Chapter 4

Aging in Ferromagnets with Quenched Disorder

4.1 Introduction

The evolution of a multi-phase system, which is quenched to the ordered phase is characterized by the emergence and growth of domains. The domain scale grows with time as $L(t) \sim t^{1/2}$ for the case with Glauber (or nonconserved) dynamics, and as $L(t) \sim t^{1/3}$ for the case with Kawasaki (or conserved) dynamics (see Chapter 1). A key concept in this regard is that many time-dependent properties of coarsening systems can be understood in terms of a simple scaling picture [1]. In the previous chapters, we have focused on the morphologies and growth laws which characterize phase ordering systems. These features have been traditionally well-studied in the literature. Recently, motivated by spin glass theory, many studies have also been devoted to understanding slow relaxation and aging phenomena in domain growth problems [1, 2]. In this chapter, we focus on aging phenomena in the ordering dynamics of ferromagnets with quenched disorder.
Consider a paramagnet → ferromagnet transition, as was discussed in Chapter 1. The phase diagram of a magnet in the temperature \( (T) \) and magnetic field \( (h) \) plane is shown in Fig. 1.1. At high temperatures \( (T > T_c) \) and zero magnetic field \( (h = 0) \), the spins are randomly oriented, and the resulting magnetization is zero. Below the critical temperature \( (T < T_c) \), the system prefers to be in a spontaneously magnetized state, even if there is no magnetic field. In domain growth problems, we study the evolution of a system, which is rapidly quenched from \( T > T_c \) to \( T < T_c \).

For simplicity, consider a system which is initially at infinite temperature \( (T_I = \infty) \), with correlation length \( \xi_I = 0 \). There are three possible classes of quenches:

(a) Quenches to \( T > T_c \) with correlation length \( 0 < \xi < \infty \);
(b) Quenches to \( T = T_c \) with \( \xi = \infty \) (for an infinite system);
(c) Quenches to \( T < T_c \) with \( \xi = \infty \) (for an infinite system).

Thus, the correlation length grows from \( \xi_I = 0 \) to a final value \( \xi > 0 \). For quenches to \( T > T_c \), there is a finite equilibration time. On the other hand, for quenches to \( T > T_c \), the system is always nonequilibrium as \( \xi(t) \sim t^{1/z} \), \( z \) being the dynamical exponent. This fact is responsible for slow relaxation and aging in quenches to or below \( T_c \). Aging usually means that older systems relax slower and younger ones faster [3]. Aging is generally described by two-time observables, such as the order-parameter autocorrelation function \( C(t, t_w) \) and the auto-response function \( R(t, t_w) \). (These quantities will be defined at the appropriate place.) The shortest time after the quench \( (t_w \geq 0) \) and the longest time \( (t \geq t_w) \) are conventionally called the waiting time and the observation time, respectively. Thus, the value of \( t_w \) sets the age of the system.

This chapter is organized as follows. In Sec. 4.2, we discuss domain growth in systems with quenched disorder. In Sec. 4.3, we present detailed numerical results for ordering dynamics in disordered systems. We will present results for the relevant
morphological features and the aging properties. Finally, we conclude this chapter with a summary and discussion in Sec. 4.4.

4.2 Domain Growth in Disordered Systems

Consider the two-state Ising model with quenched bond disorder, i.e., the random-bond Ising model (RBIM). The associated Hamiltonian is given by

\[ H = - \sum_{(ij)} J_{ij} S_i S_j, \quad S_i = \pm 1, \tag{4.1} \]

where the exchange couplings \( J_{ij} \) are quenched random variables. Here, we will study the case where the bonds are always ferromagnetic, \( J_{ij} > 0 \). They are uniformly distributed in the interval \([1 - \epsilon, 1 + \epsilon]\), where \( 0 \leq \epsilon \leq 1 \). This model shows a second-order phase transition at a critical temperature \( T_c(\epsilon) > 0 \). Note that the critical temperature \( T_c(\epsilon) \approx T_c(0) \approx 2.269 \) and it does not depend strongly upon the disorder amplitude.

We associate Glauber spin-flip dynamics with the RBIM, starting from a homogeneous initial configuration. For \( T < T_c(\epsilon) \), ordered domains form and the system evolves with a characteristic length scale \( L(t) \). In Fig. 4.1, we show evolution pictures for the 2-d Glauber-RBIM after a quench from \( T = \infty \) to \( T = 1.0 \). The snapshots correspond to different disorder amplitudes \( -\epsilon = 0 \) (pure case) and \( \epsilon = 1 \) (case with maximum disorder). It is clear from the snapshots in Fig. 4.1 that the evolution is slower for higher amplitudes of disorder. This will be quantified via the corresponding domain growth laws, and the scaling form of the correlation function and structure factor.
Figure 4.1: Evolution snapshots of a spin-1/2 ferromagnet with quenched disorder, evolving from a random initial configuration. These snapshots are obtained from a Monte Carlo (MC) simulation of the RBIM with spin-flip Glauber kinetics (see Sec. 1.2). The system size is $4096^2$, and periodic boundary conditions are applied in both directions. The quench temperature is $T = 0.5$. Time is measured in units of Monte Carlo steps (MCS). The black regions denote up spins and white regions denote down spins. The snapshots on the left correspond to $\epsilon = 0$ (pure case), and the snapshots on the right correspond to $\epsilon = 1$ (case with maximum disorder).
There is a good understanding of the growth laws for pure and isotropic systems with nonconserved order parameter – we have seen that the ordering system obeys the Allen-Cahn growth law, $L(t) \sim t^{1/2}$ (see Sec. 1.4.2). However, the presence of disorder in the system traps the domain boundaries, resulting in slower domain growth. In Sec. 3.3.2, we had discussed the Lai-Mazenko-Valls (LMV) [4] classification of domain growth laws in nonconserved systems. Recall that the growth of domains driven by a curvature-reduction mechanism can be written as [cf. Eq. (3.7)]

$$\frac{dL}{dt} = \frac{a(L, T)}{L}. \tag{4.2}$$

Here, $a(L, T)$ is a diffusion constant and it depends upon the domain length scale $L(t)$ and the quench temperature. In pure systems, the diffusion constant is independent of the length scale, i.e., $a(L, T) = a_0$, and the corresponding growth law can be written as $L(t) \sim (a_0 t)^{1/2}$.

Let us next consider systems with quenched disorder ($J_{ij} > 0$). It is known that length scales are small at early times, and the growth of domains is not affected by disorder, i.e., the growth law is the same as that for the pure case at early times. However, at late times, the domains are trapped by disorder sites which create a barrier $E_B(L)$ to domain growth. Thus, in the late-stage dynamics, thermal activation over the disorder barriers plays a crucial role during domain growth. The diffusion constant $a(L, T)$ is related to the energy barrier $E_B$ as

$$a(L, T) \simeq a_0 \exp(-\beta E_B), \tag{4.3}$$

where $\beta = T^{-1}$ ($k_B = 1$). Equation (4.3) can now be used to understand domain growth laws with disorder, when the form of $E_B(L)$ is known. Huse and Henley (HH)
argued that the coarsening domains are trapped by energy barriers $E_B(L) \simeq \epsilon L^\psi$, where $\epsilon$ is the disorder amplitude and the exponent $\psi = 1/4$ [5]. As a consequence, the late-time behavior of $L(t)$ is

$$L(t) \simeq \left[ \frac{T}{\epsilon} \ln \left( \frac{t}{t_0} \right) \right]^{1/\psi},$$

$$t_0 \simeq \frac{1}{a_0 \psi} \left( \frac{T}{\epsilon} \right)^{2/\psi}. \quad (4.5)$$

As the early-time dynamics is described by the Allen-Cahn growth law, we can reformulate the early-time and late-time behaviors as

$$R(t) = R_0(T, \epsilon) h \left( \frac{t}{t_0} \right), \quad (4.6)$$

where

$$R_0(T, \epsilon) = \left( \frac{T}{\epsilon} \right)^{1/\psi}, \quad (4.7)$$

and

$$h(x) = \begin{cases} 
\left( \frac{2}{\psi} x \right)^{1/2} & \text{if } x \ll 1, \\
(\ln x)^{1/\psi} & \text{if } x \gg 1.
\end{cases} \quad (4.8)$$

In spite of several numerical simulations [6, 7, 8] and experiments [9, 10, 11], the HH growth law has not been satisfactory proved. In recent work, Paul et al. [14] used Monte Carlo (MC) simulations to argue for an algebraic dependence of $L(t)$ in the asymptotic time limit. Our MC results are also consistent with power-law domain growth with exponents which depend upon the disorder amplitude and temperature. These exponents can be understood in the framework of a logarithmic (rather than
power-law) $L$-dependence of the energy barriers $E_B$. In the context of the dilute Ising model (DIM), Henley [15] and Rammal and Benoit [16] have argued that the fractal nature of domain boundaries results in a logarithmic $L$-dependence of the energy barriers. Thus, the scaling form of the energy barriers is

$$E_B(L) \simeq \epsilon \ln(1 + L). \quad (4.9)$$

Equation (4.2) can then be written as [using Eq. (4.9)]

$$\frac{dL}{dt} = \frac{a_0}{L} (1 + L)^{\epsilon/T}. \quad (4.10)$$

The solution of Eq. (4.10) is

$$L(t) = \begin{cases} 
(2a_0t)^{1/2}, & t \ll t_0, \\
[(2 + \frac{\epsilon}{T})a_0t]^\theta, & t \gg t_0,
\end{cases} \quad (4.11)$$

where $\theta$ is the asymptotic growth exponent:

$$\theta(\epsilon, T) = \frac{1}{2 + \epsilon/T}. \quad (4.12)$$

Thus, the growth exponent $\theta$ depends explicitly on $T$ and $\epsilon$.

### 4.3 Numerical Results

#### 4.3.1 Dynamical Scaling and Growth Laws

The details of the simulations are as follows. Our simulations were done on a RBIM with $N = 4096^2$ spins, using the standard Glauber spin-flip algorithm (see Sec. 1.2.1).
The initial configurations were homogeneous mixtures of up and down spins, corresponding to the disordered state at $T = 0$. The system is quenched to the final temperature $T = 0.5$. We average over at least 6 runs with different initial states and different realizations of noise. In this subsection, we mainly focus on disorder amplitudes $\varepsilon = 0$ (pure case) and $\varepsilon = 1$ (maximally disordered case).

The domain growth for $\varepsilon = 0, 1$ is shown in Fig. 4.1. In Fig. 4.2, we demonstrate the dynamical scaling of the correlation function and structure factor. In Fig. 4.2(a), we superpose data for the scaled correlation function $[C(r, t) \text{ vs. } r/L]$ for the pure case ($\varepsilon = 0$). Figure 4.2(b) shows the corresponding data for $\varepsilon = 1$. In both cases, the data sets at different times collapse onto a master function. Figures 4.2(c) and (d) are the scaling plots for the structure factor $[L^{-d}S(k, t) \text{ vs. } kL]$. Notice that the maximally disordered case ($\varepsilon = 1$) still exhibits Porod's law, $S(k, t) \sim k^{-(d+1)}$ as $k \to \infty$. Thus, even in the presence of disorder, the interfaces remain sharp and do not acquire a fractal shape.

In Fig. 4.3(a), we plot the scaled correlation function at the same time, but for different disorder amplitudes. The scaling functions collapse onto a master function, indicating that they are independent of the disorder amplitude. We refer to this as super-universality of the correlation function [12, 13]. Figure 4.3(b) shows the corresponding super-universality of the scaled structure factor.

In Fig. 4.4, we show the time-dependence of the length-scale, $L(t) \text{ vs. } t$. For the pure case ($\varepsilon = 0$), the data is consistent with the Allen-Cahn growth law, $L(t) \sim t^{1/2}$. For the disordered case ($\varepsilon = 1$), the data is consistent with power-law growth but with a reduced exponent $\theta \simeq 0.195$. For all disorder amplitudes, we find power-law growth but with an exponent which depends on $\varepsilon$ and $T$ as in Eq. (4.12). This is consistent with the results of Paul et al. [14].
Figure 4.2: (a) Scaled correlation function $[C(r, t) \text{ vs. } r/L]$ for the evolution of the Glauber-RBIM at $T = 0.5$. This plot is for the pure case, $\epsilon = 0$. (b) Analogous to (a), but for the maximally disordered case, $\epsilon = 1$. (c) Scaled structure factor $[L^{-d}S(k, t) \text{ vs. } kL]$ for $\epsilon = 0$. The solid line denotes the Porod tail in $d = 2$ with slope $-3$, $S(k, t) \sim k^{-(d+1)}$ as $k \to \infty$. (d) Analogous to (c) but for $\epsilon = 1$. 
Figure 4.3: Superscaling of correlation function and structure factor. (a) We plot equal time correlation function for $\varepsilon = 0, 0.5$ at time $t = 3000$. The data for $\varepsilon = 1$ is at time $t = 10^6$ MCS. The growth law for $\varepsilon = 1$ is much slower than growth law for $\varepsilon = 0, 0.25$ [see Fig. 4.4], hence a late time data is used to ensure the scaling. The solid line is OJK function. (b) Similar plot of structure factor for $\varepsilon = 0, 0.5, 1$. The fourier transform of OJK function is also plotted.
Figure 4.4: Domain growth laws in 2-d RBIM with Glauber kinetics at quench temperature $T = 0.5$. All statistical data is obtained on 4096$^2$ systems, as an average over 6 independent runs. The disorder values are $\varepsilon = 0$ (pure case) and $\varepsilon = 1$ (case with maximum disorder).

4.3.2 Autocorrelation and Response Functions

Definitions and Limiting Behaviors

Next, we focus on the autocorrelation and response functions. The *autocorrelation function* is defined as

$$ C(t, t_w) = \frac{1}{N} \sum_{i=1}^{N} [(S_i(t_w)S_i(t)) - \langle S_i(t_w) \rangle \langle S_i(t) \rangle]. \quad (4.13) $$

The *response function* is defined as follows. If a small field $h(\vec{r}, t)$ is turned on for the interval $(t_1, t_2)$ after the quench, then the magnetization for $t \geq t_2$ is

$$ \langle m(\vec{r}, t) \rangle_h = \langle m(\vec{r}, t) \rangle_{h=0} + \int d\vec{r}' \int_{t_1}^{t_2} dt' R(\vec{r} - \vec{r}', t, t') h(\vec{r}', t') + O(h^2). \quad (4.14) $$
Then, the linear response function is

\[ R(\vec{r} - \vec{r'}, t, t_w) = \left. \frac{\delta(m(\vec{r}, t))_h}{\delta h(\vec{r'}, t_w)} \right|_{h=0}. \] (4.15)

The corresponding autoresponse function is

\[ R(t, t_w) \equiv R(0, t, t_w). \] (4.16)

Before we show numerical results for these quantities, let us make a few general observations. One important property of aging phenomena is the separation of time-scales. Let us assume that \( t_w \) and the time-step \( \tau = t - t_w \) are sufficiently large. Then, the time-step can be categorized into two parts: (1) short times, i.e., \( \tau \ll t_w \), and (2) late times, i.e., \( \tau \gg t_w \). For short \( \tau \), the system appears locally equilibrated at the final temperature, i.e.,

\[ C(t, t_w) = C_{eq}(\tau, T), \]
\[ R(t, t_w) = R_{eq}(\tau, T). \] (4.17)

The two quantities are time-translation-invariant (TTI), and exhibit the same behavior as if equilibrium at the final temperature of the quench has been reached.

At the other limit, i.e., for \( \tau \gg t_w \), the true off-equilibrium behavior of the system is observed and the scaling form of the quantities can be written as

\[ C(t, t_w) = C_{ag}(t, t_w) = t_w^{-b} h_C \left( \frac{t}{t_w} \right), \]
\[ R(t, t_w) = R_{ag}(t, t_w) = t_w^{-(1+a)} h_R \left( \frac{t}{t_w} \right), \] (4.18)
where \( a, b \) are non-negative exponents and \( h_C, h_R \) are scaling functions. The scaling form of Eq. (4.18) also illustrates that relaxation at late stages is controlled by \( t_w \) only. Thus, the existence of fast and slow degrees of freedom becomes characteristic of aging or slow relaxation dynamics.

In the context of domain growth in a ferromagnetic system, the fast degrees of freedom are responsible for the thermal fluctuations within the ordered domains, and the slow ones can be thought of as labels of domains. For a given \( t_w \), the typical size of a domain is \( L(t_w) \sim t_w^{1/z} \), and it takes an interval of time \( \tau \sim t_w \) for a domain wall to sweep the whole domain. Thus, in the short-time regime, the slow degrees of freedom do not evolve much and the main contribution comes from the fast degrees of freedom in the decay of two-time quantities, yielding the behavior in Eq. (4.17). Similarly, in the asymptotic state (i.e., at \( \tau \gg t_w \)), the off-equilibrium behavior is dominated by the motion of domain walls and the scaling form is given by Eq. (4.18).

However, the interpretation of aging behavior in terms of fast and slow degrees of freedom is less clear when the system is quenched to the critical temperature \( T_c \). In the quench to \( T_c \), stationary and aging behaviors match multiplicatively, whereas in the quench below \( T_c \), the matching is additive [1].

For the quench to \( T_c \), the analytical calculation of the exponents \( a, b \) and the scaling functions \( h_C, h_R \) is done through expansion methods, e.g., the \( \epsilon \)-expansion [17, 18, 19]. On the other hand, for quenches below \( T_c \), a Gaussian auxiliary field (GAF) approximation of the OJK type in Sec. 1.4.5 is successfully applied [20, 21, 22, 23, 24]. However, the computation of the autoresponse function \( R(t, t_w) \) remains very complicated. For this reason, the study of aging in quenches below \( T_c \) relies heavily on numerical simulations, although the accurate numerical computation of \( R(t, t_w) \) is not straightforward [25, 26].
Autocorrelation Function

In Fig. 4.5, we show the time-dependence of the autocorrelation function $C(t, t_w)$ for $\epsilon = 0, 1$. In Figs. 4.5(a) and (b), we plot $C(t, t_w)$ vs. $t$. In Figs. 4.5(c) and (d), we plot $C(t, t_w)$ vs. $t/t_w$. In the latter case, a nice data collapse is observed, which is consistent with simple aging. Our scaling plot implies that $b = 0$, which is also found in the pure case. Also note that the observed scaling form of the autocorrelation function is consistent with the algebraic form of the growth laws.

Figure 4.5: (a) Plot of autocorrelation function, $C(t, t_w)$ vs. $t$, for $\epsilon = 0$. The waiting times are (from left to right) $t_w = 100, 200, 500, 1000, 2000$. (b) Analogous to (a), but for the case $\epsilon = 1.0$. The waiting times are $t_w = 100, 300, 1000, 3000, 10000and30000$. (c) Scaling plot of data in (a), $C(t, t_w)$ vs. $t/t_w$. (d) Scaling plot of data in (b), $C(t, t_w)$ vs. $t/t_w$. 

111
Response Function and Related Quantities

Let us next consider the autoresponse function, which was defined in Eq. (4.15). With a time-independent external field, Eq. (4.14) takes the form

\[
\langle m(\vec{r}, t) \rangle_h = \langle m(\vec{r}, t) \rangle_{h=0} + \int d\vec{r}' \zeta(\vec{r} - \vec{r}', t, t_1, t_2)h(\vec{r}') + O(h^2),
\]

(4.19)

where \(\zeta(\vec{r} - \vec{r}', t, t_1, t_2)\) is the integrated linear response function:

\[
\zeta(\vec{r} - \vec{r}', t, t_2, t_1) = \int_{t_1}^{t_2} dt' R(\vec{r} - \vec{r}', t, t').
\]

(4.20)

We can derive two important quantities from the definition of the integrated linear response function. The first is the thermoremanent magnetization (TRM), corresponding to the choice \(t_1 = 0, t_2 = t_w\):

\[
\rho(\vec{r}, t, t_w) = \zeta(\vec{r}, t, t_w, 0).
\]

(4.21)

The second quantity is the zero-field-cooled (ZFC) susceptibility with \(t_1 = t_w, t_2 = t\):

\[
\chi(\vec{r}, t, t_w) = \zeta(\vec{r}, t, t, t_w).
\]

(4.22)

A direct numerical calculation of the autoresponse function \(R(t, t_w)\) is considerably more difficult than the evaluation of \(C(t, t_w)\). The reason is that \(R(t, t_w)\) is much more noisy than \(C(t, t_w)\). One way to handle this difficulty is to study less noisy integrated response functions, e.g., TRM [Eq. (4.21)], ZFC susceptibility [Eq. (4.22)]. Below the critical temperature \(T_c\), using the additive structure of the response function, the
ZFC susceptibility can be written as

\[ \chi(t, t_w) = \chi_{eq}(t, t_w) + \chi_{ag}(t, t_w), \quad (4.23) \]

where the first component satisfies stationarity, and the second part is the contribution of aging. Again, \( \chi_{ag} \) can be written as

\[ \chi_{ag}(t, t_w) = \int_{t_w}^{t} dt' R_{ag}(t, t'). \quad (4.24) \]

Hence, the scaling form of this quantity can be written as [cf. Eq. (4.18)]

\[ \chi_{ag}(t, t_w) = t_w^{-a} h_x(x), \quad (4.25) \]

and for large \( x \), \( h_x(x) \sim x^{-a} \). This implies that the asymptotic form can be written as

\[ \chi_{ag}(t, t_w) \sim t^{-a}. \quad (4.26) \]

An intuitive argument, due to Barrat [27], predicts that \( \chi_{ag}(t, t_w) \) should be proportional to the density of defects at the time \( t_w \), which behaves as \( L(t_w)^{-m} \), where \( m = 1 \) and 2 for a scalar and vector order parameter, respectively. However, analytical results which involve exact solutions of the large-\( N \) model [28], \( d = 1 \) Ising model [29], as well as other approximate calculations show that the exponent \( a \) has a linear dependence on \( d \). The relevant expression is [1]

\[ a = \frac{m}{z} \left( \frac{d - d_L}{d^* - d_L} \right), \quad (4.27) \]
where $d_L$ is the lower critical dimension, and $d^*$ is the upper critical dimension. For the 2-$d$ Ising system without disorder, we have $d = 2$, $d^* = 3$, $d_L = 1$ and $z = 2$, $m = 1$. Thus the product $az = 1/2$. In this chapter, we study the corresponding relationship for the RBIM.

Let us next present results for the exponent $a$ and the form of the scaling function. In our simulations, we measure the quantity

$$\chi_{ag}(t, t_w) = \frac{1}{Nh_0^2} \sum_{i=1}^{N} \langle S_i \rangle h_i,$$  \hspace{1cm} (4.28)$$

where $h_i$ is a quenched configuration of uncorrelated random fields with values $\pm h_0$ and $h_0 = 0.05$. The angular brackets denote an average over independent initial conditions. The overbar denotes an average over random field configurations. The simulation has been done at temperature $T = 0.5$. (Note that the critical temperature $T_c \approx 2.269$.) We have also extended the calculation of $\chi_{ag}(t, t_w)$ taking into consideration the presence of quenched disorder. First, we obtain the value of $a$ by plotting $\chi_{ag}(t, t_w)$ vs. $t_w$ for a fixed ratio of $x = t/t_w = 10$ in Fig. 4.6. A clean power-law behavior is observed, which enables us to identify $a$. The values of $t_w$ in Fig. 4.6 are 50, 100, 200, 500, 1000. The statistical data is obtained as an average over 100 independent initial conditions and disorder configurations.

Next, we present results for the scaling function $\chi_{ag}$. In Fig. 4.7, we plot $T\chi_{ag}$ vs. $t$ for the pure case, i.e., $\epsilon = 0$. The asymptotic power-law decay provides an independent value of $a \approx 0.30$, which is consistent with the value from Fig. 4.6(a).

In Figs. 4.8 and 4.9, we plot $T\chi_{ag}$ vs. $t$ for the disorder values $\epsilon = 0.5, 0.75$.

Finally, we obtain data for $z$ [14] and the value of $a$ from our numerical calculation, to ascertain the validity of Eq. (4.27). Our understanding is that the product $az$ is independent of the disorder, and $az = 0.5$. The plot of $az$ vs. $\epsilon$ is shown in Fig. 4.10.
Figure 4.6: Log-log plot of $\chi_{ag}(t, t_w)$ vs. $t_w$ for the Glauber-RBIM with fixed $x = t/t_w = 10$. The various frames correspond to (a) $\epsilon = 0$ (pure case); (b) $\epsilon = 0.25$; (c) $\epsilon = 0.5$; and (d) $\epsilon = 0.75$; The best linear fits to the data are shown as dashed lines – these determine the exponent $a$. 

\begin{align*}
\text{(a)} & \quad \epsilon = 0 \\
\text{a} & \quad 0.32 \\
\text{(b)} & \quad \epsilon = 0.25 \\
\text{a} & \quad 0.23 \\
\text{(c)} & \quad \epsilon = 0.50 \\
\text{a} & \quad 0.15 \\
\text{(d)} & \quad \epsilon = 0.75 \\
\text{a} & \quad 0.147
\end{align*}
Figure 4.7: Plot of $T_{\chi_{agg}}(t, t_w)$ vs. $\tau = t - t_w$ for disorder value $\epsilon = 0$. The asymptotic is power law with exponent $a \approx 0.30$. The simulation has been done on a $4096^2$ lattice with an average over 15 independent runs. We show data for waiting times $t_w = 100, 1000$.

Figure 4.8: Analogous to Fig. 4.7 but for $\epsilon = 0.5$. The simulation has been done on a $4096^2$ lattice with an average over 85 runs. The waiting times are $t_w = 200, 500, 1000, 2500$. 
Figure 4.9: Analogous to Fig. 4.7 but for $\epsilon = 0.75$. The simulation has been done on a $4096^2$ lattice with an average over 85 runs. The waiting times are $t_w = 200, 500, 1000, 2500$.

Figure 4.10: Plot of $ax$ vs. $\epsilon$. 
4.4 Summary and Discussion

We conclude this chapter with a summary and discussion of the results presented here. In this chapter, we have focused on domain growth in ferromagnets with quenched disorder. We modeled this system via a random-bond Ising model (RBIM) with Glauber spin-flip kinetics. The disorder amplitude was such that the bonds were always ferromagnetic. In this chapter, we have studied the morphological features of domain growth, as well as aging and slow relaxation.

We found that the scaled correlation functions and structure factors for the Glauber-RBIM are independent of the disorder amplitude. These are numerically indistinguishable from the corresponding functions for the pure case. We refer to this as super-universal scaling of the correlation function and structure factor. The domain growth laws show a power-law behavior, $L(t) \sim t^\theta$, with an exponent $\theta$ which depends on the disorder amplitude and temperature. Our results for domain growth are consistent with a scenario where the coarsening domains are trapped by quenched disorder sites with an energy barrier which grows logarithmically with the domain size.

We also studied two-time functions like the autocorrelation function and the response function. We have obtained detailed numerical results for the behavior of these quantities at late times. In particular, we have quantified the exponent which governs the power-law decay of the autoresponse function in the asymptotic regime.
Bibliography


