CHAPTER-6

DYNAMICS OF STRONGLY NONLINEAR WAVES IN GRAVOVISCOUS ASTROCLOUDS

Abstract: A theoretical evolutionary model is developed to analyze the dynamics of strongly nonlinear waves in inhomogeneous gravoviscous complex bipolar astroclouds on the Jeans spatiotemporal scales.† It compositionally consists of warm lighter electrons and ions; and cold massive bipolar dust grains fluids alongside vigorous neutral dynamics in quasi-neutral hydrodynamic equilibrium. Application of the Sagdeev pseudo-potential method reduces the intercoupled structure equations into a pair of intermixed forced Korteweg-de Vries-Burgers (f-KdVB) equations. A numerical illustrative method shows that the electrostatic waves evolve as compressive dispersive shock-like eigenmodes. A unique transition from quasi-monotonic to non-monotonic oscillatory compressive shock-like patterns is found to exist. But, the self-gravitational perturbations grow purely as non-monotonic compressive oscillatory shock-like structures with no such transitory features. It is noted that the referral frame velocity acts as amplitude-reducing agent (stabilizing source) for the electrostatic fluctuations solely. A comparison of our results in the light of varied earlier satellite-based observations and in-situ measurements is offered together with a future astronomical scope.

6.1 INTRODUCTION

The dynamics of nonlinear waves has long been a widely interesting area of research due to their diversified roles played in varied interstellar space and cosmic plasma environs [4-5, 152]. A rich modified variety of such waves develop normally due to the presence of the atypical massive heteropolar charged dust grains in the contact plasma background [5, 152]. In other words, the presence of charged dust grains interestingly conforms the collective waves and instabilities, and also, introduces new saturation patterns of the normal dust-modified waves in the form of diversified instability eigenstructures, such as solitary waves, shocks, double layers, etc [103, 105, 153]. Such eigenpatterns in self-gravity play an important role via a unique source mechanism responsible for various astro-space-cosmic phenomena. It includes particle acceleration to high-energy regime, material transportation and energy-momentum transfer processes in interstellar space, thereby leading to the formation of different bounded astrostructures [64-65, 103, 152].

The evolutionary dynamics of astrospace eigenmodes has previously been investigated theoretically [103, 105, 154] as well as experimentally [155] in diverse plasma systems. Their signatures and dynamical features have also been confirmed by various multispace satellite-based observations [74, 156-158]. In this context, worth mentioning instances are Freja, Polar, FAST, Vela 3, etc. It can be seen that most of the earlier studies have assumed simplified models constituted of electrons, ions, negatively charged massive dust grains exclusively [3-4, 105, 154]. Moreover, the positively charged grains are known to play vital roles in reorganizing the wave-kinetic processes in cometary tails [80], Jupiter’s magnetosphere [80-81], Earth’s mesosphere [78], molecular clouds [159], etc.

Many researchers have extensively studied the wave dynamics in bipolar grainy plasmas both with [86] and without [85, 133, 160] self-gravity in the past. A group of researchers have theoretically investigated the basic features of dust acoustic (DA) shock waves with the help of reductive perturbation method in unmagnetized viscous bipolar grainy plasma with no gravity [133]. Likewise, another group has also studied the strongly nonlinear characteristics of the DA solitary waves with opposite-polarity adiabatic dust grains, nonthermal electrons and ions in the Sagdeev-framework [85]. Later, the excitation of finite-amplitude DA solitary waves has also been studied in bipolar dusty plasma system in the small-wavelength limit with self-gravity [86]. In this direction, the dynamics of neutrals and positively charged grains have never been included simultaneously to the best of our knowledge in the past. It indicates that the evolution of fully nonlinear waves in gravoviscous bipolar complex dust clouds in active neutral gaseous background with all the possible driving factors taken into account still remains as an open problem yet to be well explored.

In this work, after being motivated by the above lacunae, we propose a simplistic theoretical model to investigate the evolutionary dynamics of the strongly nonlinear realistic gravito-electrostatic fluctuations. A modified fluid formalism is constructed to derive a new pair of gravito-electrostatically coupled energy integral equations on the basis of the Sagdeev pseudo-potential approach [53]. The Jeans-normalized coupled governing equations are further reduced to a unique pair of intermixed forced Korteweg-de Vries-Burgers (f-KdVB) equations. It is numerically seen that the fluctuations coevolve as electrostatic compressive dispersive shock-like structures with a unique transition from quasi-monotonic profile to non-monotonic oscillatory compressive shock-like patterns and gravitational non-monotonic compressive dispersive oscillatory shock-
Dynamics of strongly nonlinear waves in gravoviscous astroclouds

like structures. The main implications and applications of our new results in the complex astrophysical context are briefly indicated.

6.2 PHYSICAL AND MATHEMATICAL FORMALISMS

We consider an astrophysical cloud model composed of gravoviscous bipolar multicomponent dust fluidic species amid active neutral gaseous background in a planar geometry (1-D). The basic justification behind the plane-geometry approximation is that the considered model extension (~Jeans length) is much greater than all the characteristic plasma scale lengths. The cloud model is presumed to exist in a global quasi-neutral hydrodynamic equilibrium. The electrons and ions are assumed to be inertialess on the observational Jeans scales of space and time. The cold dust grains constitute adiabatic fluids, with equal polytropic indices, $\gamma_{d+} = \gamma_{d-} = \gamma_{de} = \gamma = \gamma = 3$, where $\gamma = (2+D)D^{-1}=3$ for problem dimension, $D=1$ [85, 161]. The considered model, furthermore, ignores complications, such as turbulence, nonthermal chemical kinetics, plasma-neutral collisions, etc. Finally, the Jeans swindle [51], which is preponderantly useful in assuming the system initially to be in homogenous equilibrium, is also relaxed. This is because plasma fluids in the presence of gravity-induced (mass-dependant) stratification effects are indeed inhomogeneous in nature.

We begin our study by using the continuity, momentum, pressure and coupling electro-gravitational Poisson equations. The electron-ion dynamics with all the conventional notations in dimensional form in planar geometry [2-4] are respectively given as

\begin{equation}
n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right),
\end{equation}

\begin{equation}
n_i = n_{i0} \exp\left(-\frac{e\phi}{T_i}\right).
\end{equation}

The inertial dust dynamics in the same customary symbols [133-134] is described by

\begin{equation}
\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x} (n_j u_j) = 0,
\end{equation}

\begin{equation}
\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = -\frac{q_j}{m_j} \frac{\partial \phi}{\partial x} - \frac{1}{m_j n_j} \frac{\partial p_j}{\partial x} + \nu_j \frac{\partial^2 u_j}{\partial x^2},
\end{equation}

\begin{equation}
\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + \gamma p_j \frac{\partial u_j}{\partial x} = 0.
\end{equation}
Dynamics of strongly nonlinear waves in gravoviscous astroclouds

The closing electro-gravitational Poisson equations for the density-sourced potential distributions in all the conventional notations [133-134] are respectively presented as

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e \left[ n_e - n_j + Z_d.n_{d-} - Z_d.n_{d+} \right],
\]

(6.6)

\[
\frac{\partial^2 \psi}{\partial x^2} = 4\pi G \left[ m_{d-}n_{d-} + m_{d+}n_{d+} + m_{d0}n_{d0} \right].
\]

(6.7)

A strategic combination of equations (6.6)-(6.7) enables us to see the effective nature of the coupled gravito-electrostatic potential [162] given as

\[
\frac{\partial^2 \psi}{\partial x^2} \left[ \psi - \left( \frac{q_{d+}}{m_{d-}} + \frac{q_{d-}}{m_{d+}} \right) \phi \right] = 4\pi \left[ G \left\{ m_{d-}n_{d-} + m_{d+}n_{d+} + m_{d0}n_{d0} \right\} - \left\{ \frac{q_{d+}}{m_{d+}} + \frac{q_{d-}}{m_{d-}} \right\} e \left[ n_e - n_j \right] + Z_d.n_{d-} - Z_d.n_{d+} \right].
\]

(6.8)

Here, \( n_{e0} \) and \( n_{j0} \) are the equilibrium population densities of electrons and ions; respectively. The notations \( n_j, m_j, u_j, p_j \) and \( u_j \) are the population density, mass, flow velocity, adiabatic pressure and coefficient of viscosity of the \( j^{th} \) species; respectively. Here, \( j = d^+ \) for “positive dust grains”, \( d^- \) for “negative dust grains” and \( d^0 \) for “neutral dust grains”. The term, \( q_j = Z_j|e| \), is the grain charge with \( e \) as the electronic charge unit and \( Z_j \) as the charge number. Besides, \( \phi \) and \( \psi \) are respectively the electrostatic and gravitational potentials developed at the cost of local density fields.

The normalized form of equations (6.1)-(6.8) is respectively set out as

\[
N_e = N_{e0} \exp \left( + \Phi \right),
\]

(6.9)

\[
N_i = N_{i0} \exp \left( - \Phi \right),
\]

(6.10)

\[
\frac{\partial N_j}{\partial T} + \frac{\partial}{\partial X} \left( N_j M_j \right) = 0,
\]

(6.11)

\[
\frac{\partial M_j}{\partial T} + M_j \frac{\partial M_j}{\partial X} = - \delta_{d-} \left( \frac{q_j}{e} \right) \frac{\partial \Phi}{\partial X} - 3 \delta_{d-} \left( \frac{T_i}{T_p} \right) N_j \frac{\partial N_j}{\partial X} - \frac{\partial \psi}{\partial X} + \kappa_j \frac{\partial^3 M_j}{\partial X^3},
\]

(6.12)

\[
\frac{\partial P_j}{\partial T} + M_j \frac{\partial P_j}{\partial X} + \gamma P_j \frac{\partial M_j}{\partial X} = 0,
\]

(6.13)

\[
\frac{\partial^2 \Phi}{\partial X^2} = \left[ \frac{e^2}{\rho_0 m_{d+} G} \right] \left[ n_{e0} n_e - n_{i0} n_i + Z_{d-} n_{d-0} N_{d-} - Z_{d+} n_{d+0} N_{d+} \right],
\]

(6.14)

\[
\frac{\partial^2 \psi}{\partial X^2} = \frac{1}{\rho_0} \left[ m_{d-} n_{d-0} N_{d-} + m_{d+} n_{d+0} N_{d+} + m_{d0} n_{d00} N_{d0} \right],
\]

(6.15)
\[
\frac{\partial^2 \theta}{\partial X^2} = \frac{4\pi}{\omega_j^2} \left[ G[m_{d,0}n_{d,0}N_{d,0} + m_{d,0}n_{d,0}N_{d,0} + m_{d,0}n_{d,0}N_{d,0}] - 2 \{n_{e,0}N_e - n_{i,0}N_i + Z_d n_{d,0}N_{d,0}) \right].
\] (6.16)

The parameters \(N_e\), \(N_i\), and \(N_j\) are the normalized population densities of electrons, ions and dust species; normalized by their respective equilibrium values \(n_{e,0}\), \(n_{i,0}\) and \(n_{j,0}\), respectively. The independent variables \(X\) and \(T\) are normalized by the Jeans wavelength \(\lambda_j\) and Jeans time scale \(\omega_j^{-1} = (c_{sa}/\lambda_j)^{-1}\), respectively. The parameter \(M_j\) is the normalized fluid velocity, normalized by the dust acoustic phase speed \(c_{sa} = (T_p/m_{d,0})^{1/2}\), where \(T_p \sim T_i = T_p\) is the plasma temperature (in eV). Further, \(T_p >> T_j\), where \(T_j\) is the temperature (in eV) of the \(j\)th species. Moreover, \(P_j = P_j/P_{j,0} = N_j^\gamma\) denotes the normalized adiabatic pressure, where, \(P_{j,0} = n_j T_j\) is the equilibrium isothermal pressure. The electrostatic potential \(\Phi\) and gravitational potential \(\Psi\) are normalized by the cloud thermal potential \(T_p/e\) and acoustic potential \((c_{sa}^2)\), respectively. Moreover, \(\theta = (\Psi - 2\Phi)\) denotes the normalized effective gravito-electrostatic potential, which gives a measure of competitive strength between \(\Phi\) and \(\Psi\). In other words, it describes the effective gravito-electrostatic force field experienced by the dust particles with unit mass and unit charge in the considered cloud model. Besides, the viscosity coefficient, \(\kappa_j\), is normalized by the Jeans viscosity \(\nu_j = \omega_j \lambda_j^{-2}\) [133]. In addition, the term \(\delta_{d,j} = m_{d,j}/m_j\) represents the mass ratio of the negative to the \(j\)th dust species.

### 6.3 Sagdeev Pseudo-Potential Method

To analyze the strongly nonlinear fluctuations, we apply the Sagdeev pseudo-potential method [52]. Then, we transform all the equations (equations (6.9)-(6.16)) into a time-stationary form by using the Galilean coordinate transformation, \(\xi = X - \mu T\), with \(\mu\) as the reference frame velocity. We now, for simplicity, use two integral functions as, \(f_j(\Phi) = \int_{\Phi_0}^{\Phi_j} N_j d\Phi\), approximating pure electrostatic case with \(\Psi\) treated as a constant; and \(g_j(\Psi) = \int_{\Psi_0}^{\Psi_j} N_j d\Psi\), assuming pure self-gravitational case with \(\Phi\) treated as a constant.
In the electrostatic analysis, a simplifying situation with small but non-zero $\kappa_j$ is considered. We use the boundary conditions as, \( N_e \rightarrow 1, \ N_i \rightarrow 1, \ N_j \rightarrow 1, \ M_j \rightarrow 0, \ \Phi \rightarrow 0, \ \Psi \rightarrow 0, \) and \( \partial \Phi / \partial \xi \rightarrow 0, \) at \( \xi \rightarrow \pm \infty \) in equations \((6.9)-(6.13)\) with the \( O(\kappa_j) \) -retention to get

\[
N_{d+} = \mu (3\alpha_1 \delta_{d-,d+})^{1/2} \left\{ 1 - \left( \frac{Z_{d+} \delta_{d-,d+} \Phi + \Psi - \frac{3}{2} \alpha_1 \delta_{d-,d+}}{\mu^2} \right) \left[ 1 - \frac{3}{2} \alpha_1 \delta_{d-,d+} \mu^{-4} \left( 4Z_{d+} \delta_{d-,d+} \Phi + \Psi \right) \right] \left[ 1 + \frac{3}{2} \alpha_1 \delta_{d-,d+} \mu^{-2} \left( 4 \left( Z_{d+} \delta_{d-,d+} \Phi + \Psi - \frac{3}{2} \alpha_1 \delta_{d-,d+} \right) \mu^{-2} \right) \right] \right\},
\]

\[
N_{d-} = \mu (3\alpha_2)^{1/2} \left\{ 1 + \left( \frac{Z_{d-} \Phi - \Psi + \frac{3}{2} \alpha_2}{\mu^2} \right) \left[ 1 - \frac{3}{2} \alpha_2 \mu^{-4} \left( 4 \left( Z_{d-} \Phi - \Psi + \frac{3}{2} \alpha_2 \right) \mu^{-2} \right) \right] \left[ 1 + \frac{3}{2} \alpha_2 \mu^{-2} \left( 4 \left( Z_{d-} \Phi - \Psi - \frac{3}{2} \alpha_2 \right) \mu^{-2} \right) \right] \right\},
\]

where, \( \alpha_1 = T_{d+}/T_p \) and \( \alpha_2 = T_{d-}/T_p \) represent the temperature ratios of the positive and negative grains to the plasma temperature.

Now, we substitute the derived expressions for \( N_e, \ N_i \) and \( N_j \) from equations \((6.9)-(6.10), \ (6.17) \) and \((6.18) \) in equation \((6.14)\), and then, multiply both sides of equation \((6.14)\) by \( \partial \Phi / \partial \xi \). Thereafter, we integrate it under the boundary conditions as \( N_e \rightarrow 1, \ N_i \rightarrow 1, \ N_j \rightarrow 1, \ M_j \rightarrow 0, \ \Phi \rightarrow 0, \ \Psi \rightarrow 0 \) and \( \partial \Phi / \partial \xi \rightarrow 0 \) at \( \xi \rightarrow \pm \infty \) for local disturbance. The outcome of the exercise is the electrostatic energy integral equation given as

\[
\frac{1}{2} \left( \frac{\partial \Phi}{\partial \xi} \right)^2 + V_e (\Phi, \Psi) = 0,
\]

where, the electrostatic Sagdeev potential, \( V_e (\Phi, \Psi) \) is derived as

\[
V_e (\Phi, \Psi) = \left[ \frac{e^2}{\rho_0 G m_{d+}} \right] \left[ n_{d+} e^{\phi} + n_{d-} e^{-(\phi)} + Z_{d+} n_{d+} f_{d+} (\Phi) - Z_{d-} n_{d-} f_{d-} (\Phi) \right]
\]

73
\[ + \left( \frac{e^2}{\rho_0 G m_d} \right) \left[ n_{e0} + n_{i0} + Z_d n_{d-0} f_{d-0} \left( \Phi \right) \bigg|_{\Phi=0, \Phi_i=0} - Z_d n_{d-0} f_{d-0} \left( \Phi \right) \bigg|_{\Phi=0, \Phi_i=0} \right]. \quad (6.20) \]

The analytical solution of equation (6.19) can be obtained by the analytic integration as

\[ f - \left[ \left( \frac{2e^2}{\rho_0 G m_d} \right) \left[ n_{e0} e^{i\phi} + n_{i0} e^{i\phi} + Z_d n_{d-0} f_{d-0} \left( \Phi \right) - Z_d n_{d-0} f_{d-0} \left( \Phi \right) \right] \right] \]

\[ + \left( \frac{2e^2}{\rho_0 G m_d} \right) \left[ n_{e0} + n_{i0} + Z_d n_{d-0} f_{d-0} \left( \Phi \right) \bigg|_{\Phi=0, \Phi_i=0} - Z_d n_{d-0} f_{d-0} \left( \Phi \right) \bigg|_{\Phi=0, \Phi_i=0} \right] \]

\[ d\Phi = \xi + C_E. \quad (6.21) \]

where, \( C_E \) is a new constant of integration having electrostatic origin.

It is seen that the mathematical shape of equation (6.21) is highly nonlinear and complicated in nature due to the presence of various functionals. So, it is non-integrable and unsolvable analytically, thereby paving the way for numerical techniques. Now, to see the exact structural evolutions, we execute analytical tests to check the existential conditions for the possible nonlinear coherent structures. It is seen that equation (6.20) satisfies the following conditions intended for the electrostatic compressive shock-like structures [99] as

\[ V_E (\Phi, \Psi) = 0, \frac{\partial V_E (\Phi, \Psi)}{\partial \Phi} \neq 0, \text{ at } \Phi = 0, \Psi = 0, \quad (6.22a) \]

\[ \frac{\partial^2 V_E (\Phi, \Psi)}{\partial \Phi^2} < 0, \text{ at } \Phi = 0, \Psi = 0, \quad (6.22b) \]

\[ V_E (\Phi, \Psi) \neq 0, \text{ at } \Phi = \Phi_{max}, \quad (6.22c) \]

\[ V_E (\Phi, \Psi) < 0, \text{ at } 0 < |\Phi| < |\Phi_{max}|. \quad (6.22d) \]

We now differentiate equation (6.14) with respect to \( \xi \) and apply some normal simplistic approximations as \( \alpha << 1, \alpha_z << 1, Z_{d \pm} >> 1, \mu > 1 \) and \( \Phi^1, \Psi^1 \approx 0 \). Our exercise finally results in the following \( f \)-KdVB equation for the electrostatic disturbance given as

\[ \frac{\partial \Phi}{\partial \xi} + A_1 \Phi \frac{\partial \Phi}{\partial \xi} + A_2 \frac{\partial^2 \Phi}{\partial \xi^2} + A_3 \frac{\partial^3 \Phi}{\partial \xi^3} = F_E (\Phi, \Psi), \quad (6.23) \]

where, the involved coefficients, namely, the nonlinear convective coefficient \( (A_1) \), dissipative coefficient \( (A_2) \), dispersive coefficient \( (A_3) \) and self-consistent nonlinear driving force \( (F_E (\Phi, \Psi)) \) are sensitively dependent on the diverse plasma parameters as

\[ A_1 = \left[ n_{e0} - n_{i0} \right] + \left\{ 4(3\alpha_2)^{1/2} Z_{d-}^{3} n_{d-0} \right\} \mu^{-5} - \left\{ 4(3\alpha_1\delta_{d-0})^{1/2} Z_{d-}^{3} n_{d+0} \right\} \mu^{-5} \]
\[
\begin{align*}
&\left[ (n_{e0} + n_{i0}) + Z_{d^0} n_{i0} (3 \alpha_z)^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5} \right\} \right] \\
&+ Z_{d^0} n_{i0} (3 \alpha_z \delta_{d^0})^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right]\right)^{-1}, \\
&\text{(6.24)}
\end{align*}
\]

\[
A_2 = \frac{1}{2} \left[ Z_{d^0} n_{i0} \kappa_{d^0} \right] \mu^{-3} \left\{ 2 - 3 \alpha_z \mu^{-2} \right\} \left( \frac{1}{2} \left[ Z_{d^0} n_{i0} \kappa_{d^0} \right] \mu^{-3} \left\{ 2 - 6 \alpha_z \delta_{d^0} \mu^{-2} \right\} \right.
\]

\[
+ 9 \alpha_z \delta_{d^0} \mu^{-2} \left\{ (n_{e0} + n_{i0}) + Z_{d^0} n_{i0} (3 \alpha_z)^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5} \right\} \right]
\]

\[
+ Z_{d^0} n_{i0} (3 \alpha_z \delta_{d^0})^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \left\} \right)^{-1},
\text{(6.25)}
\]

\[
A_3 = - (\rho_0 Gm_d) e^{-2} \left[ (n_{e0} + n_{i0}) + Z_{d^0} n_{i0} (3 \alpha_z)^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5} \right\} \right]
\]

\[
+ Z_{d^0} n_{i0} (3 \alpha_z \delta_{d^0})^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right\} \right)^{-1},
\text{(6.26)}
\]

\[
A_4 = \left[ Z_{d^0} n_{i0} (3 \alpha_z)^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5} \right\} + Z_{d^0} n_{i0} (3 \alpha_z \delta_{d^0})^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right]
\]

\[
+ Z_{d^0} n_{i0} (3 \alpha_z \delta_{d^0})^{1/2} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right] \right)^{-1},
\text{(6.27)}
\]

\[
\begin{align*}
&\text{\quad}\text{\quad}A_5 = \left[ 4 (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \mu^{-1} + 4 (3 \alpha_z \delta_{d^0})^{1/2} Z_{d^0} n_{i0} \mu^{-1} \right]\left\{ (n_{e0} + n_{i0}) + (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \right\} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5} \right\} \right]
\]

\[
+ \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5}\right\} \right\} \right)^{-1},
\text{(6.28)}
\]

\[
\begin{align*}
&\text{\quad}\text{\quad}A_6 = \left[ 4 (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \mu^{-1} + 4 (3 \alpha_z \delta_{d^0})^{1/2} Z_{d^0} n_{i0} \mu^{-1} \right]\left\{ (n_{e0} + n_{i0}) + (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \right\} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right]
\]

\[
+ \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5}\right\} \right\} \right)^{-1},
\text{(6.29)}
\]

\[
\begin{align*}
&\text{\quad}\text{\quad}A_7 = \left[ -4 (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \mu^{-1} + 4 (3 \alpha_z \delta_{d^0})^{1/2} Z_{d^0} n_{i0} \mu^{-1} \right]\left\{ (n_{e0} + n_{i0}) + (3 \alpha_z)^{1/2} Z_{d^0} n_{i0} \right\} \left\{ (3 \alpha_z)^{1/2} \mu^{-1} + \frac{3}{2} \delta_{d^0} \mu^{-3} + 6 \alpha_z \delta_{d^0} \mu^{-5} \right\} \right]
\]

\[
+ \frac{3}{2} \mu^{-3} + 6 \alpha_z \mu^{-5}\right\} \right\} \right)^{-1},
\text{(6.30)}
\]

\[
F_{\lambda}(\Phi, \Psi) = A_1 \frac{\partial \Psi}{\partial \xi} + A_2 \Phi \frac{\partial \Psi}{\partial \xi} + A_3 \Phi \frac{\partial \Phi}{\partial \xi} + A_4 \Psi \frac{\partial \Phi}{\partial \xi} + A_5 \Psi \frac{\partial \Psi}{\partial \xi}.
\text{(6.31)}
\]
Similarly, for the self-gravitational counterparts, we apply the boundary conditions as $N_e \to 1$, $N_j \to 1$, $N_t \to 1$, $M_j \to 0$, $\Phi \to 0$, $\Psi \to 0$ and $\partial \Psi / \partial \xi \to 0$ at $\xi \to \pm \infty$ and consider the similar $\kappa_j$-behaviors as before. The excise reduces equations (6.11)-(6.13) into

\[ N_{d+} = \mu(3\alpha_2)\frac{1}{2}\left\{ 1 - \left( Z_{d+} \delta_{d-\xi} \Phi + \Psi - \frac{3}{2} \alpha_2 \delta_{d-\xi} \right) \mu^{-2} \right\}^{1/2} \left[ 1 - \frac{3}{2} \alpha_2 \delta_{d-\xi} \mu^{-4} \left\{ 1 - 4 \left( 1 - \frac{3}{2} \alpha_2 \delta_{d-\xi} \right) \mu^{-2} \right\} \right] 
\]

(6.32)

\[ N_{d-} = \mu(3\alpha_2)\frac{1}{2}\left\{ 1 + \left( Z_{d-} \Phi - \Psi + \frac{3}{2} \alpha_2 \right) \mu^{-2} \right\}^{1/2} \left[ 1 - \frac{3}{2} \alpha_2 \mu^{-4} \left\{ 1 - 4 \left( Z_{d-} \Phi - \Psi + \frac{3}{2} \alpha_2 \right) \mu^{-2} \right\} \right] 
\]

(6.33)

\[ N_{d\phi} = \mu(3\alpha_2)\frac{1}{2}\left\{ 1 - \left( \Psi - \frac{3}{2} \alpha_2 \delta_{d-\phi} \right) \mu^{-2} \right\}^{1/2} \left[ 1 - \frac{3}{2} \alpha_2 \delta_{d-\phi} \mu^{-4} \left\{ 1 + 4 \left( \Psi - \frac{3}{2} \alpha_2 \delta_{d-\phi} \right) \mu^{-2} \right\} \right] 
\]

(6.34)

where, $\alpha_3 = T_{d\phi}/T_p$ is the temperature ratio between the neutral grains and plasma.

Now, we replace the derived expression for $N_j$ from equations (6.32)-(6.34) in equation (6.15), and multiply both sides of equation (6.15) by $\partial \Psi / \partial \xi$. We then integrate equation (6.15) with the appropriate boundary conditions, i.e.,$N_{d+} \to 1$, $N_j \to 1$, $N_t \to 1$, $M_j \to 0$, $\Phi \to 0$, $\Psi \to 0$ and $\partial \Psi / \partial \xi \to 0$ at $\xi \to \pm \infty$ for the local disturbances to evolve. This exercise, finally, gives the self-gravitational energy integral equation as
\[
\frac{1}{2} \left( \frac{\partial \Psi}{\partial \rho} \right)^2 + V_G(\Phi, \Psi) = 0,
\]
where, the self-gravitational Sagdeev potential, \( V_G(\Psi) \) is described as
\[
V_G(\Phi, \Psi) = -\frac{1}{\rho_0} \left[ m_d n_d g_d G_1(\Psi) + m_d n_d g_d G_2(\Psi) + m_d n_d g_d G_3(\Psi) \right] + \frac{1}{\rho_0} \left[ m_d n_d g_d G_4(\Psi) \right]_{\Psi = 0, \xi = 0} - \frac{1}{\rho_0} \left[ m_d n_d g_d G_5(\Psi) \right]_{\Psi = 0, \xi = 0} + m_d n_d g_d G_6(\Psi)_{\Psi = 0, \xi = 0}.
\]

The analytical solution of equation (6.35) can be obtained via direct integration as
\[
\left[ -2\rho_0^{-1} \left[ m_d n_d g_d G_1(\Psi) + m_d n_d g_d G_2(\Psi) + m_d n_d g_d G_3(\Psi) \right] + 2\rho_0^{-1} \left[ m_d n_d g_d G_4(\Psi) \right]_{\Psi = 0, \xi = 0} + m_d n_d g_d G_6(\Psi)_{\Psi = 0, \xi = 0} \right] d\Psi = \xi + C_G,
\]
where, \( C_G \) is a new constant of integration having self-gravitational origin.

It is now clear that the mathematical structure of equation (6.37) is also highly complicated and nonlinear in nature, as before in the case of equation (6.21), thereby driving us for numerics. Now, equation (6.36) fulfills the following conditions for existence of self-gravitational compressive shock-like structures [54, 99] as
\[
V_G(\Phi, \Psi) = 0, \quad \frac{\partial V_G(\Phi, \Psi)}{\partial \Psi} \neq 0, \text{ at } \Phi = 0, \Psi = 0,
\]
\[
\frac{\partial^2 V_G(\Phi, \Psi)}{\partial \Psi^2} < 0, \text{ at } \Phi = 0, \Psi = 0,
\]
\[
V_G(\Phi, \Psi) = 0, \text{ at } \Psi = \Psi_{\text{max}},
\]
\[
V_G(\Phi, \Psi) < 0, \text{ at } 0 < |\Psi| < |\Psi_{\text{max}}|.
\]

Moreover, differentiating equation (6.15) with respect to \( \xi \) and using the same analytical approximations already mentioned above with \( \alpha_5 < 1 \), we get
\[
\frac{\partial^3 \Psi}{\partial \xi^3} + B_1 \frac{\partial \Psi}{\partial \xi} + B_2 \left( \frac{\partial^2 \Psi}{\partial \xi^2} \right) + B_3 \left( \frac{\partial^3 \Psi}{\partial \xi^3} \right) = F_G(\Phi, \Psi).
\]
This is the self-gravitational f-KdVB equation governing the considered fluctuations. The various involved coefficients are the nonlinear convective coefficient \( (B_1) \), dissipative coefficient \( (B_2) \), dispersive coefficient \( (B_3) \) and self-consistent nonlinear driving force \( (F_G(\Phi, \Psi)) \) having plasma multiparametric dependencies given as
\[
B_1 = \left[ -4m_d n_d (3\alpha_1 \delta_{d, d^*}) \right]^{1/2} \mu^{-5} + 4m_d n_d (3\alpha_2) \right]^{1/2} \mu^{-5} + 4m_d n_d (3\alpha_3 \delta_{d^*, d}) \right]^{1/2} \mu^{-5}
\]
Dynamics of strongly nonlinear waves in gravoviscous astrophysics

\[
\left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left\{ \left( 3\alpha_2 \delta_{d,-d} \right)^i \mu^i + \frac{5}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \right\} - m_d n_{d0} (3\alpha_2)^2 \left( 3\alpha_2 \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) ]^{-1}, \quad (6.40)
\]

\[
B_2 = \left[ \frac{1}{2} \left( m_d n_{d0} \kappa_{d,-d} \right)^0 \left( -2 - 21 (\alpha_2 \delta_{d,-d}) \right) \mu^{-2} + \frac{1}{2} \left( m_d n_{d0} \kappa_{d,-d} \right)^0 \left( -2 + 3 (\alpha_2 \delta_{d,-d}) \right) \mu^{-2} \right] \left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^{-1} + \frac{5}{2} \mu^3 \right) + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^{-1} \right]^{-1} \right], \quad (6.41)
\]

\[
B_3 = \left[ \left( m_d n_{d0} \delta_{d,-d} \right)^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right] \left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right]^{-1}, \quad (6.42)
\]

\[
B_4 = \left[ \left( m_d n_{d0} \delta_{d,-d} \right)^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right] \left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right]^{-1}, \quad (6.43)
\]

\[
B_5 = \left[ \left( m_d n_{d0} \delta_{d,-d} \right)^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right] \left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 - \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right]^{-1}, \quad (6.44)
\]

\[
B_6 = \left[ \left( m_d n_{d0} \delta_{d,-d} \right)^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right] \left[ m_d n_{d0} (3\alpha_2 \delta_{d,-d}) \right]^0 \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \left( 3\alpha_2 \delta_{d,-d} \right)^0 \mu^4 + \frac{3}{2} \mu^3 + \left( 6\alpha_2 \delta_{d,-d} \right)^0 \mu^5 \right) \right]^{-1}, \quad (6.45)
\]
Dynamics of strongly nonlinear waves in gravoviscous astroclouds

\[ B_2 = 4m_{d}n_{d} \left[ 3(3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} - \frac{3}{2}) Z_{d, \mu} \right] + 4m_{d}n_{d} \left( 3\alpha_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-5} \]

\[ \left[ - \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right)^{1/2} \mu^{-1} + \frac{5}{2} \mu^{-3} + \left( 6\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-5} \right] - m_{d}n_{d} \left( 3\alpha_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \]

\[ + \left( 6\alpha_2 \mu^{-3} \right) - m_{d}n_{d} \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \]

\[ \{ \left[ \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \right] \mu^{-1} \} - \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \]

\[ \quad \{ \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \} \] \[ \quad \{ \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \} \]

\[ \quad \{ \left( 3\alpha_2 \delta_2 \gamma_{d,d} \gamma_{d,d} \right) \mu^{-1} + \frac{3}{2} \mu^{-3} \} \]

\( F_0(\Phi, \Psi) = B_2 \frac{\partial \Phi}{\partial \xi} + B_2 \frac{\partial \Phi}{\partial \xi} + B_2 \frac{\partial \Phi}{\partial \xi} + B_2 \frac{\partial \Phi}{\partial \xi}. \) \[ (6.47) \]

It is now clearly evident that the strongly nonlinear fluctuations satisfy all the analytic conditions (equation (6.22) and equation (6.38)) needed for the compressive shock-like pulse patterns to evolve. It may be noted from equation (6.23) and equation (6.39) that, if the nonlinear convective effects are balanced under the combined action of dispersion and dissipation, then the fluctuations evolve as dispersive shock-like patterns [163]. In contrast, otherwise, the fluctuations propagate as non-dispersive shock-like eigenmodes. The analytical tests show two-fold explicit possibilities for the fluctuations to propagate either as compressive dispersive shock-like or compressive non-dispersive shock-like patterns.

### 6.4 RESULTS AND DISCUSSIONS

The proposed theoretical work is mainly focused to study the evolutionary dynamics of strongly nonlinear gravito-electrostatic waves reinforced in multicomponent gravoviscous dust clouds by using the modified Sagdeev pseudo-potential techniques [53]. With a view to see the exact eigenpatterns, we numerically analyze the developed model dynamics (equations (6.14)-(6.15), equation (6.19) and equation (6.35)) by using the fourth-order Runge-Kutta (RK-IV) method [164] in astrophysical judicious multiparametric conditions [5-7, 152]. The results obtained in the sensible parametric domains are given in figures 6.1-6.3.

It may be noteworthy here that, it is only the supersonic domain of the referral frame (\( \mu > 1 \)) that allows our numerical analysis to run. We take the diverse input parametric values of the dust grain properties relevant in the cold (\( T_d = 10^3 - 10^2 \) eV) interstellar medium [5-7]. It is pertinent to add further that the hydrodynamical approximation (here, on the Jeans scale) is based on vanishingly small mean free path, and hence, small viscosity [165]. Therefore, the numerical integration here deals only
with small viscosity scenarios of the H II clouds (infrared clouds) including heterogeneous cloud complexes [6, 154].

Figure 6.1: Profile of the normalized electrostatic (a) Sagdeev potential \( V_\xi(\Phi, \Psi) \) and (b) physical potential \( \Phi \) for the different \( \mu \)-values. Various lines correspond to (A) \( \mu = 2.90 \) (blue solid line), (B) \( \mu = 2.94 \) (red dashed line) and (C) \( \mu = 2.98 \) (black dotted line), respectively. The different input and initial values used in the analysis are given in the text.

In figure 6.1, we see the profiles of the normalized electrostatic (a) Sagdeev potential \( V_\xi(\Phi, \Psi) \) and (b) physical (real) potential \( \Phi \) on the \( \xi \)-space for the different \( \mu \)-values. Various lines correspond to (A) \( \mu = 2.90 \) (blue solid line), (B) \( \mu = 2.94 \) (red dashed line), and (C) \( \mu = 2.98 \) (black dotted line), respectively. Different input values used are \( (\bar{\xi}_i) = 1.00 \times 10^{-2} \) with \( \Delta \bar{\xi} = 1.00 \times 10^{-2} \), \( (\Phi)_i = 2.00 \times 10^0 \), \( (\Psi)_i = 1.00 \times 10^{-11} \), \( (\Psi')_i = 1.00 \times 10^{-4} \), and \( (\Psi''_\xi)_i = 1.00 \times 10^{-3} \). The other parameters kept fixed are \( n_{\bar{\xi}0} = 5.00 \times 10^3 \text{ m}^{-3} \), \( n_{\bar{\tau}0} = 5.00 \times 10^3 \text{ m}^{-3} \), \( n_{d-0} = 7.00 \times 10^{-1} \text{ m}^{-3} \), \( n_{d+0} = 1.00 \times 10^{-1} \text{ m}^{-3} \), \( n_{d\tau0} = 9.00 \times 10^{-1} \text{ m}^{-3} \), \( Z_{d-} = 1.50 \times 10^2 \), \( Z_{d+} = 1.00 \times 10^2 \), \( m_{d-} = 2.80 \times 10^{-8} \text{ kg} \), \( m_{d+} = 1.00 \times 10^{-8} \text{ kg} \), \( m_{d\tau} = 1.00 \times 10^{-11} \text{ kg} \), \( \alpha_1 = 1.10 \times 10^{-2} \), \( \alpha_2 = 1.20 \times 10^{-2} \), \( \alpha_3 = 1.00 \times 10^{-2} \), \( \kappa_{d-} = 2.00 \times 10^{-2} \), \( \kappa_{d+} = 2.00 \times 10^{-2} \), and \( \kappa_{d\tau} = 1.00 \times 10^{-2} \) [6-7, 164]. It is seen that \( V_\xi(\Phi, \Psi) \) satisfies all the approximate analytic conditions, equation (6.22), mentioned before, with minor deviations, in the context of equation (6.19) for the evolution of compressive shock-like fluctuation structures. The corresponding \( \Phi \) (figure 6.1(b)) evolves as quasi-monotonic compressive dispersive shock-like structure for \( \mu = 2.90 \). The \( \Phi \)-amplitude decreases with increase in \( \mu \), and
Dynamics of strongly nonlinear waves in gravoviscous astroclouds

vice-versa. It is interestingly noted that, when \( \mu \geq 2.94 \), there exists a unique transition from the quasi-monotonic type to non-monotonic oscillatory compressive shock-like patterns at \( \xi = 2.50 \). The physics behind such transition is attributable to the Doppler-shifting mechanism enhancing the resonant mode-mode coupling and anti-resonant mode-mode decoupling mechanisms, producing thereby consonances (crests) and dissonances (troughs) via adiabatic energy exchange processes among the background spectral wave components, respectively.

![Figure 6.2: Same as figure 6.1, but for the self-gravitational wave dynamics.](image)

Figure 6.2 depicts the normalized self-gravitational (a) Sagdeev potential \([ V_0(\Phi, \Psi) ]\) and (b) physical potential \((\Psi)\) under the same conditions as figure 6.1. Here, \( V_0(\Phi, \Psi) \) satisfies all the analytic conditions, equation (6.38), thereby fulfilling the germination of compressive shock-like structures. Analogously, the corresponding \( \Psi \)-fluctuations evolve as non-monotonic compressive oscillatory shock-like structures (figure 6.2(b)). Here, we see that the \( \Psi \)-amplitude increases with increase in \( \mu \), and vice-versa. It may, analogously, be attributable to the relativistic Doppler effects in the supersonic range thereby causing \( \Psi \) to grow with the dust mass as an outcome of enhancement in \( \mu \).

In figure 6.3, we lastly depicts the evolutionary phase diagram (in 3-D) of the effective gravito-electrostatic potential \((\theta)\) mapped as an explicit function of the electrostatic real potential \((\Phi)\) and self-gravitational real potential \((\Psi)\). It simply depicts the reproduced \( \theta \)-evolution in the defined potential phase plane constructed from the above results (figures 6.1(b)-6.2(b)). Different input and initial values used here are the same as figure 6.1, but with \( \mu = 2.90 \) only. Here, we see that \( \theta \) decreases with increase
in $\Phi$, but increases with increase in $\Psi$. This further confirms that the formation of bounded structures is possible if and only if the gravitational attraction is at least comparable with the effective strength of electrostatic repulsion among the diverse dust grains in the astroclouds prevailing in the galaxies.

![Phase diagram of the effective gravito-electrostatic potential ($\psi$) evolving as a function of electrostatic real potential ($\Phi$) and self-gravitational real potential ($\Psi$). Different input and initial values here are the same as figure 6.1, but with $\mu = 2.90$ only.]

Figure 6.3: Phase diagram of the effective gravito-electrostatic potential ($\psi$) evolving as a function of electrostatic real potential ($\Phi$) and self-gravitational real potential ($\Psi$). Different input and initial values here are the same as figure 6.1, but with $\mu = 2.90$ only.

While comparing with the existing closely related works, the analysis presented here deals strategically with the modelled massive viscous bipolar dust clouds in dynamic neutral dusty gaseous background in the modified Sagdeev framework evolving as diverse shock-like patterns. In the formation mechanism of such eigenstructures, the nonlinear steepening effects are attributable to fluid convection; whereas, dissipative effects, to fluid viscosity, as widely seen in the literature [99, 105]. A quantitative glimpse on the basis of existing normal cloud parameters [5, 7] may be drawn as the following. In our analysis, the physical strength of the $\Phi$-fluctuations comes out as $\sim 2 \text{ V}$ for $T_p \sim 10^4 \text{ K}$; while, that of the $\Psi$-fluctuations is $\sim 10^{-10} \text{ J kg}^{-1}$ for $m_{d_\infty} = 10^{-8} \text{ kg}$ and $T_p \sim 10^4 \text{ K}$. The smallness in the strength is a subject to the chosen set of diverse plasma properties considered herein. Our investigation, however, differs from the other reports depicting weakly nonlinear fluctuations with self-gravity [86] and strongly nonlinear analyses without self-gravity [85, 160]. Nevertheless, the obtained findings are quite similar with the Vela-3 observations [74] and in-situ measurements [158].
Dynamics of strongly nonlinear waves in gravoviscous astroclouds

6.5 CONCLUSIONS
In summary, a theoretical model analysis is presented to explore the strongly nonlinear waves supported in inhomogeneous complex gravoviscous astroclouds in the Sagdeev pseudo-potential [53] framework. It reveals the excitation of electrostatic compressive dispersive shock-like structures undergoing a unique transitory behavior from quasi-monotonic to non-monotonic oscillatory compressive shock-like patterns. The self-gravitational and effective gravito-electrostatic fluctuations evolve as non-monotonic compressive oscillatory shock-like patterns. The main concluding remarks from our investigation are highlighted as follows.

1. A theoretical strategic model to study the excitation physics of strongly nonlinear fluctuations in complex viscous astroclouds with active neutral gas dynamics taken into account is methodologically constructed in the Sagdeev-potential framework.

2. The fluctuations are sourced by the atypical redistribution of massive (Newtonian) positively-negatively charged (Columbic) dust grains amid active neutrals (Newtonian) on the relevant astrophysical fluid scales of space and time.

3. The fluctuations investigated here are quite similar with the multispace satellite-based observations reported before [74, 158].

4. Finally, the investigated results, despite some analytic simplifications, can be useful to see diverse wave-instabilities and eigenmode patterns leading to the early phase formation of large-scale bounded structures via re-distributed transfer of cloud mass, momentum and energy in diversified astrospace and cosmic plasma environments.

It is, finally, admitted that the proposed investigation highlights a fully nonlinear wave spectrum excitable merely in a pure (external field-free) gravito-electrostatic cloud fluid. The nonlocal effects stemming in the secular instabilities due to diversified dissipative mechanisms are also ignored. The eigenspectral purity would likely be bewildered resulting in additional spectral plethora [5, 7], if other intrinsically influential factors are considered, such as grain magnetizations, grain distributions, rotational (Coriolis) effects, temperature distribution, collective correlative dynamics, and so forth. Despite the analytic model simplifications, a base test-bed for experimental reliability checking in the domain of practical validity of the proposed shock theory via scale-invariant shock physics in laboratory plasma devices, may be set out. Apart from this, spontaneous scale-invariant triggering processes of astronomical bounded structure formation, via wave-induced material transport mechanisms in sensible microgravity conditions [155], may also be extensively established.