is higher than the reported results for downward facing hot plate in an infinite ambient liquid. The reason for this difference can be attributed to the presence of a heat sink at the side wall which lowers the plate edge temperature and hence increases the convective velocity near the edge of the plate. For a hot plate in a pool of liquid, the velocity at the edge of the plate is not enhanced by a temperature difference as in this geometry but rather from buoyancy of the hotter liquid tending to rise due to gravity. Predicted local heat transfer coefficient as a function of radial distance is shown in Fig. 4.12. It is observed that near the top extreme edge of the cavity, the local heat transfer coefficient is high because of the enhanced natural circulation. This contributes to the
higher proportionality constant in the present side wall cooled enclosure. Nevertheless, it is observed that the power law dependence is the same for all the results.

![Fig. 4.12 Radial variation of local heat transfer coefficient](image)

### 4.4.4 Isoflux Top Surface

In this part of the study, a constant heat flux is imposed on the top wall of the cylindrical enclosure. All the other conditions are the same as in the previous case. Rigoleur and Tenchine (1982) have developed an empirical correlation using modified Boussinesq number $Bo^*$ as

$$Nu = 0.59(Bo^*)^{1/5} \quad \text{for } 8.6 \times 10^5 < Gr \ Pr^2 < 3 \times 10^6 \quad (4.7)$$

where

$$Bo^* = Gr^* \ Pr^2 = g \beta q L^4 / (\kappa \alpha^2)$$

In the case of constant heat flux boundary in the present study, $Bo^*$ is varied from $4 \times 10^4$ to $2 \times 10^9$. Based on the computed temperature distribution, the Nusselt number is seen to have 1/10 power law dependence on Boussinesq number. The Nusselt number is seen to correlate with modified Boussinesq number as per the relation,

$$Nu = 2.9 (Bo^*)^{0.096} \quad \text{for } 4 \times 10^4 < Bo^* < 2 \times 10^9 \quad (4.8)$$
with the regression coefficient of 0.98. The difference between the correlations represented by Eqn. (4.7) and Eqn. (4.8) is attributed to the fact that the present study is for an enclosure with hot top wall and cold side wall, while that of Rigoleur is for a downward facing hot plate immersed in a large pool. The comparison of present result with the empirical correlations of Rigoleur and Tenchine (1982) and Shereiff and Davies (1979) is given in Fig. 4.13.

![Fig. 4.13 Nusselt number vs Boussinesq number for constant flux heating](image)

4.5 RESULTS AND DISCUSSION FOR THICK PLATE ENCLOSURE

Results presented in the previous sections correspond to thin plate consideration. However in reactor designs, the grid plate is of finite thickness. For example in BN 800 reactor, the grid plate thickness is 11 cm. The corresponding value in Indian FBR is 5 cm (Chetal et al., 2006). The finite thickness of the grid plate necessitates a conjugate heat transfer analysis. Therefore, conjugate heat transfer analysis has been carried out for transient as well as steady state conditions for the geometry depicted in Fig 4.1(b). As explained in Section 3.2, low Reynolds number $k-\varepsilon$ model is used to account for turbulence effects. A typical plot of $y^+$ close to the top horizontal wall is given in
Fig. 4.14. The values ranging from 0.2 to 3 indicate that the laminar sub-layer is well resolved by the adopted mesh and gradients along the wall are adequately captured without the use of wall functions.

4.5.1 Transient and Steady State Analysis for Sudden Heating of Top Plate

Usually steady state heat transfer correlations serve a majority of practical purposes, barring a few applications namely, start up and shut down of heat transfer equipments and in processes with time varying boundary conditions. In such cases transient analysis assumes importance. Here, we have considered transient analysis to understand if finite thickness of the plate has any bearing on the transient stage of heat transfer. The evolution of heat transfer coefficient for the case of plate thickness 50 mm and Boussinesq number $7.9 \times 10^{10}$ is given in Fig. 4.15.

Due to the resistance offered by the plate to heat transfer, there is a time delay before the effect of heating on top surface of the plate is first felt at its bottom surface. Time for heat penetration to the first node in liquid sodium is found to be 12 s. At 12 s, the heat transfer coefficient ($h$) is 2500 W/m$^2$K and it reduces rapidly with increasing time. At steady state, when the plate temperature is 894 K, this value is 780 W/m$^2$K.
Fig. 4.15 Transient heat transfer coefficient

To understand the dependence of heat transfer coefficient with time in the initial unsteady state, a power law fit with respect to time was attempted for computed $h$ value. The graph indicates that it varies inversely as square root of time. The predicted value of $h$ is found to compare well with the equivalent heat transfer coefficient representing transient heat conduction defined as

$$h_{eq} = \frac{\kappa_{Na}}{\sqrt{\alpha t}}$$

upto the first 120 s.

This clearly indicates that conduction is the dominant heat transfer mechanism during this time interval. Equivalently, it can be stated that Nusselt number is inversely proportional to square root of dimensionless time

$$Nu = \left( \frac{t}{\tau} \right)^{\frac{1}{2}}$$

(4.10)

where, $\tau$ is the time constant of the entire sodium column (of height $z$). Transient heat transfer coefficient is primarily a characteristic of the liquid and is not dependent on the
wall thickness or temperature variation - whether it is steady or changing. This is true for a thin wall approximation case also. But the boundary wall temperature and its variation do have an influence on the final steady state heat transfer. By carrying out a parametric study by varying the source temperature, it is found that for thick plate configuration the steady state Nusselt number is correlated with Boussinesq number as

\[ Nu = 0.4(Bo)^{0.2} \]  \hspace{1cm} (4.11)

The hot plate temperature used in arriving at this correlation is the average temperature of the lower surface of the hot plate. This final steady state heat transfer correlation is compared with that calculated using an empirical correlation suggested by Rigoleur for a downward facing hot plate in liquid sodium which is given by

\[ Nu = 0.6(Bo)^{0.2} \]  \hspace{1cm} (4.12)

Comparing Eqn. 4.11 with Eqn. 4.12, it is evident that the present Nusselt number is \(~33\%\) lower than that of Rigoleur’s prediction. The reason for this deviation is that the empirical correlation is valid for plates at uniform steady temperatures only in a pool geometry. In this particular configuration of thick top plate, the temperature of the surface in contact with the liquid is a slowly increasing one due to the resistance offered by the thickness of the plate. On comparing the correlation in Eqn. 4.11 for thick top plate with that of thin top plate given by Eqn. (4.6), it is found that the reduction in heat transfer is about \(50\%\) in case of the thick plate. The velocity vectors obtained for the same Boussinesq number for thick and thin plates shown in Figs. 4.16 and 4.17 respectively. The comparison of velocity vectors qualitatively explains the reduced heat transfer coefficient with the thick plate. It is seen that natural convective velocity is almost one half of that in the case of a thin plate and the reduced convective velocity in the case of a thick plate, leads to a lower heat transfer coefficient.
4.5.2 Steady State Analysis

An attempt has been made to develop a single correlation for the Nusselt number combining the influence of defining parameters, viz., Boussinesq number, conductivity ratio and height ratio by carrying out another parametric study by varying the source temperature, material of the plate and thickness of the plate one by one. The aspect ratio (height/diameter) of the cavity is 0.4 and is preserved in all the cases. Height ratio or non-dimensional plate thickness is defined as the ratio between the thickness of the plate and
the height of the cylindrical enclosure. Conductivity ratio is the ratio of the thermal conductivity of the plate to that of sodium liquid.

Steady state analysis is carried out for different conductivity ratio between the solid plate and the liquid. Three different metals are considered and predicted \( \text{Nu} - \text{Bo} \) dependence is presented in Fig. 4.18.

![Graph showing the variation of the Biot number with Nusselt number](image)

**Fig. 4.18** Effect of conductivity ratio on Nu-Bo Correlation

The thickness of the plate is also varied as a parameter ranging between 10 mm and 50 mm. A graph showing the variation of the Biot number with Nusselt number is given in Fig. 4.19. It is observed that when the Biot number is large, the temperature drop across the plate increases. Therefore, the temperature at the bottom of the plate is low which leads to low heat transfer coefficient and hence small Nusselt number.

![Graph showing the Nu-Bi relationship for varying thickness of plate](image)

**Fig. 4.19** Nu-Bi relationship for varying thickness of plate
The developed correlation indicates the reduced significance of Boussinesq number on heat transfer in case of thick plates. While developing this correlation, the constant temperature imposed on top surface of the hot plate is considered as the plate temperature, so that the influence of the thickness of the plate is included in the analysis. The correlation developed using multi variable regression analysis is given below.

\[ Nu = 0.15(Bo)^{0.07}(l^*)^{-0.6}(\kappa^*)^{0.27} \]  

(4.13)

The goodness of fit of the correlation is shown in Fig. 4.20. The correlation coefficient between the exact and the fitted data is 0.99. To further check the validity and applicability of the above correlation, two cases are chosen with different conductivity ratio \( \kappa^* \) and one case with a different scale and conjugate heat transfer analysis is carried out to estimate heat transfer coefficient. The estimated heat transfer coefficient is compared with that obtained using the correlation and the error was found to be less than 3% as shown in Table 4.2.

![Fig. 4.20 Comparison of computed and correlated Nusselt numbers](image_url)

This numerical study clearly points out that using thin plate approximated correlations for thick plates can lead to erroneous results. Hence, it is prudent to carry out
a conjugate heat transfer study accounting for the conduction heat transfer in the plate or to use correlations that are specific for such thick plate configuration. The set of correlations developed in the present study for thin and thick plates and the ranges of their validity in terms of corresponding independent non-dimensional numbers are summarized in Table 4.3. For thick plates, the reduced influence of Boussinesq number on convective heat transfer is evident from the developed correlations.

Table 4.2 Validation of the developed correlation

<table>
<thead>
<tr>
<th>Bo</th>
<th>Heat Transfer Coefficient (W/m²K)</th>
<th>% deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conjugate heat transfer study</td>
<td>Predicted by Eqn. (4.13)</td>
</tr>
<tr>
<td>5.48×10⁷</td>
<td>927</td>
<td>910</td>
</tr>
<tr>
<td>1.19×10⁸</td>
<td>990</td>
<td>966</td>
</tr>
<tr>
<td>3.76×10⁹</td>
<td>200</td>
<td>198</td>
</tr>
</tbody>
</table>

Table 4.3 Correlations developed and their ranges of validity

<table>
<thead>
<tr>
<th>Plate</th>
<th>Top boundary condition</th>
<th>Correlation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin</td>
<td>Isothermal</td>
<td>$Nu = 3.6811(Bo)^{0.1022}$</td>
<td>$2 \times 10^3 &lt; Bo &lt; 2 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Nu = 0.8715(Bo)^{0.2029}$</td>
<td>$2 \times 10^6 &lt; Bo &lt; 2 \times 10^{11}$</td>
</tr>
<tr>
<td>Thin</td>
<td>Isoflux</td>
<td>$Nu = 2.9(Bo)^{0.096}$</td>
<td>$4 \times 10^4 &lt; Bo &lt; 2 \times 10^9$</td>
</tr>
<tr>
<td>Thick</td>
<td>Isothermal</td>
<td>$Nu = 0.15(Bo)^{0.07}(l^<em>)^{-0.6}(\kappa^</em>)^{0.27}$</td>
<td>$3.6 \times 10^7 &lt; Bo &lt; 2 \times 10^8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0.27 &lt; \kappa^* &lt; 5.4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$6 &lt; l^* &lt; 60$</td>
</tr>
</tbody>
</table>
4.6 CLOSURE

Natural convection heat transfer studies have been carried out numerically for a top surface heated and side wall cooled sodium filled cylindrical enclosures. Nusselt number correlations are obtained both for thin and thick plates forming the top boundary. The study on the thick top plate enclosure has revealed that conductivity ratio and height ratio also significantly affect the Nusselt number other than the Boussinesq number. In the thick top plate configuration, lower sodium plenum of the reactor main vessel is also investigated and there can be reduction in heat transfer upto even 50% because of the sluggish sodium natural convection velocity. A few of these developed correlations are further to be used in the core-melt relocation studies to be discussed in the next chapter.