CHAPTER 7

AN OPTIMAL INVENTORY POLICY FOR ITEMS HAVING QUADRATIC TIME DEPENDENT DEMAND AND CONSTANT DETERIORATION RATE WITH TRADE CREDIT
7.1 Introduction

In most of the classical inventory models demand is considered as constant, but in most of the practical situation the demand changes with time, quantity etc. In this model, we have studied an order level inventory problem with demand rate represented by a continuous quadratic function of time and constant deterioration rate. The effect of permissible delay is also taken and incorporated in this model.

One of the challenging problems for the marketing researchers and practitioners is to study and analyze the inventory systems as the proper inventory systems not only reduce the costs, but also reduce stock-outs and improve customer satisfactions. Thus, proper inventory systems can improve the profitability and help in the survival of an organization. The researchers have considered various types of demands, viz. a linear time-dependent demand, stock dependent demand, price dependent demand etc. Time dependent demand indicates a uniform change in the demand rate of the product per unit time which is an unrealistic phenomenon and it rarely occurs in the market. On the other hand, exponentially time-varying demand indicates very rapid change in demand rate which is also unrealistic because the demand of any product can not undergo a rate which is so high as exponential demand is.

The concept of permissible delay is not new, even when currency was not in circulation, then also permissible delay was provided by suppliers to buyers. In general practice, suppliers are known to offer their customers a fixed period of time and do not charge any interest for this period. However, a higher interest is charged if the payment is not settled by the end of credit period. The permissible delay in payment produces three advantages to the supplier

- It helps to attract new customer.
- It helps in the bulk sale of goods.
It may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions.

In this chapter, an effort has been made to analyze an EOQ model for deteriorating item considering time-dependent quadratic demand rate and permissible delay in payment. Among the various time-varying demand in EOQ models, the more realistic demand approach is to consider a quadratic time dependent demand rate because it represents both accelerated and retarded growth in demand. The demand rate in this case is of the form, Here, \( c = 0 \) represents linear demand rate and \( b = 0, c = 0 \) represent the constant demand rate. Thus, it helps retailer to decide its optimal ordering quantity under the constraints of constant deterioration rate and quadratic time dependent demand.

7.2 Assumptions and Notations

The following are the assumptions

(i) \( a = \) initial demand rate.

(ii) \( b = \) rate with which the demand rate increases.

(iii) \( c = \) rate of change in the demand rate itself changes at a rate \( c \).

(iv) \( D(t) = a + bt + ct^2, \) the demand rate.

(v) \( A = \) ordering cost for items per order of inventory.

(vi) \( C = \) unit cost of the item

(vii) \( D_T = \) total demand during cycle period \( T \)
(viii) \( i \) = inventory carrying charge and expressed as \% of the average inventory

(ix) \( C_D \) = cost of deterioration per cycle per unit time

(x) \( C_H \) = holding cost per cycle per unit time

(xi) \( I(t) \) = inventory at any time \( t \).

(xii) \( \theta(t) = 0 \), constant rate of deterioration

(xiii) \( M \) = permissible delay time

(xiv) \( P_T \) = profit earned on the item at any time \( T \)

(xv) \( I_T \) = interest earned per cycle.

(xvi) \( C_{VT} \) = total variable cost per cycle.

(xvii) \( C_T(P, T) \) = variable cost per unit time.

(xviii) \( s \) = the time at which the inventory level reaches zero in the replacement cycle.

(xix) \( I_P \) = interest earned at time period \( P \).

(xx) \( I_C \) = interest payable at constant rate.

7.3 Mathematical model

In the development of this model, we assume that the variable rate of demand is considered with the variable rate of deterioration. Depletion of the inventory occurs due to demand as well as due to deterioration which occurs only when there is inventory i.e. in the period \([0, T]\). So, the required differential equation is given by

\[
\frac{di(t)}{dt} + \theta(t)I(t) = -D(t) \quad 0 \leq t \leq T
\]
Where, \( I(0) = I_0 \) and \( I(T) = 0 \).

\[ \ldots (7.1) \]

Solving the above linear differential equation (7.1) with the boundary conditions and putting constant = \( I_0 \), we get

\[
I(t) = e^{\frac{-\theta t^2}{2}} (I_0 - at - \frac{b t^2}{2} - \frac{c t^3}{3} - \frac{a t^3 \theta}{6} - \frac{b t^4 \theta}{8} - \frac{c t^5 \theta}{10}) \ldots (7.2)
\]

It is obvious that at \( t = T \) i.e. at the end of the cycle period, \( I(T) = 0 \). So (7.2) gives

\[
I_0 = aT + \frac{b T^2}{2} + \frac{c T^3}{3} + \frac{a T^3 \theta}{6} + \frac{b T^4 \theta}{8} + \frac{c T^5 \theta}{10} \ldots (7.3)
\]

So substituting the value of \( I_0 \) from the equation (7.3) in the equation (7.2), expanding the exponential power and multiplying and neglecting the higher power of \( \theta \), we get

\[
= -at - \frac{b t^2}{2} - \frac{c t^3}{3} + aT + \frac{b T^2}{2} + \frac{c T^3}{3} + \frac{a t^3 \theta}{6} + \frac{b t^4 \theta}{8} + \frac{c t^5 \theta}{10} - \frac{a t^2 T \theta}{2}
\]

\[
- \frac{b T^2 t^2 \theta}{4} + \frac{a T^3 \theta}{6} - \frac{c T^3 t^2 \theta}{6} + \frac{b T^4 \theta}{8} + \frac{c T^5 \theta}{10}
\]

\[ 0 \leq t \leq T \quad \ldots (7.4) \]

The total demand during cycle period \( T \) is given by \( \int_0^T D(t) \, dt \). Thus it can easily be seen that the amount of items deteriorates during one cycle is given by

\[
D_T = I_0 - \int_0^T D(t) \, dt
\]

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\[ = aT + \frac{b T^2}{2} + \frac{c T^3}{3} + \frac{a T^3 \theta}{6} + \frac{b T^4 \theta}{8} + \frac{c T^5 \theta}{10} - aT - \frac{b T^2}{2} - \frac{c T^3}{3} \]

\[ = \frac{a T^3 \theta}{6} + \frac{b T^4 \theta}{8} + \frac{c T^5 \theta}{10} \]

... (7.5)

So the total cost of deterioration is given by putting the value of \( D_T \) from the equation (7.5), \( C_D \) is computed.

\[ C_D = C D_T \]

\[ C_D = C \left( \frac{a T^3 \theta}{6} + \frac{b T^4 \theta}{8} + \frac{c T^5 \theta}{10} \right) \]

... (7.6)

The holding cost of the inventory is given by

\[ C_H = iC \int_0^T I(t) dt \]

Putting the value of \( I(t) \) from the equation number (7.4) in the above equation, we have

\[ = i C \left\{ \frac{a T^2}{2} + \frac{b T^3}{3} + \frac{c T^4}{4} + \frac{a T^4 \theta}{12} + \frac{b T^5 \theta}{15} + \frac{c T^6 \theta}{18} \right\} \]

... (7.7)

At any time \( t \) the cost of the unpaid inventory is the cost of the current inventory at that time, minus profit of amount sold during time \( M \), minus the interest earned from
the sales revenue during time $M$ the extra amount that can be paid can be
determined by profit on the amount sold after the permissible delay time $M$.
Therefore, the interest payable per cycle for the inventory not being sold after due
date is given by

$$P_T = I_p \int_{0}^{M} \left\{ CI(t) - (s - C) \int_{0}^{M} D(t) \, dt - s l_e \int_{0}^{M} D(t) \, dt \right\} \, dt - (s - C) I_p \int_{0}^{M} D(t) \, dt$$

$$= -\frac{2}{3} b C l_p M^3 + \frac{3}{2} b C l_p M^2 P - \frac{2}{3} c C l_p M^3 P - b C l_p M P^2 + \frac{3}{2} c C l_p M^2 p^2$$

$$+ \frac{1}{6} b C l_p P^3 - c C l_p M P^3 + \frac{1}{6} c C l_p P^4 + \frac{1}{2} a I_p M^2 s$$

$$+ \frac{5}{6} b l_p M^3 s + \frac{1}{2} a l_c l_p M^3 s + \frac{1}{12} c l_p M^4 s + \frac{1}{3} b l_c l_p M^4 s$$

$$+ \frac{1}{4} c l_c l_p M^5 s - \frac{3}{2} b l_p M^2 P s - \frac{1}{2} a l_c l_p M^2 P s + \frac{2}{3} c l_p M^3 P s$$

$$- \frac{1}{3} b l_c l_p M^3 P s - \frac{1}{4} c l_c l_p M^4 P s - \frac{1}{2} a I_p P^2 s + b l_p M^2 P s$$

$$- \frac{3}{2} c l_p M^2 P^2 s - \frac{1}{3} b l_p P^3 s + c l_p M P^3 s - \frac{1}{4} c l_p P^4 s - \frac{1}{4} c l_p P^4 s$$

$$- a C l_p M T + a C l_p P T - \frac{1}{2} b C l_p M T^2 + \frac{1}{2} b C l_p P T^2$$

$$- \frac{1}{3} c C l_p M T^3 + \frac{1}{3} c C l_p P T^3 - \frac{1}{12} a C l_p M^4 \theta - \frac{1}{40} b C l_p M^5 \theta$$

$$- \frac{1}{90} c C l_p M^6 \theta + \frac{1}{12} a C l_p P^4 \theta + \frac{1}{40} b C l_p P^5 \theta + \frac{1}{90} c C l_p P^6 \theta$$

$$+ \frac{1}{6} a C l_p M^3 T \theta - \frac{1}{4} a C l_p P^3 T \theta + \frac{1}{12} b C l_p M^3 T^2 \theta$$

$$- \frac{1}{12} b C l_p P^3 T^2 \theta - \frac{1}{6} a C l_p M T^3 \theta + \frac{1}{18} c C l_p M^3 T^3 \theta$$

$$+ \frac{1}{6} a C l_p P T^3 \theta - \frac{1}{18} c C l_p P^3 T^3 \theta - \frac{1}{8} b C l_p M T^4 \theta$$

$$+ \frac{1}{8} b C l_p P T^4 \theta - \frac{1}{10} c C l_p M T^5 \theta + \frac{1}{10} c C l_p P T^5 \theta$$

... (7.8)
Interest earned per cycle, \( I_T \) is the interest earned during the positive inventory is given by

\[
I_T = s l_c \left\{ \int_0^M D(t) dt + \int_0^{T-p} D(t) dt \right\}
\]

\[
= \frac{1}{2} al_c M^2 s + \frac{1}{3} bl_c M^3 s + \frac{1}{4} cl_c M^4 s + \frac{1}{2} al_c P^2 s - \frac{1}{3} bl_c P^3 s + \frac{1}{4} cl_c P^4 s - a l_c P s T
\]
\[
+ b l_c P^2 s T - c l_c P^3 s T + \frac{1}{2} a l_c s T^2 - b l_c P s T^2 + \frac{3}{2} c l_c P^2 s T^2
\]
\[
+ \frac{1}{3} b l_c s T^3 - c l_c P s T^3 + \frac{1}{4} c l_c s T^4
\]

... (7.9)

The total variable cost is the sum of ordering cost, carrying cost, deterioration cost and the interest payable cost minus the interest earned and is given by substituting the respective values in the above equation number (7.10) from (7.6), (7.7), (7.8) and (7.9), we can find the value of \( C_{VT} \).

\[
C_{VT} = A + C_D + C_H + P_T - I_T
\]

... (7.10)

And hence the average cost per unit time is given by

\[
C_T(P, T) = \frac{C_{VT}}{T}
\]
\[ a \cdot c \cdot l_p (P - M) + a \cdot l_c \cdot P \cdot s - b \cdot l_c \cdot P^2 \cdot s + c \cdot l_c \cdot P^3 \cdot s + \frac{A}{T} + \frac{3}{2} \cdot b \cdot c \cdot l_p \cdot M^2 \cdot P + \frac{2}{3} \cdot c \cdot l_p \cdot M^3 \cdot P \]

\[ \frac{b \cdot c \cdot l_p \cdot M^2 \cdot P}{T} + \frac{3}{3} \cdot c \cdot l_p \cdot M^2 \cdot P^2 \]

\[ + \frac{2}{3} \cdot b \cdot c \cdot l_p \cdot M^3 + \frac{b \cdot c \cdot l_p \cdot P^3}{c \cdot l_p \cdot M \cdot P^3} \]

\[ + \frac{c \cdot l_p \cdot P^4}{6} \cdot a \cdot (l_c - l_p) \cdot M^2 \cdot S \] 

\[ + \frac{b \cdot l_c \cdot M^3 \cdot s}{5 \cdot b \cdot l_c \cdot M^3 \cdot s} + \frac{a \cdot l_c \cdot l_p \cdot M^3 \cdot s}{T} \]

\[ + \frac{c \cdot l_c \cdot M^4 \cdot s}{4 \cdot T} + \frac{c \cdot l_p \cdot M^4 \cdot s}{3 \cdot T} + \frac{b \cdot l_c \cdot l_p \cdot M^4 \cdot s}{4 \cdot T} + \frac{c \cdot l_c \cdot l_p \cdot M^4 \cdot P \cdot s}{2 \cdot T} \]

\[ - \frac{a \cdot l_c \cdot l_p \cdot M^2 \cdot P \cdot s}{2} + \frac{2 \cdot c \cdot l_p \cdot M^3 \cdot P \cdot s}{3 \cdot T} - \frac{b \cdot l_c \cdot l_p \cdot M^3 \cdot P \cdot s}{2 \cdot T} - \frac{c \cdot l_c \cdot l_p \cdot M^4 \cdot P \cdot s}{4 \cdot T} \]

\[ + \frac{c \cdot l_p \cdot M^3 \cdot s}{4 \cdot T} - \frac{1}{2} \cdot a \cdot c \cdot T - \frac{1}{2} \cdot b \cdot c \cdot l_p \cdot (M - P) \cdot T - \frac{1}{2} \cdot a \cdot l_c \cdot s \cdot T \]

\[ + b \cdot l_c \cdot P \cdot s \cdot T - \frac{3}{2} \cdot b \cdot l_c \cdot P^2 \cdot s \cdot T + \frac{1}{3} \cdot b \cdot c \cdot i \cdot T^2 - \frac{1}{3} \cdot c \cdot c \cdot l_p \cdot M \cdot T^2 + \frac{1}{3} \cdot c \cdot c \cdot l_p \cdot P \cdot T^2 \]

\[ - \frac{1}{3} \cdot b \cdot s \cdot l_c \cdot T^2 + c \cdot l_c \cdot P \cdot s \cdot T^2 + \frac{1}{4} \cdot c \cdot c \cdot i \cdot T^3 - \frac{1}{4} \cdot c \cdot l_c \cdot s \cdot T^3 \]

\[ + \theta \left( \frac{1}{6} \cdot a \cdot c \cdot l_p \cdot (M^3 - P^3) - \frac{a \cdot c \cdot l_p \cdot (M^4 - P^4)}{12 \cdot T} - \frac{b \cdot c \cdot l_p \cdot (M^5 - P^5)}{40 \cdot T} \right) \frac{c \cdot l_p \cdot (M^6 - P^6)}{90 \cdot T} \]

\[ + \frac{b \cdot c \cdot l_p \cdot (M^3 - P^3) \cdot T}{12} + \frac{1}{6} \cdot a \cdot c \cdot T^2 - \frac{b \cdot c \cdot l_p \cdot M \cdot T^2}{6} \]

\[ + \frac{1}{18} \cdot c \cdot c \cdot l_p \cdot M^3 \cdot T^2 + \frac{1}{6} \cdot a \cdot c \cdot l_p \cdot P \cdot T^2 - \frac{1}{18} \cdot c \cdot c \cdot l_p \cdot P^3 \cdot T^2 + \frac{b \cdot c \cdot T^3}{8} \]

\[ + \frac{1}{12} \cdot a \cdot c \cdot i \cdot T^3 - \frac{1}{8} \cdot b \cdot c \cdot l_p \cdot (M - P) \cdot T^3 + \frac{1}{10} \cdot c \cdot c \cdot T^4 + \frac{1}{15} \cdot b \cdot c \cdot i \cdot T^4 \]

\[ - \frac{1}{10} \cdot c \cdot c \cdot l_p \cdot (M - P) \cdot T^4 + \frac{1}{18} \cdot c \cdot c \cdot i \cdot T^5 \]

\[ \ldots (7.11) \]

The average values of P and T will give us the optimal solution and is solved by Newton’s Technique. We have to find a minimum value of \( C_T(P, T) \) corresponding values of T and P. So the necessary conditions of \( C_T(P, T) \) to be minimum are

\[ \frac{\partial C_T(P, T)}{\partial T} = 0 \]
\[ 
\Phi - \frac{A}{T^2} = \frac{3bc l_p M^2 P}{2T^2} + \frac{2c C l_p M^3 P}{3T^2} + \frac{b C l_p M^2 P}{T^2} - \frac{3c C l_p M^2 P^2}{2T^2} - \frac{b C l_p P^3}{6T^2} \\
+ \frac{c C l_p M P^3}{T^2} - \frac{5b l_p M^2 s}{6T^2} - \frac{a l_c l_p M^3 s}{2T^2} + \frac{2c l_c M^4 s}{3T^2} - \frac{c l_p M^4 s}{12T^2} - \frac{b l_c l_p M^4 s}{3T^2} \\
- \frac{c l_c l_p M^3 s}{2T^2} + \frac{3b l_p M^2 P s}{2T^2} + \frac{a l_c l_p M^2 s}{2T^2} - \frac{2c l_p M^3 P s}{12T^2} \\
+ \frac{b l_c l_p M^3 P s}{3T^2} + \frac{4l_c l_p M^4 P s}{3T^2} + \frac{a (l_c + l_p) P^2 s}{2T^2} - \frac{b l_p M P^2 s}{6T^2} \\
+ \frac{3c l_p M^2 P^2 s}{T^2} - \frac{b (l_c - l_p) P^3 s}{3T^2} - \frac{c l_p M^3 P^3 s}{12T^2} + \frac{c (l_c + l_p) P^4 s}{4T^2} \\
- \frac{1}{2} a i c - \frac{1}{2} b C l_p (M - P)T - \frac{1}{2} a l_c s + b l_c P s - \frac{3}{2} c l_c P^2 s \\
+ \frac{1}{3} b C i T - \frac{2}{3} C l_p M T + \frac{2}{3} C l_p P T - \frac{1}{3} b s l_c T + 2c l_c P s T \\
+ \frac{1}{4} c C i T^2 - \frac{3}{4} c l_c s T^2 \\
+ \theta \left\{ \frac{a C l_p (M^4 - P^4)}{12T^2} + \frac{b C l_p (M^5 - P^5)}{40T^2} + \frac{c C l_p (M^6 - P^6)}{90T^2} \\
+ \frac{b C l_p (M^3 - P^3)}{12} + \frac{a C T - a C l_p M T}{3} + \frac{a C l_p P T}{3} - \frac{1}{9} C l_p (P^3) \\
- \frac{M^3}{T} + \frac{3b C T^2}{8} + \frac{1}{4} a C i T^2 - \frac{3}{8} b C l_p (M - P)T^2 + \frac{2}{5} c C T^3 \\
+ \frac{4}{15} b C i T^3 - \frac{2}{5} c C l_p (M - P)T^3 + \frac{5}{18} c C i T^4 \right\} = 0 
\]

\[ \frac{\partial C_T(P,T)}{\partial P} = 0 \]
\[ a \cdot c \cdot I_p + a \cdot l_c \cdot s - 2 \cdot b \cdot l_c \cdot P \cdot s + 3 \cdot c \cdot l_c \cdot P^2 \cdot s + \frac{3 \cdot b \cdot c \cdot I_p \cdot M^2}{2 \cdot T} - \frac{2 \cdot c \cdot C \cdot I_p \cdot M^3}{3 \cdot T} - \frac{b \cdot C \cdot I_p \cdot M^2}{T} + \frac{3 \cdot c \cdot I_p \cdot M^2 \cdot P}{T} + \frac{3 \cdot b \cdot C \cdot I_p \cdot P^2}{2 \cdot T} - \frac{3 \cdot c \cdot I_p \cdot M^2}{2 \cdot T} - \frac{b \cdot I_p \cdot M^2}{T} - \frac{a \cdot l_c \cdot I_p \cdot M^2 \cdot s}{2 \cdot T} + \frac{2 \cdot c \cdot I_p \cdot M^3}{2 \cdot T} - \frac{b \cdot I_c \cdot I_p \cdot M^3 \cdot s}{3 \cdot T} - \frac{c \cdot I_c \cdot I_p \cdot M^4 \cdot s}{4 \cdot T} - \frac{a \cdot (l_c + l_p) \cdot P \cdot s}{T} + \frac{2 \cdot b \cdot I_p \cdot M \cdot P \cdot s}{T} - \frac{3 \cdot c \cdot I_p \cdot M^2 \cdot P \cdot s}{T} + \frac{b \cdot (l_c - l_p) \cdot P^2 \cdot s}{T} + \frac{3 \cdot c \cdot I_p \cdot M \cdot P^2 \cdot s}{T} - \frac{c \cdot I_p \cdot P^3 \cdot s}{T} + \frac{1}{2} \cdot b \cdot C \cdot I_p \cdot T + b \cdot l_c \cdot s \cdot T - 3 \cdot c \cdot I_p \cdot P \cdot s \cdot T + \frac{1}{3} \cdot c \cdot C \cdot I_p + c \cdot l_c \cdot s \cdot T^2 + \theta \left\{ -\frac{1}{2} \cdot a \cdot c \cdot I_p \cdot P^2 + \frac{a \cdot C \cdot I_p \cdot P^3}{3 \cdot T} + \frac{b \cdot C \cdot I_p \cdot P^4}{8 \cdot T} + \frac{c \cdot C \cdot I_p \cdot P^5}{15 \cdot T} - \frac{b \cdot C \cdot I_p \cdot P^2 \cdot T}{4} \right\} = 0 \]

\[ \cdots (7.12.2) \]

Solving the equation (7.12.1) and (7.12.2) for \( T \) and \( P \). The values of \( T \) and \( P \) will be optimal i.e. \( T = T^* \) and \( P = P^* \) Hessian determinant is greater than zero

\[ H = \begin{vmatrix} \frac{\partial^2 C_T(P,T)}{\partial P^2} & \frac{\partial^2 C_T(P,T)}{\partial T \partial P} \\ \frac{\partial^2 C_T(P,T)}{\partial T \partial P} & \frac{\partial^2 C_T(P,T)}{\partial P^2} \end{vmatrix} \]

Corresponding minimum average cost \( C_T^*(P^*, T^*) = C_T(P, T) \) \( (T^*, P^*) \)

7.4 Solution Procedure

The total average cost \( C_T(P, T) \) given above, is a function of the two variables \( T \) and \( P \). The necessary condition for \( C_T(P, T) \) to be minimum is given by the equations
These equations are highly non-linear in nature, which can be easily solved by Newton Method of higher variables when the values of the parameters are prescribed. The optimal solutions of the equations (7.12.1) and (7.12.2) are \( T^* \) and \( P^* \) provided these values satisfy the conditions \( H > 0 \). Subsisting these values in equation (7.12), the optimal average cost \( C_{T}(P, T) \) \( (T^*, P^*) \) can be obtained.

### 7.5 Numerical examples

To illustrate the obtained result following are some examples

Considering, \( A = 250 \) units, \( s = 46, I_p = 0.18, I_c = 0.15, a = 12, b = 7.3, c = 0.9, \theta(t) = 0.23, i = 0.26 \) and \( C = 1.72 \) units

**Case I**, for \( M = 0 \),

\[ T^* = 1.9 \text{ year, and } P^* = 1.43 \text{ year and } C_{T}(P, T)^{*} = 45.71 \text{ units} \]

**Case II**, for \( M = 0.3 \) year,

\[ T^* = 2.05 \text{ year, and } P^* = 1.7 \text{ year and } C_{T}(P, T)^{*} = 27.88 \text{ units} \]
7.6 Conclusion

In this chapter, some realistic features are considered. These features are likely to be associated with an inventory of consumer goods. The assumption of a quadratic time-dependent demand rate and permissible delay in payments are very realistic in the market. The deterioration rate is also assumed to be constant. From the numerical example, we observe that for $0 < a, b, c \leq 1$, the total average cost is minimum. If we put $c = 0$ in equation number (7.11), we get the total average cost as in the model of (Singh and Singh, 2009) with demand rate $D(t) = a + bt$. To illustrate the model two numerical examples are done, with different situations.

The above model can be converted into constant demand model by taking ‘$b=0$, $c=0$’, linear demand model by taking ‘$c=0$’, or for items having no deterioration by taking ‘$0(t)=0$’. This study can further be extended for items having various types of other demand patterns like periodic time dependent demand, stock dependent demand, exponential demand rate as well as the effect of inflation and time value of money also can be incorporated in this model to make it more realistic for the business environment. Thus, this kind of model will also help the retailers or buyers in deciding their optimal order quantity to have minimum inventory cost, payment time, and benefits of permissible delay in payments.
CHAPTER 8

AN EOQ MODEL WITH LINEAR TIME DEPENDENT DETERIORATION RATE AND PERIODIC TIME DEPENDENT DEMAND UNDER PERMISSIBLE DELAY IN PAYMENT
8.1 Introduction

In the classical EOQ (Economic Order Quantity) model developed in 1915, the demand rate of an item was assumed as constant. Thereafter, many models were developed in the inventory literature considering constant demand. However, in the real market, the demand rate of any product is always in a dynamical state. Many inventory modelers have paid their attention for various types of time-dependent demand viz. Constant Demand, Linear Demand, Quadratic Demand, Exponential Demand etc. A linearly time-varying demand indicates a uniform change in demand rate of the item per unit time which seldom occurs in the real market. On the other hand, exponentially time dependent demand and quadratic time dependent demand indicates very rapid change in demand rate which is also unrealistic because the demand of any product can not undergo a rate which is so high as exponential demand is. Among the various time-varying demand in EOQ models, the more realistic demand approach is to consider a periodic time dependent demand rate because it represents both accelerated and retarded growth in demand.

In this chapter, an EOQ model is developed with deteriorating item having periodic time dependent demand when delay in payment is permissible. The deterioration rate is assumed to be a linear function of time. Different mathematical models are derived with different situations, i.e. if the

\[ \Rightarrow \text{Credit period is less than the cycle time for settling the account.} \]
\[ \Rightarrow \text{Credit period is equal to the cycle time for settling the account.} \]
\[ \Rightarrow \text{Credit period is greater than the cycle time for settling the account.} \]

The above models are illustrated with numerical examples. Also the sensitivity analysis of the model is examined for changes in the parameters.
8.2 Assumptions and Notations

In this chapter, we discussed an inventory model with the following assumptions:

(i) The demand rate for the item is represented by a periodic function of time.

(ii) Shortage is not allowed.

(iii) The deterioration rate is linearly time dependent.

(iv) Time horizon is infinite.

(v) Delay in payments is permissible for fixed time.

The following notations are used in developing the model:

1) The demand rate \( D(t) = a + bt + csint \), where \( t \geq 0 \), \( a \geq 0 \), \( b \neq 0 \) and \( c \neq 0 \).
   Also assumed that \( |b| < a \) and \( |c| < a \) to ensure that the demand rate is non-negative.

2) \( U_p \) is the unit purchase cost of item.

3) \( C_h \) is the inventory holding per unit per unit time.

4) \( \theta(t) = \alpha + \beta t \), is the rate of deterioration of an item.

5) \( V_p \) is the unit deterioration cost of an item

6) \( A \) is the replenishment cost.

7) \( I_p \) is the interest charges per rupee investment in stock per unit time.

8) \( I_e \) is the interest earned per rupee per unit time.

9) \( T \) is the total time period.
$10)$ \(t_1\) is permissible period (in unit of time) of delay in settling the accounts with the supplier.

### 8.3 Mathematical Model

The instantaneous inventory level \(Q(t)\) at any time \(t\) during the cycle time \(T\) is given by the following equation

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -(a + bt + csint) \quad 0 \leq t \leq T
\]

\[\text{... (8.A.1)}\]

Where \(Q(0) = Q_0\) and \(Q(T) = 0\)

Solving equation (8.A.1), we get

\[
Q(t) = -a \frac{t}{2} - \frac{b t^2}{2} + a T + \frac{b T^2}{2} + \frac{a t^2 \beta}{2} + \frac{b t^3 \beta}{6} - a t T \beta + \frac{a T^2 \beta}{2} + \frac{b t T^2 \beta}{2} \\
+ \frac{b T^3 \beta}{3} + \frac{a t^3 \beta}{3} + \frac{b t^4 \beta}{8} - \frac{a t^2 T \beta}{2} - \frac{b t^2 T^2 \alpha}{4} + \frac{a T^3 \alpha}{6} \\
+ \frac{b T^4 \alpha}{8} + c \alpha \cos t - c \alpha \cos \frac{T}{2} + c T \frac{\alpha \cos t}{2} - c T \frac{\alpha \cos \frac{T}{2}}{2} - c T \frac{\alpha \sin t}{2} - c \alpha \frac{\alpha \sin t}{2} - c T \frac{\alpha \sin T}{2} - c \alpha \frac{\alpha \sin \frac{T}{2}}{2} \text{... (8.A.2)}
\]

Using the initial value and neglecting the higher powers of \(\beta\) and \(\alpha\), we have
\[ Q_0 = c + aT + \frac{bT^2}{2} + \frac{aT^2 \beta}{2} + \frac{bT^3 \beta}{3} - c\alpha + \frac{aT^3 \alpha}{6} + \frac{bT^4 \alpha}{6} + \frac{bT^4 \alpha}{4} \]

\[-c\cos T - cT\beta \cos T + C\alpha \cos T - \frac{cT^2\alpha \cos T}{2} + c\beta \sin T + cT\alpha \sin T \]

\[ \cdots \text{(8.A.3)} \]

Total demand during the cycle period \([0, T]\) is given by

\[ \int_0^T D(t)dt = c + aT + \frac{bT^2}{2} - c\cos T \]

\[ \cdots \text{(8.A.4)} \]

Number of deteriorating Items = (Initial Inventory at time \(t = 0\)) - (Total demand during the time period \([0, T]\))

\[ = Q_0 - \int_0^T D(t)dt \quad \text{[Using (8.A.3) and (8.A.4)]} \]

\[ = \frac{aT^2 \beta}{2} + \frac{bT^3 \beta}{3} - c\alpha + \frac{aT^3 \alpha}{6} + \frac{bT^4 \alpha}{8} - cT\beta \cos T + C\alpha \cos T + 2c\cos T \]

\[-\frac{cT^2\alpha \cos T}{2} + c\beta \sin T + cT\alpha \sin T \]

\[ \cdots \text{(8.A.5)} \]

Therefore the deteriorating cost is given by \((V_p)(Q_0 - \int_0^T D(t)dt) \quad \text{[Using (8.A.5)]}\)

\[ V_p \left( \frac{aT^2 \beta}{2} + \frac{bT^3 \beta}{3} - c\alpha + \frac{aT^3 \alpha}{6} + \frac{bT^4 \alpha}{8} - cT\beta \cos T + C\alpha \cos T + 2c\cos T \right) \]

\[-\frac{cT^2\alpha \cos T}{2} + c\beta \sin T + cT\alpha \sin T \right) \]

\[ \cdots \text{(8.A.6)} \]
Total holding cost of the items for the period \([0, T]\) is given by

\[
C_h \{ - c x + \frac{1}{120} (60 a T^2 + 40 b T^3 + 20 a T^3 \beta + 15 b T^4 \beta + 10 a T^4 \alpha \\
+ 8 b T^5 \alpha - 120 c T \cos T + 120 c \beta \cos T - 60 c T^2 \beta \cos T \\
+ 240 c T \alpha \cos T - 40 c T^3 \alpha \cos T + 120 c \sin T + 120 c T \beta \sin T \\
- 240 c \alpha \sin T + 120 c T^2 \alpha \sin T) \}
\]

... (8.A.7)

Interest earned in the cycle period \([0, T]\) is given by

\[
= U_p \times I_c \times \int_0^T t \, D(t) \, dt
\]

\[
= \left( I_c \right) \left( U_p \right) \left\{ \frac{1}{6} T^2 (3 a + 2 b T) - c T \cos T + c \sin T \right\}
\]

... (8.A.8)

Now the above model can be classified into three different cases viz. \(T > t_1; T < t_1; T = t_1\).

**Case I** 'When \(T > t_1\)'

Interest payable during the time \([t_1, T]\) is given by,

\[
= U_p \times I_p \times \int_{t_1}^T Q(t) \, dt
\]

[Using (8.A.2)]
\[
\begin{align*}
= I_p \times U_p \times \left(\frac{1}{12c}\right) [ & (60 a T^2 + 40 b T^3 + 20 a T^3 \beta + 15b T^4 \beta + \\
10 a T^4 \alpha + 8 b T^5 y - 120 c T CosT + 120 c \beta CosT - 60 c T^2 \beta CosT + \\
240 c T \alpha CosT - 40 c T^3 \alpha 1' CosT + 120 c SinT + 120 c T \beta SinT - \\
240 c \alpha SinT + 120 c T^2 \alpha SinT) + (-120 a T t_1 - 60 a T^2 t_1 + 60 a t_1^2 + \\
20 b t_1^3 - 60 a T^2 t_1 \beta - 40 b T^3 t_1 \beta + 60 a T t_1^2 \beta + 30 b T^2 t_1^2 \beta - \\
20 a t_1^3 \beta - 5 b t_1^4 \beta - 20 a T^3 t_1 \alpha - 15 b T^4 t_1 \alpha + 20 a T t_1^3 \alpha + 10 b T^2 t_1^3 \alpha - \\
10 a t_1^4 \alpha - c b t_1^5 \alpha + 120 c t_1 CosT + 120 c T t_1 \beta CosT - 60 c t_1^2 \beta CosT - \\
120 c t_1 \alpha Cost_1 + 60 c T^2 t_1 \alpha CosT - 20 c t_1^3 \alpha CosT - 120 c \beta Cost_1 - \\
120 c t_1 \alpha Cost_1 - 120 c t_1 \beta SinT - 120 c T t_1 \alpha SinT - 120 c SinT + 240 c a Sint_1) ]
\end{align*}
\]
\[\text{... (8.A1.9)}\]

The total cost per unit time is = [Replenishment cost + Inventory Holding Cost + Deterioration Cost + Interest payable during Delay period – Interest earned during the cycle]

The total average cost per unit time is = \(\frac{1}{T}\) \times [Replenishment cost + Inventory Holding Cost + Deterioration Cost + Interest payable during Delay period – Interest earned during the cycle]

\[\text{... (8.A1.10)}\]

By using the equations (8.A.6), (8.A.7), (8.A.8) and (8.A1.9) in equation (8.A1.10) and expanding the values of SinT, CosT, Sint_1, Cost_1 and simplifying, we have

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\[ TAC_{A1} = \frac{A}{T} + \frac{a C_h}{3} - \frac{(b + c) C_h}{3} + \frac{a}{2} \left( I_c - I_p \right) T U_p - \frac{a}{3} \left( I_c - I_p \right) T U_p - \frac{(b + c) I_c T^2 U_p}{3} + \frac{(b + c) I_p T^2 U_p}{3} - \frac{a I_p t_1 U_p}{2} - \frac{(b + c) I_p T t_1 U_p}{2} + \frac{a I_p t_1^2 U_p}{2 T} + \frac{(b + c) I_p t_1^3 U_p}{6 T} + \frac{a C_h T^2 \beta}{6} + \frac{b C_h T^3 \beta}{8} + \frac{c C_h T^3 \beta}{12} + \frac{a I_p T^2 U_p \beta}{6} + \frac{b I_p T^3 U_p \beta}{8} + \frac{c I_p T^3 U_p \beta}{12} - \frac{a I_p T t_1 U_p \beta}{2} + \frac{(b + c) I_p t_1 U_p \beta}{4} + \frac{a I_p t_1^3 U_p \beta}{6 T} + \frac{b I_p t_1^3 U_p \beta}{24 T} + \frac{a T V_p \beta}{2} + \frac{(b + c) T^2 V_p \beta}{3} + \frac{a C_h T^3 \alpha}{12} + \frac{b C_h T^4 \alpha}{15} + \frac{a I_p T^3 U_p \alpha}{12} + \frac{b I_p T^4 U_p \alpha}{15} - \frac{a I_p t_1 U_p \alpha}{6} - \frac{b I_p T^3 t_1 U_p \alpha}{8} - \frac{c I_p T^3 t_1 U_p \alpha}{12} + \frac{a I_p t_1^3 U_p \alpha}{6} + \frac{(b + c) a I_p T t_1^3 U_p \alpha}{12} - \frac{a I_p t_1^4 U_p \alpha}{12 T} - \frac{b I_p t_1^5 U_p \alpha}{40 T} + \frac{a T^2 V_p \alpha}{6} + \frac{b T^3 V_p \alpha}{8} + \frac{c T^3 V_p \alpha}{12} \]

\[ \text{To find the minimum value of total average cost (TAC}_{A1}) \text{, so we have} \]

\[ \frac{d(TAC_{A1})}{dT} = 0 \]
$$\Phi = -\frac{A}{T^2} + \frac{a C_h}{2} + \frac{2(b + c)C_h T}{3} - \frac{a (I_e - I_p) U_p}{2} - \frac{2(b + c)I_e T U_p}{3} + \frac{2(b + c)I_p T U_p}{3} - \frac{(b + c)I_p t_1 U_p}{2} - \frac{a I_p t_1^2 U_p}{2 T^2} + \frac{(b + c)I_p t_1^3 U_p}{6 T^2} + \frac{a C_h T \beta}{3} + \frac{3 b C_h T^2 \beta}{8} + \frac{c C_h T^2 \beta}{4} + \frac{a I_p T U_p \beta}{3} + \frac{3 b I_p T^2 U_p \beta}{8} + \frac{c I_p T^2 U_p \beta}{4} - \frac{a I_p t_1 U_p \beta}{2} + \frac{2 (b + c)I_p T t_1 U_p \beta}{3} + \frac{(b + c)I_p t_1 U_p \beta}{4} + \frac{a I_p t_1^3 U_p \beta}{6 T^2} + \frac{b I_p t_1^3 U_p \beta}{24 T^2} + \frac{a V_p \beta}{2} + \frac{2 (b + c)T V_p \beta}{3} + \frac{a C_h T^2 \alpha}{4} + \frac{4 b C_h T^3 \alpha}{15} + \frac{a I_p T^2 U_p \alpha}{4} + \frac{4 b I_p T^3 U_p \alpha}{15} - \frac{a I_p T t_1 U_p \alpha}{3} - \frac{3 b I_p T^2 t_1 U_p \alpha}{8} - \frac{c I_p T^2 U_p \alpha}{4} + \frac{(b + c)I_p t_1^3 U_p \alpha}{12 T^2} + \frac{a I_p t_1^4 U_p \alpha}{12 T^2} + \frac{b I_p t_1^5 U_p \alpha}{40 T^2} + \frac{a T V_p \alpha}{3} + \frac{3 b T^2 V_p \alpha}{8} + \frac{c T^2 V_p \alpha}{4} = 0$$

... (8.A1.12)

By solving the equation (8.A1.12) by Newton Rapson Method, we can have the value of T. Then ‘T’ will be optimal i.e. T*, if the following condition (8.A1.13) will be satisfied with T=T*.

$$\frac{d^2(TAC_{A1})}{dT^2} > 0$$

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\[
\frac{2A}{T^3} + \frac{2(b + c)C_h}{3} - \frac{2(b + c)I_p}{3} U_p + \frac{2(b + c)I_p}{3} U_p + \frac{a I_p t_1^2}{T^3} U_p \\
- \frac{(b + c) I_p t_1^3}{3 T^3} U_p + \frac{a C_h \beta}{3} + \frac{3 b C_h T \beta}{4} + \frac{c C_h T \beta}{2} + \frac{a I_p U_p \beta}{3} \\
+ \frac{3 b I_p T U_p \beta}{4} + \frac{c I_p T U_p \beta}{2} + \frac{2 (b + c) I_p t_1 U_p \beta}{3} - \frac{a I_p t_1^3 U_p \beta}{3 T^3} \\
- \frac{b I_p t_1^3 U_p \beta}{12 T^3} + \frac{2 (b + c) V_p \beta}{3} + \frac{a C_h T \alpha}{2} + \frac{4 b C_h T^2 \alpha}{5} \\
+ \frac{a I_p T U_p \alpha}{2} + \frac{4 b I_p T^2 U_p \alpha}{5} - \frac{a I_p t_1 U_p \alpha}{3} - \frac{3 b I_p t_1 U_p \alpha}{4} \\
- \frac{c I_p T t_1 U_p \alpha}{2} - \frac{a I_p t_1^4 U_p \alpha}{6 T^3} - \frac{b I_p t_1^5 U_p \alpha}{20 T^3} + \frac{a V_p \alpha}{3} \\
+ \frac{3 b T V_p \alpha}{4} + \frac{c T V_p \alpha}{2} > 0
\]

… (8.A1.13)

So if \( T \) from (8.A1.12) satisfy the condition (8.A1.13) then the optimal value will be \( T = T^* \) and thus the minimum value of \( TAC_{A1}^* \) can be obtained from the equation (8.A1.11).

**Case II 'When \( T < t_1 \)**

In this case the customer earns interest on the sales revenue upto the permissible delay period and no interest is payable during the period for the item kept in stock.

So interest earned for the permissible delay period \([T, t_1]\) is given by

\[
= U_p \times I_e \times (t_1 - T) \int_0^T d(t) \, dt \\
= U_p \times I_e \times (t_1 - T) \left( c + a T + \frac{b T^2}{2} - c \cos T \right)
\]

… (8.A2.14)
So in this case the total cost per unit time is \( = [\text{Replenishment cost} + \text{Inventory Holding Cost} + \text{Deterioration Cost} - \text{Interest payable during Delay period} - \text{Interest earned during the cycle}] \)

The total average cost per unit time is \( = \frac{1}{t} \times [\text{Replenishment cost} + \text{Inventory Holding Cost} + \text{Deterioration Cost} + \text{Interest payable during Delay period} - \text{Interest earned during the cycle}] \)

\[ \ldots (8.2.15) \]

By using the equations (8.A.6), (8.A.7), (8.A.8) and (8.A2.14) in equation (8.A2.15) and expanding the values of \( \sin T, \cos T, \sin t_1, \cos t_1 \) and simplifying, we have
\[
TAC_{A2} = \frac{A}{T} + \frac{a C_h T}{2} + \frac{(b + c) C_h T^2}{3} - \frac{a (l_e - l_p) T U_p}{2} + \frac{(b + c) l_e T^2 U_p}{3} \\
- \frac{(b + c) l_p T^2 U_p}{3} - a l_e t_1 U_p + a l_p t_1 U_p + \frac{(b + c) l_p T t_1 U_p}{2} \\
- \frac{(b + c) l_e T t_1 U_p}{2} - \frac{a l_p t_1^2 U_p}{2 T} - \frac{(b + c) l_p t_1^3 U_p}{6 T} + \frac{a C_h T^2 \beta}{6} \\
+ \frac{b C_h T^3 \beta}{8} + \frac{c C_h T^3 \beta}{12} - \frac{a l_p T^2 U_p \beta}{6} - \frac{b l_p T^3 U_p \beta}{8} - \frac{c l_p T^3 U_p \beta}{12} \\
+ \frac{a l_p T t_1 U_p \beta}{2} + \frac{(b + c) l_p T^2 t_1 U_p \beta}{6} - \frac{a l_p t_1^2 U_p \beta}{2} \\
- \frac{(b + c) l_p T t_1 U_p \beta}{4} + \frac{a l_p t_1^3 U_p \beta}{6 T} + \frac{b l_p t_1^3 U_p \beta}{24 T} + \frac{a T V_p \beta}{2} \\
+ \frac{(b + c) T^2 V_p \beta}{3} + \frac{a C_h T^3 \alpha}{12} + \frac{b C_h T^4 \alpha}{15} - \frac{a l_p T^3 U_p \alpha}{12} \\
- \frac{b l_p T^4 U_p \alpha}{15} + \frac{a l_p T^2 t_1 U_p \alpha}{6} + \frac{b l_p T^3 t_1 U_p \alpha}{8} + \frac{c l_p T^3 t_1 U_p \alpha}{12} \\
- \frac{a l_p t_1^3 U_p \alpha}{6} - \frac{(b + c) a l_p T t_1^3 U_p \alpha}{12 T} + \frac{a l_p t_1^4 U_p \alpha}{12} \\
+ \frac{b l_p t_1^5 U_p \alpha}{40 T} + \frac{a T^2 V_p \alpha}{6} + \frac{b T^3 V_p \alpha}{8} + \frac{c T^3 V_p \alpha}{12} \\
\]

To find the minimum value of total average cost \(TAC_{A2}\), so we have

\[
\frac{d(TAC_{A2})}{dT} = 0
\]
\[ \Phi = -\frac{A}{T^2} + \frac{a}{2} \frac{C_h}{3} \frac{3}{2} + \frac{2(b + c)C_h T}{3} + \frac{a}{2} \left( \frac{Ic - Ip}{2} \right) U_p + \frac{2(b + c)Ic T U_p}{3} - \frac{2(b + c)I_p T U_p}{3} + \frac{b + c}{3} I_p t_1 U_p + \frac{a I_p t_1^2 U_p}{2} + \frac{a I_p T U_p}{3} + \frac{b I_p T^2 U_p}{8} + \frac{c I_p T^2 U_p}{4} + \frac{a I_p t_1 U_p}{2} + \frac{2(b + c)I_p T t_1 t_1 U_p}{3} \]

\[ = \frac{b I_p t_1^3 U_p}{24 T^2} + \frac{a V_p}{2} + \frac{2(b + c)T V_p}{3} + \frac{a C_h T^2}{4} + \frac{4b C_h T^3}{15} \frac{a I_p T^2 U_p}{4} - \frac{4b I_p T^3 U_p}{15} + \frac{a I_p T t_1 U_p}{3} + \frac{3b I_p T^2 t_1 U_p}{8} + \frac{c I_p T^2 t_1 U_p}{4} - \frac{(b + c) a I_p t_1^3 U_p}{12} + \frac{a I_p t_4 U_p}{12 T^2} - \frac{b I_p t_1^5 U_p}{40 T^2} + \frac{a T V_p}{3} + \frac{3b T^2 V_p}{8} + \frac{c T^2 V_p}{4} = 0 \]

\[ \text{... (8.A2.17)} \]

By solving the equation (8.A2.17) by Newton Rapson Method, we can have the value of T. Then 'T' will be optimal i.e. T*, if the following condition (8.A2.18) will be satisfied with T=T*.

\[ \frac{d^2(TAC_{A2})}{dT^2} > 0 \]
\[ -\frac{A}{T^2} + \frac{2(b + c)C_h}{3} - \frac{2(b + c)I_e}{3} U_p - \frac{2(b + c)I_p}{3} U_p - \frac{a I_p}{T^3} t_1^2 U_p \]

\[ - \frac{(b + c) I_p}{3} t_1^3 U_p - \frac{3 b I_p}{4} U_p - \frac{a C_h}{4} T \beta + \frac{3 b C_h}{2} T \beta - \frac{a I_p}{3} U_p \beta \]

\[ + \frac{b I_p}{12} t_1^3 U_p \beta - \frac{2(b + c) V_p}{3} \beta + \frac{a C_h}{2} T \alpha + \frac{4 b C_h}{5} T^2 \alpha \]

\[ + \frac{a I_p}{2} U_p \alpha - \frac{4 b I_p}{3} T U_p \alpha + \frac{a I_p}{3} t_1 U_p \alpha + \frac{3 b I_p}{4} T t_1 U_p \alpha \]

\[ + \frac{c I_p}{2} t_1 U_p \alpha + \frac{a I_p}{6} t_1^4 U_p \alpha + \frac{b I_p}{20} t_1^5 U_p \alpha + \frac{a V_p}{3} T \alpha \]

\[ + \frac{3 b}{4} T V_p \alpha + \frac{c T}{2} V_p \alpha > 0 \]

... (8.A2.18)

So if T from (8.A2.17) satisfy the condition (8.A2.18) then the optimal value will be

\( T = T^* \)

and thus the minimum value of \( (TAC^*_{a2}) \) can be obtained from the equation (8.A2.16).

**Case III ‘When T = t_1’**

For \( T = t_1 \), if we replace the value of \( t_1 \) in any of the equation (8.A1.11) or in (8.A2.16), we will get the required result i.e. \( I_c = I_p = 0 \).

\[ \frac{A}{t_1} + \frac{a C_h}{2} t_1^2 + \frac{(b + c)C_h}{6} t_1^2 \beta + \frac{b C_h}{8} t_1^3 \beta + \frac{c C_h}{12} t_1^3 \beta + \frac{a t_1}{2} V_p \beta \]

\[ + \frac{(b + c)t_1^2 V_p}{3} \beta + \frac{a C_h}{12} t_1^3 \alpha + \frac{b C_h}{15} t_1^4 \alpha + \frac{a t_1^2}{6} V_p \alpha + \frac{b t_1^3}{8} V_p \alpha \]

\[ + \frac{c t_1^3 V_p \alpha}{12} \]

... (8.A3.19)
The result obtained from the equation by putting the values of the parameters used in equation (8.A3.19) will be feasible if the value of $TAC_{A3} > 0$.

### 8.4 Numerical examples

Let us consider the example for $a=55$ units per month; $b=10$ units per month; $c=5$ units per month; $A=Rs \ 25.00$ per order; $I_p=0.15$ per month; $I_e=0.13$ per month; $C_b=Rs \ 0.12$ per month; $U_p=Rs \ 1.80$ per month; $V_p=Rs \ 0.50$ per month; $\alpha=0.15$; $\beta=0.20$.

<table>
<thead>
<tr>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T &gt; t_1 \ (=0.25 \text{ Month})$</td>
<td>$T &lt; t_1 \ (=1.25 \text{ Month})$</td>
<td>$T = t_1 \ (=0.98 \text{ Month})$</td>
</tr>
<tr>
<td>$\Rightarrow T = 1.28 \text{ Month}$</td>
<td>$\Rightarrow T = 0.87 \text{ Month}$</td>
<td>$\Rightarrow T = 0.98 \text{ Month}$</td>
</tr>
<tr>
<td>$\Rightarrow \frac{\partial^2(TAC_{A1})}{\partial T^2} = 33.47 &gt; 0$</td>
<td>$\Rightarrow \frac{\partial^2(TAC_{A2})}{\partial T^2} = 46.86 &gt; 0$</td>
<td>$\Rightarrow \frac{\partial^2(TAC_{A3})}{\partial T^2} = 57.16 &gt; 0$</td>
</tr>
<tr>
<td>$\Rightarrow TAC_{A1} = Rs \ 29.49 \text{ per Month}$</td>
<td>$\Rightarrow TAC_{A2} = Rs \ 28.68 \text{ per Month}$</td>
<td>$\Rightarrow TAC_{A3} = Rs \ 33.46 \text{ per Month}$</td>
</tr>
</tbody>
</table>

Now if we consider the deterioration rate as constant i.e. $'\beta = 0'$ and $\theta(t) = \alpha$, then the instantaneous inventory level $Q(t)$ at any time $t$ during the cycle time $t$ is given by the following equation

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -(a + bt + csint), \quad 0 \leq t \leq T$$

*... (8.B.20)*

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Where $Q(0) = Q_0$ and $Q(T) = 0$

Solving equation (8.B.20), we get,

$$Q(t) = -a t - \frac{b t^2}{2} + a T + \frac{b T^2}{2} + \frac{a t^3}{3} + \frac{b t^4}{8} - \frac{a T^3}{6} - \frac{b T^4}{4} + c \alpha T - c \alpha Cost - c \alpha Cost - c \alpha Cost + c \alpha Cost + c \alpha Cost + c \alpha Cost$$

$$+ \frac{c t^2 \alpha CosT}{2} - \frac{c T^2 \alpha CosT}{2} - c t \alpha Sint + c T \alpha SinT$$

$$\cdots (8.B.21)$$

Using the initial value and neglecting the higher powers of $\alpha$, we have

$$Q_0 = c + a T + \frac{b T^2}{2} - c \alpha + \frac{a T^3}{6} + \frac{b T^4}{4} - c \alpha Cost + c \alpha Cost$$

$$- \frac{c T^2 \alpha CosT}{2} + c T \alpha SinT$$

$$\cdots (8.B.22)$$

Total demand during the cycle period $[0, T]$ is given by

$$\int_0^T D(t)dt = c + a T + \frac{b T^2}{2} - c \alpha Cost$$

$$\cdots (8.B.23)$$

Number of deteriorating Items = (Initial Inventory at time $t = 0$) - (Total demand during the time period $[0, T]$)

$$= Q_0 - \int_0^T D(t)dt \quad \text{[Using (8.B.22) and (8.B.23)]}$$

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\[
= -c \alpha + \frac{a T^3 \alpha}{6} + \frac{b T^4 \alpha}{8} + C \alpha \cos T + 2 c \cos T - \frac{c T^2 \alpha \cos T}{2} + c T \alpha \sin T
\]

\ldots \text{(8.B.24)}

Therefore the deteriorating cost is given by \( (V_p)(Q_0 - \int_0^T D(t) \, dt) \) [Using (8.B.24)]

\[
V_p \left( -c \alpha + \frac{a T^3 \alpha}{6} + \frac{b T^4 \alpha}{8} + C \alpha \cos T + 2 c \cos T - \frac{c T^2 \alpha \cos T}{2} + c T \alpha \sin T \right)
\]

\ldots \text{(8.B.26)}

Total holding cost of the items for the period \([0, T]\) is given by

\[
= C_h \times \int_0^T Q(t) \, dt,
\]

[Using (8.B.21)]

\[
C_h \left\{ \frac{1}{120} \left( 60 a T^2 + 40 b T^3 + 10 a T^4 \alpha + 8 b T^5 \alpha - 120 c T \cos T + 240 c T \alpha \cos T - 40 c T^3 \alpha \cos T + 120 c \sin T - 240 c \alpha \sin T + 120 c T^2 \alpha \sin T \right) \right\}
\]

\ldots \text{(8.B.26)}

Interest earned in the cycle period \([0, T]\) is given by

\[
= U_p \times I_e \times \int_0^T t \, D(t) \, dt
\]

\[
= \left( I_e \right) \left( U_p \right) \left\{ \frac{1}{6} T^2 (3 a + 2 b T) - c T \cos T + c \sin T \right\}
\]

\ldots \text{(8.B.27)}

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Now the above model can be classified into three different cases viz. \( T > t_1; T < t_1; T = t_1. \)

**Case I 'When \( T > t_1 \)**

Interest payable during the time \([t_1, T]\) is given by,

\[
= U_p \times I_p \times \int_{t_1}^{T} Q(t) \, dt \\
= l_p \times U_p \times \left( \frac{1}{120} \right) \left[ \{ 60 \, a \, T^2 + 40 \, b \, T^3 + 10 \, a \, T^4 \, \alpha + 8 \, b \, T^5 \, \alpha - 120 \, c \, T \, \cos{T} + 240 \, c \, T \, \alpha \, \cos{T} - 40 \, c \, T^3 \, \alpha \, \cos{T} + 120 \, c \, \sin{T} - 240 \, c \, \alpha \, \sin{T} + 120 \, c \, T^2 \, \alpha \, \sin{T} \} + \{-120 \, a \, T \, t_1 - 60 \, a \, T^2 \, t_1 + 60 \, a \, t_1 \, t_1^2 + 20 \, b \, t_1^3 - 20 \, a \, T^3 \, t_1 + 15 \, b \, T^4 \, t_2 + 20 \, a \, T \, t_1^2 \, t_1 + 10 \, b \, T^2 \, t_1^3 \, \alpha - 10 \, a \, t_1^4 \, \alpha - c \, b \, t_1^5 \, \alpha + 120 \, c \, t_1 \, \cos{T} - 120 \, c \, t_1 \, \alpha \, \cos{T} + 60 \, c \, T^2 \, t_1 \, \alpha \, \cos{T} - 20 \, c \, t_1^3 \, \alpha \, \cos{T} - 120 \, c t_1 \, \alpha \, \cos{T} - 120 \, c \, T \, t_1 \, \alpha \, \sin{T} - 120 \, c \, \sin{T} t_1 + 240 \, c \, \alpha \, \sin{T} t_1 \} \right] \]

\[ \cdots (8.8.28) \]

The total cost per unit time is \( = \) \[ \text{[Replenishment cost + Inventory Holding Cost + Deterioration Cost + Interest payable during Delay period – Interest earned during the cycle]} \]

The total average cost per unit time is \( = \frac{1}{T} \times \) \[ \text{[Replenishment cost + Inventory Holding Cost + Deterioration Cost + Interest payable during Delay period – Interest earned during the cycle]} \]

\[ \cdots (8.8.29) \]
By using the equations (8.B.25), (8.B.26), (8.B.27), and (8.B.28) in equation (8.B1.29) and expanding the values of SinT, CosT, SinT1, Cost1 and simplifying, we have

\[
TAC_{B1} = \frac{A}{T} + \frac{a c h T}{2} + \frac{(b + c) c h T^2}{3} - \frac{a(I_p - I_p) T U_p}{2} - \frac{(b + c) I_p T^2 U_p}{3} + \frac{(b + c) I_p T^2 U_p}{2} - \frac{a I_p T U_p}{2} + \frac{a I_p T^2 U_p}{2 T} + \frac{(b + c) I_p T^2 U_p}{3} - \frac{a I_p t_1 T U_p}{2} - \frac{a I_p T^2 U_p}{2 T} + \frac{(b + c) I_p T^2 U_p}{3} - \frac{a C h T^3 \alpha}{12} + \frac{b C h T^4 \alpha}{15} + \frac{a I_p T^3 U_p \alpha}{12} + \frac{b I_p T^4 U_p \alpha}{6} - \frac{a I_p T^2 U_p \alpha}{6} - \frac{b I_p T^3 U_p \alpha}{8} - \frac{c I_p T^3 U_p \alpha}{12} + \frac{a I_p t_1^3 U_p \alpha}{6} + \frac{(b + c) a I_p T t_1^3 U_p \alpha}{12} + \frac{a I_p t_1^4 U_p \alpha}{12 T} - \frac{b I_p t_1^5 U_p \alpha}{40 T} + \frac{a T^2 V_p \alpha}{6} + \frac{b T^3 V_p \alpha}{8} + \frac{c T^3 V_p \alpha}{12}
\]

\( \ldots \) (8.B1.30)

To find the minimum value of total average cost \( TAC_{B1} \), so we have

\[
\frac{d(TAC_{B1})}{dT} = 0
\]
\[ -\frac{A}{T^2} + \frac{a \, C_h}{2} + \frac{2(b + c) C_h \, T}{3} - \frac{a \, (l_e - l_p) \, U_p}{2} - \frac{2(b + c) l_e \, U_p}{3} + \frac{2(b + c) l_p \, T \, U_p}{3} + \frac{(b + c) l_p \, t_1^3 \, U_p}{6 \, T^2} - \frac{(b + c) l_p \, t_1 \, U_p}{2} - \frac{a \, l_p \, t_1^2 \, U_p}{2 \, T^2} + \frac{a \, C_h \, T^2 \, \alpha}{4} + \frac{4 \, b \, C_h \, T^3 \, \alpha}{15} + \frac{a \, l_p \, T^2 \, U_p \, \alpha}{4} + \frac{4 \, b \, l_p \, T^3 \, U_p \, \alpha}{15} - \frac{a \, l_p \, T \, t_1 \, U_p \, \alpha}{3} - \frac{3 \, b \, l_p \, T^2 \, t_1 \, U_p \, \alpha}{8} + \frac{c \, l_p \, T^2 \, t_1 \, U_p \, \alpha}{12} + \frac{(b + c) a \, l_p \, t_1^3 \, U_p \, \alpha}{12 \, T^2} + \frac{a \, l_p \, t_1^4 \, U_p \, \alpha}{4} + \frac{a \, T \, V_p \, \alpha}{3} + \frac{3 \, b \, T^2 \, V_p \, \alpha}{8} + \frac{c \, T^2 \, V_p \, \alpha}{4} = 0 \]

\[ \text{By solving the equation (8.B1.31) by Newton Rapson Method, we can have the value of } T. \text{ Then 'T' will be optimal i.e. } T^*, \text{ if the following condition (8.B1.32) will be satisfied with } T=T^*. \]

\[ \frac{d^2(TAC_{B1})}{dT^2} > 0 \]

\[ -\frac{2A}{T^3} + \frac{2(b + c) C_h}{3} - \frac{2(b + c) l_e \, U_p}{3} + \frac{2(b + c) l_p \, U_p}{3} + \frac{a \, l_p \, t_1^2 \, U_p}{T^3} - \frac{(b + c) l_p \, t_1^3 \, U_p}{3 \, T^3} - \frac{(b + c) l_p \, t_1 \, U_p}{2} - \frac{a \, C_h \, T \, \alpha}{2} + \frac{4 \, b \, C_h \, T^2 \, \alpha}{5} + \frac{a \, l_p \, T \, U_p \, \alpha}{2} + \frac{4 \, b \, l_p \, T^2 \, U_p \, \alpha}{5} - \frac{a \, l_p \, t_1 \, U_p \, \alpha}{3} - \frac{3 \, b \, l_p \, T \, t_1 \, U_p \, \alpha}{4} - \frac{c \, l_p \, T \, t_1 \, U_p \, \alpha}{2} - \frac{a \, l_p \, t_1^4 \, U_p \, \alpha}{6 \, T^3} - \frac{b \, l_p \, t_1^5 \, U_p \, \alpha}{20 \, T^3} + \frac{a \, V_p \, \alpha}{3} + \frac{3 \, b \, T \, V_p \, \alpha}{4} + \frac{c \, T \, V_p \, \alpha}{2} > 0 \]

\[ \text{... (8.B1.31)} \]
So if $T$ from (8.B1.31) satisfy the condition (8.B1.32) then the optimal value will be $T = T^*$ and thus the minimum value of $TAC_{B_1}$ can be obtained from the equation (8.B1.30).

**Case II** 'When $T < t_1$'

In this case the customer earns interest on the sales revenue upto the permissible delay period and no interest is payable during the period for the item kept in stock.

So interest earned for the permissible delay period $[T, t_1]$ is given by

$$
= U_p \times I_e \times (t_1 - T) \int_0^T d(t) \, dt
$$

$$
= U_p \times I_e \times (t_1 - T) \left( c + aT + \frac{bT^2}{2} - c\cos T \right)
$$

... (8.B2.33)

So in this case the total cost per unit time is $=$ [Replenishment cost + Inventory Holding Cost + Deterioration Cost – Interest payable during Delay period – Interest earned during the cycle]

The total average cost per unit time is $=$ $\frac{1}{T} \times$ [Replenishment cost + Inventory Holding Cost + Deterioration Cost + Interest payable during Delay period – Interest earned during the cycle]

... (8.B2.34)
By using the equations (8.B.25), (8.B.26), (8.B.27) and (8.B.23) in equation (8.B.23) and expanding the values of \( \sin T \), \( \cos T \), \( \sin t_1 \), \( \cos t_1 \) and simplifying, we have

\[
TAC_{B2} = \frac{A}{T} + \frac{a C_h T}{2} + \frac{(b + c) C_h T^2}{3} - \frac{a L_e (L_e - L_p) T U_p}{2} + \frac{(b + c)L_e T^2 U_p}{3} - \frac{(b + c)L_p T^2 U_p}{3} - \frac{a L_e L_1 U_p}{2} + \frac{a L_p L_1 U_p}{2} + \frac{(b + c)L_p T L_1 U_p}{2}
\]

\[
- \frac{(b + c)L_e T L_1 U_p}{2} - \frac{a L_p L_1^2 U_p}{2} - \frac{(b + c)L_p L_1^3 U_p}{6} + \frac{a C_h T^3 \alpha}{12}
\]

\[
+ \frac{b C_h T^4 \alpha}{15} - \frac{a L_p T^3 U_p \alpha}{12} - \frac{b L_p T^4 U_p \alpha}{15} + \frac{a L_p T^2 L_1 U_p \alpha}{6}
\]

\[
+ \frac{b L_p T^3 L_1 U_p \alpha}{8} + \frac{c L_p T^3 L_1 U_p \alpha}{12} - \frac{a L_p L_1^3 U_p \alpha}{6}
\]

\[
- \frac{(b + c)a L_p T L_1^3 U_p \alpha}{12} + \frac{a L_p L_1^4 U_p \alpha}{12 T} + \frac{b L_p L_1^5 U_p \alpha}{40 T} + \frac{a T^2 V_p \alpha}{6}
\]

\[
+ \frac{b T^3 V_p \alpha}{8} + \frac{c T^3 V_p \alpha}{12}
\]

\[\cdots (8.B.23)\]

To find the minimum value of total average cost \( TAC_{B2} \), so we have

\[
\frac{d(TAC_{B2})}{dT} = 0
\]
\[
\Rightarrow - \frac{A}{T^2} + \frac{a C_h}{2} + \frac{2(b + c) C_h T}{3} + \frac{a (2 l_e - l_p) U_p}{2} - \frac{2(b + c) l_e T U_p}{3} \\
- \frac{2(b + c) l_p T U_p}{3} + \frac{(b + c) l_p t_1 U_p}{2} + \frac{a l_p t_1^2 U_p}{2 T^2} \\
+ \frac{(b + c) I_p t_1^3 U_p}{6 T^2} + \frac{a C_h T^2 \alpha}{4} + \frac{4 b C_h T^3 \alpha}{15} - \frac{a I_p T^2 U_p \alpha}{4} \\
- \frac{4 b I_p T^3 U_p \alpha}{15} + \frac{a I_p T t_1 U_p \alpha}{3} + \frac{3 b I_p T^2 t_1 U_p \alpha}{8} \\
+ \frac{c I_p T^2 t_1 U_p \alpha}{4} - \frac{(b + c) a I_p t_1^3 U_p \alpha}{12} - \frac{a I_p t_1^4 U_p \alpha}{12 T^2} \\
- \frac{b I_p t_1^5 U_p \alpha}{40 T^2} + \frac{a T V_p \alpha}{3} + \frac{3 b T^2 V_p \alpha}{8} + \frac{c T^2 V_p \alpha}{4} = 0
\]

\[\ldots (8.B2.36)\]

By solving the equation (8.B2.36) by Newton Rapson Method, we can have the value of T. Then ‘T’ will be optimal i.e. T*, if the following condition (8.A2.37) will be satisfied with T=T*.

\[
\frac{d^2(TAC_{B2})}{dT^2} > 0
\]

\[
\Rightarrow - \frac{A}{T^2} + \frac{2(b + c) C_h}{3} - \frac{2(b + c) l_e U_p}{3} - \frac{2(b + c) l_p U_p}{3} - \frac{a I_p t_1^2 U_p}{T^3} \\
- \frac{(b + c) l_p t_1^3 U_p}{3 T^3} + \frac{a C_h T \alpha}{2} + \frac{4 b C_h T^2 \alpha}{5} - \frac{a I_p T U_p \alpha}{2} \\
- \frac{4 b I_p T^2 U_p \alpha}{5} + \frac{a I_p t_1 U_p \alpha}{3} + \frac{3 b I_p T t_1 U_p \alpha}{4} + \frac{c I_p T t_1 U_p \alpha}{2} \\
+ \frac{a I_p t_1^4 U_p \alpha}{6 T^3} + \frac{b I_p t_1^5 U_p \alpha}{20 T^3} + \frac{a V_p \alpha}{3} + \frac{3 b T V_p \alpha}{4} + \frac{c T V_p \alpha}{2} > 0
\]

\[\ldots (8.B2.37)\]
So if \( T \) from (8.B2.36) satisfy the condition (8.B2.37) then the optimal value will be \( T = T^* \) and thus the minimum value of \( TAC_{B_2} \) can be obtained from the equation (8.B2.35).

**Case III ‘When \( T = t_1 \)’**

For \( T = t_1 \), if we replace the value of \( t_1 \) in any of the equation (8.B1.30) or in (8.B2.35), we will get the required result i.e. \( I_e = I_p = 0 \)

\[
TAC_{B_3} = \frac{A}{t_1} + \frac{a C_h t_1}{2} + \frac{(b + c) C_h t_1^2}{3} + \frac{a C_h t_1^3}{12} \alpha + \frac{b C_h t_1^4}{15} \alpha + \frac{a t_1^2}{6} V_p \alpha \\
+ \frac{b t_1^3}{8} V_p \alpha + \frac{c t_1^3}{12} V_p \alpha
\]

... (8.B3.38)

The result obtained from the equation by putting the values of the parameters used in equation (8.A3.19) will be feasible if the value of \( TAC_{B_3} > 0 \).

Let us consider the example for \( a = 55 \) units per month; \( b = 10 \) units per month; \( c = 5 \) units per month; \( A = Rs \ 25.00 \) per order; \( I_p = 0.15 \) per month; \( I_e = 0.13 \) per month; \( C_h = Rs \ 0.12 \) per month; \( U_p = Rs \ 1.80 \) per month; \( V_p = Rs \ 0.50 \) per month; \( \beta = 0; \alpha = 0.15. \)

**Table 8.B**

<table>
<thead>
<tr>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T &gt; t_1 ) (= 0.25 Month)</td>
<td>( T &lt; t_1 ) (= 2.25 Month)</td>
<td>( T = t_1 ) (= 1.63 Month)</td>
</tr>
<tr>
<td>( \Rightarrow \ T = 1.52 ) Month</td>
<td>( \Rightarrow \ T = 1.87 ) Month</td>
<td>( \Rightarrow \ T = 1.63 ) Month</td>
</tr>
<tr>
<td>( \Rightarrow \ \frac{\partial^2(TAC_{B_1})}{\partial T^2} = 21.77 &gt; 0 )</td>
<td>( \Rightarrow \ \frac{\partial^2(TAC_{B_2})}{\partial T^2} = 24.37 &gt; 0 )</td>
<td>( \Rightarrow \ \frac{\partial^2(TAC_{B_3})}{\partial T^2} = 16.53 &gt; 0 )</td>
</tr>
<tr>
<td>( TAC_{B_1} = Rs \ 23.51 ) per Month</td>
<td>( TAC_{B_2} = Rs \ 9.14 ) per Month</td>
<td>( TAC_{B_3} = Rs \ 25.12 ) per Month</td>
</tr>
</tbody>
</table>

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8.5 Sensitivity Analysis

In this section, we study the sensitivity analysis to examine the effect of changes in the input parameters on the optimal results obtained in the example, case I from the Table 8.A. We first find the optimal values of variables $T$ and $TVCA_1$ by changing (increasing or decreasing) one parameters by 25% and 50% and all other parameters remains unchanged. Then we calculate the percentage change of $T$ and $TVCA_1$ with respect to the other values.

### Table 8.B

<table>
<thead>
<tr>
<th>Parameters</th>
<th>% Change in Parameters</th>
<th>% Change $T$</th>
<th>% Change $TVCA_1$</th>
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<td>-6.41</td>
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</tr>
<tr>
<td>-25</td>
<td>-0.78</td>
<td>-1.32</td>
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<td>-2.32</td>
<td>-2.61</td>
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<table>
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<th></th>
<th>V_{p}</th>
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<td>+50</td>
<td>6.98</td>
<td>-9.25</td>
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<tr>
<td>+25</td>
<td>3.88</td>
<td>-4.75</td>
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<td>-3.10</td>
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<td>-7.75</td>
<td>10.34</td>
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<th></th>
<th>β</th>
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<tr>
<td>+50</td>
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<td>-8.58</td>
<td></td>
</tr>
<tr>
<td>+25</td>
<td>6.98</td>
<td>-4.37</td>
<td></td>
</tr>
<tr>
<td>-25</td>
<td>-3.10</td>
<td>4.65</td>
<td></td>
</tr>
<tr>
<td>-50</td>
<td>-6.98</td>
<td>9.53</td>
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<tr>
<th></th>
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<tr>
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<td>-3.02</td>
<td></td>
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<tr>
<td>+25</td>
<td>2.33</td>
<td>-1.56</td>
<td></td>
</tr>
<tr>
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<td>-1.55</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>-50</td>
<td>-4.65</td>
<td>3.39</td>
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<table>
<thead>
<tr>
<th></th>
<th>t_{t}</th>
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</tr>
</thead>
<tbody>
<tr>
<td>+50</td>
<td>-1.55</td>
<td>6.75</td>
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<tr>
<td>+25</td>
<td>-0.78</td>
<td>3.46</td>
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<td>1.55</td>
<td>-3.63</td>
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<tr>
<td>-50</td>
<td>2.33</td>
<td>-7.43</td>
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</table>

The following are the conclusions made from the table – 8. B given below

- We see that, T* is less sensitive to the changes in ‘c’ and t_{t}. TVC_{A1} * is less sensitive to the changes in ‘b’ and ‘c’.
- T* is slightly sensitive to the changes in ‘b’, ‘U_{p}’ and ‘y’. TVC_{A1} * is slightly sensitive to the changes in ‘U_{p}’ and ‘α’.
- T* is sensitive to the changes in ‘C_{h}’, ‘V_{p}’ and ‘β’. TVC_{A1} * is sensitive to the changes in ‘α’, ‘I_{p}’, ‘C_{h}’, ‘V_{p}’, ‘β’ and ‘t_{t}’.

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• \( T^* \) is highly sensitive to the changes in ‘a’, ‘A’, ‘\( I_p \)’, and ‘\( I_e \)’. \( TVC_{A1}^* \) is highly sensitive to the changes in ‘A’ and ‘\( I_e \)’.

### 8.6 Discussion

**Comparative study of the above two models**

Consider the example for \( a=55 \) units per month; \( b=10 \) units per month; \( A=Rs \ 25.00 \) per order; \( I_p=0.15 \) per month; \( I_e=0.13 \) per month; \( C_h=Rs \ 0.12 \) per month; \( U_p=Rs \ 1.80 \) per month; \( V_p=Rs \ 0.50 \) per month.

<table>
<thead>
<tr>
<th>( t ) (0.25 Month) ( &lt; t_i )</th>
<th>Model with linear deterioration</th>
<th>Model with Constant deterioration</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha=0.15; \beta=0.20 )</td>
<td>( T = 1.28 ) Month</td>
<td>( T = 1.52 ) Month</td>
</tr>
<tr>
<td>( \alpha=0.15; \beta=0 )</td>
<td>( \frac{\partial^2 TVC_{A1}}{\partial r^2} = 33.47 &gt; 0 )</td>
<td>( \frac{\partial^2 TVC_{R1}}{\partial r^2} = 21.77 &gt; 0 )</td>
</tr>
<tr>
<td>( TAC_{A1} = Rs \ 29.49 ) per Month</td>
<td>( TAC_{B1} = Rs \ 23.51 ) per Month</td>
<td></td>
</tr>
</tbody>
</table>

From the above table-8.C, we observe that when the deterioration rate is linearly time dependent, the total average cost is high and the permissible delay time is less. But when the deterioration rate is constant, the total average cost is low and the permissible delay time is high, as compared to the linearly time dependent deterioration rate. This is because, in real life situation, the deterioration rate is not constant, it is dependent on time. So, with the increase of time, the deterioration of the item increases and thus the total average cost of the item increases and hence the total time period of the item decreases.
8.7 Conclusion

In this chapter, some realistic features are considered. These features are likely to be associated with an inventory of consumer goods. The assumption of a periodic time-dependent demand rate and production rate is very realistic in the market. The deterioration rate is also assumed to be a linear time-dependent demand rate. If we equate ‘$\beta = 0$’, i.e. the deterioration rate is constant with all other parameters same. The sensitivity analysis of the solution to changes in the values of different parameters has been discussed.