CHAPTER-3

BIMODAL CONJUGATIONAL PAIR f-KdV DYNAMICS IN COMPLEX ASTROCLOUDS

Abstract: A theoretical formalism to see the nonlinear bimodal conjugational eigenmode fluctuations dynamics dictated by a pair of the forced Korteweg-de Vries (f-KdV) equations is constructed in a complex turbo-magnetized star-forming cloud. A standard multi-scale analysis is executed over the cloud dynamics, which, in simplification, yields the unique conjugated pair of the f-KdV equations. We numerically see the structural features of the f-KdV dynamics. Two distinctive classes of eigenmode patterns, electrostatic compressive monotonic aperiodic shock-like patterns and gravitational compressive non-monotonic oscillatory shock-like structures, are excitable. It is specifically shown that the constitutive grain-charge (grain-mass) plays as electrostatic stabilizer (gravitational destabilizer) against the cloud dynamics. Their interesting features are confirmed in a phase-plane analysis too. The relevancy of the findings in the real astro-cosmic scenarios of early phases of structure formation mechanics via the wave-driven fluid-accretive-decretive transport processes is summarily emphasized.

3.1 INTRODUCTION

The existence of a diversified spectrum of various nonlinear waves and collective eigenmodes in star-forming dusty molecular clouds is ubiquitously well known. The excitation of perturbations and waves in such interstellar fluid media, depending on the conjugational strength of fluid nonlinearity and dispersion, saturates into a number of eigenmode structures having real astronomical significance, such as shocks, solitons, vortices, and so forth [7, 10, 42, 95-97]. Besides, the gravito-thermal coupling of the usual Jeans mode (due to constitutive Newtonian grains) and electrostatic mode (due to constitutive Coulombic grains) in astrophysical partially ionized complex plasma fluids reinforces the fluctuations usually to evolve as two distinct wave classes of bimodal significance in the context of fluid matter redistribution and reorganization to the initiation of proto-structure formations via massive grain-centric material accretion processes [6-7, 10, 95-96, 98-99]. These processes are operative in dynamically structuring clouds into stellesimals, subsequently, into stars; and planetsimals into planets


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[6-7, 70, 98]. The dynamics of such bimodal mode conjugacy has been addressed in a number of recent works by different researchers [7, 10, 42, 49, 95-97, 100-101]. They have studied the linear gravitational instability in turbo-magneto active filamentary and sheetlike molecular clouds and found that the magneto-active fluid turbulence introduces a stabilizing influence against the self-gravitational cloud collapse dynamics. In the nonlinear regime, several authors have investigated different nonlinear coherent structures, like nonlinear stationary waves [49] and solitary waves [49, 96] in such molecular clouds. It may be noted that the conjoint complex effects of fluid turbulence, inhomogeneous magnetic field and collective interspecies heterogeneous collisional dynamics have never been previously incorporated concurrently in the nonlinear saturation mechanisms of such conjugational instabilities in astromedia.

In this Chapter, a theoretical model is developed to study the nonlinear eigenstructure formation in a complex turbo-magnetized collisional astrocloud. The cloud fluid perturbation dynamics is procedurally shown as being governed by a bimodal pair forced Korteweg-de Vries (f-KdV) equations derived from the cloud structure equations by using the reductive perturbation techniques (RPTs). All the unavoidable realistic key agencies are judiciously considered. The derived f-KdV system numerically depicts the electrostatic perturbations growing as compressive monotonic aperiodic shock-like patterns and gravitational fluctuations evolving as compressive non-monotonic oscillatory shock-like structures. A detailed phase plane analysis is also presented for eigen-confirmation. The non-trivial significance of our findings in astro-cosmic realm is highlighted together with future scope.

3.2 MODEL AND FORMALISM

A complex multi-fluidic quasi-neutral model configuration of a self-gravitating turbo-magnetized inhomogeneous collisional astrocloud of unbounded spatial extension is considered. The model consists of lighter electrons and ions; and heavier charged dust grains amid partial ionization. The lighter electrons and ions are inertialess and isothermal in nature, and, hence, treated in the Boltzmann fluid framework [4, 10, 42]. The partially ionized heavier dust microspheres (inertial) are morphologically assumed to be identical in geometric shape and size. These dust grains are electrically charged by the background plasma contact-electrification under dynamic constant bombardment with the variable thermal currents of the constitutive electrons and ions, or by photoelectric effects [7, 98].

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The global electrical quasi-neutral equilibrium condition is assumed to pre-exist. It is further anticipated that the complex fluid environment is embedded in a uniform magnetic field, $\vec{B} = B\hat{z}$, acting along the positive $z$-direction with the cloud centre as the referral origin. The constitutive distinct turbulent fluids composing the resultant cloud are modelled with the help of the Larson semi-empirical logatropic equation of state derived observationally by the non-thermal spectroscopic line width-size relationship in real astronomical situations [37, 49, 70, 100, 102]. This spectro-strategic nonlinear barotropic expression is conjointly applied for the net pressure correlating both the isothermal pressure (linear on density) and turbulence pressure (nonlinear on density). Here, the source of fluid turbulence lies in the nonlinear fluid advective mechanisms initiated by the velocity field fluctuations over multiple irregular scales of space and time. The conceived situations are normally realizable in most of the interstellar dust molecular clouds, fragmented cloudlets and cloud complexes [6-7, 70].

The dynamics of the considered astrocloud in a coordination space $(x, t)$ is described by the continuity, momentum and closing electro-gravitational Poisson potential distribution equations in a closed unnormalized form in usual notations [10, 49] given respectively as

\[
\begin{align*}
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{u}_e) &= -\nu_{oe} (n_e - n_o), \\
\frac{\partial \phi}{\partial t} &= -\nabla p_e - e n_e \nabla \phi - m_e n_e \nu_{oe} \vec{u}_e, \\
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{u}_i) &= -\nu_{ie} (n_i - n_o), \\
\frac{\partial \psi}{\partial t} &= -\nabla p_i + e n_i \nabla \phi - m_i n_i \nu_{ie} \vec{u}_i, \\
\frac{\partial \rho_{de}}{\partial t} + \nabla \cdot (\rho_{de} \vec{u}_{de}) &= 0, \\
m_{de} \frac{\partial}{\partial t} \left( \frac{\partial}{\partial \vec{u}_{de}} \nabla \right) \vec{u}_{de} &= -\nabla p_{de} - q_{de} n_{de} \nabla \phi - m_{de} n_{de} \nabla \psi - m_{de} n_{de} \nu_{oe} (\vec{u}_{de} - \vec{u}_e), \\
m_{de} \frac{\partial}{\partial t} \left( \frac{\partial}{\partial \vec{u}_{de}} \nabla \right) \vec{u}_{de} &= -\nabla p_{de} - m_{de} n_{de} \nabla \psi - m_{de} n_{de} \nu_{ie} (\vec{u}_{de} - \vec{u}_i), \\
\nabla^2 \phi &= -4\pi \left\{ e(n_i - n_e) - q_{de} n_{de} \right\}, \\
\nabla^2 \psi &= 4\pi G \left( m_{de} n_{de} + m_{de} n_{de} \right). 
\end{align*}
\]
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The dust grain-charge fluctuation dynamics is depicted as

\[
\left( \frac{\partial}{\partial t} + \mathbf{u}_d \cdot \nabla \right) q_d = e \left[ v_{\text{ed}} \left( \frac{n_e - n_{e0}}{n_{e0}} \right) - v_{\text{id}} \left( \frac{n_i - n_{i0}}{n_{i0}} \right) \right].
\]  

(3.11)

The dynamics of the embedded magnetic field lines in our model is governed by the magnetic induction equation with all the customary notations [91] given as

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_a \cdot \nabla \right) \mathbf{B} = \nabla \times \left( \mathbf{v}_a \times \mathbf{B} \right).
\]  

(3.12)

Here, \( n_j, m_j, u_j \) and \( T_j \) are the population density, mass, flow velocity and temperature of the \( j \)-th species, where, \( j = e \) for “electrons”, \( i \) for “ions”, \( dc \) (or, \( dc \), as in the next Chapters) for “charged dust (or, negatively charged dust)” and \( dn \) for “neutral dust”, respectively. The parameter \( n_{j0} \) depicts the equilibrium population density of the \( j \)-th species. The symbol \( p_j \) represents the total pressure which is composed of three parts: isothermal pressure \( p_j(\text{iso}) = T_j n_j \), turbulence pressure \( p_j(\text{turb}) = T_j n_{j0} \log(p_j/p_{0j}) \) and magnetic pressure \( p_j(\text{mag}) = B^2/(8\pi) \). For the mathematical simplicity, we consider \( T_e \approx T_i = T_p >> T_{dc} \approx T_{dn} = T_d \) and \( m_{dc} = m_{dn} = m_d \). The notations, \( \phi \) and \( \psi \), are the electrostatic and self-gravitational potentials, respectively. Moreover, \( \mathbf{v}_a = (\mathbf{u}_e - \mathbf{u}_i + \mathbf{u}_d) \approx (\mathbf{u}_e - \mathbf{u}_i) \) denotes the mean fluid velocity contributed by the ionic flow (\( \mathbf{u}_i \)) and electronic flow (\( \mathbf{u}_e \)) for the cold dust configuration (\( \mathbf{u}_e, \mathbf{u}_i >> \mathbf{u}_d \)). The term \( G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \) is the universal gravitational constant. Finally, the notations \( v_{\text{ed}}, v_{\text{en}}, v_{\text{nc}}, v_{\text{id}} \) and \( v_{\text{id}} \) describe the collisional momentum transfer frequencies between the different constituent species as indicated by the subscripts [10], respectively.

The normalized form of equations (3.1)-(3.12) in a standardized normalized coordination space \( (X, T) \) is respectively given as

\[
\frac{\partial N_e}{\partial T} + \frac{\partial}{\partial X} \left( N_e M_e \right) = -F_{\text{ed}}(N_e - 1),
\]  

(3.13)

\[
\frac{\partial N_e}{\partial X} + \frac{1}{N_e} \frac{\partial N_e}{\partial X} + \alpha_e B_x \frac{\partial B_x}{\partial X} + N_e \frac{\partial \Phi}{\partial X} + \beta_e F_{\text{ed}} N_e M_e = 0,
\]  

(3.14)

\[
\frac{\partial N_i}{\partial T} + \frac{\partial}{\partial X} \left( N_i M_i \right) = -F_{\text{id}}(N_i - 1),
\]  

(3.15)

\[
\frac{\partial N_i}{\partial X} + \frac{1}{N_i} \frac{\partial N_i}{\partial X} + \alpha_i B_x \frac{\partial B_x}{\partial X} - N_i \frac{\partial \Phi}{\partial X} + \beta_i F_{\text{id}} N_i M_i = 0,
\]  

(3.16)
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\[
\frac{\partial N_{dc}}{\partial T} + \frac{\partial}{\partial X} \left( N_{dc} M_{dc} \right) = 0, \quad (3.17)
\]

\[
\left[ \frac{\partial M_{dc}}{\partial T} + M_{dc} \frac{\partial M_{dc}}{\partial X} \right] = -\left( \frac{T_d}{T_p} \right) \frac{1}{N_{dc}} \frac{\partial N_{dc}}{\partial X} - \left( \frac{T_d}{T_p} \right) \frac{1}{N_{dc}^2} \frac{\partial N_{dc}}{\partial X} - \frac{\partial \Psi}{\partial X} \left( M_{dc} - M_{d0} \right), \quad (3.18)
\]

\[
\frac{\partial N_{dc}}{\partial T} + \frac{\partial}{\partial X} \left( N_{dc} M_{dc} \right) = 0, \quad (3.19)
\]

\[
\left[ \frac{\partial M_{dc}}{\partial T} + M_{dc} \frac{\partial M_{dc}}{\partial X} \right] = -\left( \frac{T_d}{T_p} \right) \frac{1}{N_{dc}} \frac{\partial N_{dc}}{\partial X} - \alpha_3 \frac{1}{N_{dc}} B_0 \frac{\partial B_x}{\partial X}
\]

\[-\left( \frac{q_{de}}{e} \right) Q_\alpha \frac{\partial \Phi}{\partial X} - \frac{\partial \Psi}{\partial X} - F_{\alpha \alpha} \left( M_{dc} - M_{d0} \right), \quad (3.20)\]

\[
\frac{\partial^2 \Phi}{\partial X^2} = -C_p \left[ \epsilon \left( n_{de} N_i - n_{de} N_e \right) - n_{de} q_{de} Q_x N_{dc} \right], \quad (3.21)\]

\[
\frac{\partial^2 \Psi}{\partial X^2} = \frac{1}{\rho_0} m_j \left( n_{de} N_{dc} + n_{de} N_{d0} \right). \quad (3.22)\]

The normalized grain-charging equation in generic notations [7] is cast as

\[
\left( \frac{\partial}{\partial T} + M_{dc} \frac{\partial}{\partial X} \right) Q_x = \frac{e}{q_{de}} \left[ \frac{n_{de}}{n_{de} + n_{d0}} F_x(N_e - 1) \right] \left[ \frac{n_{de}}{n_{de} + n_{d0}} F_x(N_i - 1) \right]. \quad (3.23)\]

Finally, the normalized magnetic induction equation [91], depicting the spatiotemporal evolutionary dynamics of the magnetic field lines frozen together with the ultra-weakly magnetized turbulent cloud fluid, is given as

\[
\frac{\partial B_x}{\partial T} = -\frac{\partial}{\partial X} \left( M_{dc} B_x \right). \quad (3.24)\]

Here, \( X \) and \( T \) are the space and time coordinates normalized by the Jeans scale length \( \lambda_\odot = c_\odot / w_\odot \) and Jeans time \( w_\odot^{-1} = (4\pi \rho_0 G)^{-1/2} \), respectively. The symbols \( N_j \) and \( M_j \) denote the population density and fluid velocity of the \( j \)th species, normalized by the respective equilibrium population density \( (n_{j0}) \) and dust acoustic phase speed \( (c_\odot = (T_p / m_j)^{1/2}) \); where, the subscript \( j = \) “e” for electrons, “i” for ions, “d” for neutral dust grains and “dc” for charged dust grains; respectively. The new parameters, \( \alpha_1 = B_0^2 / 4\pi \rho_0 T_p \), \( \alpha_2 = B_0^2 / 4\pi \rho_0 T_p \) and \( \alpha_3 = B_0^2 / 4\pi \rho_0 T_p \) represent the relative strengths of the interspecies magneto-thermal interactions, respectively. Further, \( \beta_1 = m_e / m_d \) and \( \beta_2 = m_i / m_d \) symbolize the mass ratios of electrons and ions relative to dust mass, respectively. Besides, the notation \( C_p = \epsilon/(\rho_0 m_d G) \) defines a new coupling parameter.
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arising due to the long-range electro-gravitational interactions. Here, $\rho_0 = m_e (n_{da} + n_{dc})$ is the net equilibrium material density of the astrocloud and $G = 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ is the universal gravitational constant. Moreover, $Q_d$ is the grain-charge normalized by the equilibrium grain charge $q_{de} = Z_{de} e$, where $Z_{de}$ is the equilibrium dust surface charge number and $e$ is the electronic charge unit. The parameters, $\Phi$ and $\Psi$ denote the electrostatic and gravitational potentials, normalized by the plasma thermal potential $(T_p/e)$ and square of the dust acoustic phase speed ($c_{sa}^2$), respectively. Moreover, $F_{eda}$, $F_{eas}$, $F_{me}$, $F_{mdc}$ and $F_{id}$ are the Jeans-normalized interspecies collisional momentum transfer frequencies [10]. Likewise, the magnetic field $B_N$ is normalized by the equilibrium magnetic field value ($B_0$). Finally, the mean flow velocity, $M_s \approx (M_c - M_i)$, is normalized by the dust acoustic phase speed ($c_{sa} = (T_p/m_p)^{1/2}$).

### 3.3 DERIVATION OF THE PAIR $f$-KdV EQUATIONS

We study the weakly nonlinear bimodal fluctuation dynamics with the help of a derived conjugational pair of forced Korteweg-de Vries ($f$-KdV) equations. As a first step, we apply a multiscale analysis [2, 95, 97, 103] in a new stretched coordinate system defined by $\xi = \varepsilon^{1/2} (x - \mu T)$ for space and $\tau = \varepsilon^{1/2} \tau$ for time. Here, $\varepsilon$ is a smallness order parameter and $\mu$ is the $c_{sa}$-normalized referral frame velocity [96-97, 101]. The dependant physical variables of physical relevance are now nonlinearly expanded in the $\varepsilon$-powers as

$$Z = Z_0 + \sum_{k=1}^{\infty} \varepsilon^k Z_k, \quad (3.25)$$

$$Z = [N_e \ N_i \ N_{dc} \ M_{dc} \ M_{dc} \ \Phi \ \Psi]^T, \quad (3.26)$$

$$Z_0 = [1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0]^T, \quad (3.27)$$

$$Z_k = [N_{ek} \ N_{ik} \ N_{dek} \ M_{dek} \ M_{dek} \ \Phi_k \ \Psi_k]^T. \quad (3.28)$$

A systematic order-by-order analysis of equations (3.13)-(3.24), after equation (3.25) adopted by a decompositional simplification, yields the $f$-KdV equations given respectively as

$$\frac{\partial \Phi_1}{\partial \tau} + A_1 \Phi_1 \frac{\partial \Phi_1}{\partial \xi} + A_2 \frac{\partial^3 \Phi_1}{\partial \xi^3} = F_k (\Phi, \Psi), \quad (3.29)$$
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\[
\frac{\partial \Psi}{\partial \tau} + B_1 \frac{\partial \Psi}{\partial \xi} + B_2 \frac{\partial^3 \Psi}{\partial \xi^3} = F_o(\Phi, \Psi),
\]

(3.30)

where,

\[
F_e(\Phi, \Psi') = A_1 \frac{\partial \Phi}{\partial \xi} + A_1 \frac{\partial \Phi}{\partial \xi} + A_2 \frac{\partial \Psi'}{\partial \xi},
\]

(3.31)

\[
F_o(\Phi, \Psi') = B_1 \frac{\partial \Phi}{\partial \xi} + B_2 \frac{\partial \Phi}{\partial \xi} + B_3 \frac{\partial \Phi}{\partial \xi} + B_4 \frac{\partial \Psi'}{\partial \tau}.
\]

(3.32)

The involved coefficients, such as the nonlinear convective coefficients (A1 and B1); dispersion coefficients (A2 and B2) and the self-consistent nonlinear driving forces (F_e(\Phi, \Psi') and F_o(\Phi, \Psi')) are clearly dependent on plasma parameters respectively given as

\[
A_1 = \mu 2 (2 - \alpha_i) \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \left[ \frac{2e (2 - \alpha_i) \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \right]^{-1} \left\{ -n_{i0} + n_{e0} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] \right\} \right]^{-1},
\]

(3.33)

\[
A_2 = -\mu \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \left[ \frac{2e (2 - \alpha_i) \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \right]^{-1} \left\{ -n_{i0} + n_{e0} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] \right\} \right]^{-1},
\]

(3.34)

\[
A_3 = q_{ed} n_{ed} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \left[ \frac{2e (2 - \alpha_i) \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \right]^{-1} \left\{ -n_{i0} + n_{e0} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] \right\} \right]^{-1},
\]

(3.35)

\[
A_4 = q_{ed} n_{ed} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \left[ \frac{2e (2 - \alpha_i) \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] + \alpha_i \right]^{-1} \left\{ -n_{i0} + n_{e0} \left[ \frac{F_{ed}}{F_{ed}} \right] \left[ \frac{n_{i0}}{n_{e0}} \right] \right\} \right]^{-1},
\]

(3.36)
\[ A_t = q_d n_d \mu \left( \frac{T_d}{T_p} \right)^{2 \mu} \left( 2 - \alpha_t \right) \left( \frac{F_d}{F_d} \right)^{n_d} \alpha_t \left( \frac{F_d}{F_d} \right)^{n_d} + q_d n_d \left( \frac{n_d}{n_d} \right) \left( \frac{F_d}{F_d} \right)^{n_d}, \]  
(3.37)

\[ B_t = \left[ -2 \rho \mu (m_d n_d) \right] \left[ 2 \left( \frac{T_d}{T_p} \right) - \mu \right] \left[ 1 - \left( \frac{n_d}{n_d} \right) \right], \]  
(3.38)

\[ B_2 = \left[ \rho \mu (m_d n_d) \right] \left[ 2 \left( \frac{T_d}{T_p} \right) - \mu \right] \left[ 1 + \left( \frac{n_d}{n_d} \right) \right], \]  
(3.39)

\[ B_3 = 2 \mu \left[ 2 \left( \frac{T_d}{T_p} \right) - \mu \right]^{2 \mu} \left[ 1 + \left( \frac{n_d}{n_d} \right) \right]^{2 \alpha} \left[ 2 - \alpha_t \right] \left( \frac{F_d}{F_d} \right)^{n_d} \left( \frac{F_d}{F_d} \right)^{n_d} \left( 1 - \left( \frac{F_d}{F_d} \right)^{n_d} \right), \]  
(3.40)

\[ B_4 = -2 \mu \left[ 2 \left( \frac{T_d}{T_p} \right) - \mu \right]^{2 \mu} \left[ 1 - \left( \frac{n_d}{n_d} \right) \right]^{2 \alpha} \left[ 2 - \alpha_t \right] \left( \frac{F_d}{F_d} \right)^{n_d} \left( \frac{F_d}{F_d} \right)^{n_d} \left( 1 - \left( \frac{F_d}{F_d} \right)^{n_d} \right), \]  
(3.41)

\[ B_5 = 2 \mu \left[ 2 \left( \frac{T_d}{T_p} \right) - \mu \right]^{2 \mu} \left[ 1 - \left( \frac{n_d}{n_d} \right) \right]^{2 \alpha} \left[ 2 - \alpha_t \right] \left( \frac{F_d}{F_d} \right)^{n_d} \left( \frac{F_d}{F_d} \right)^{n_d} \left( 1 - \left( \frac{F_d}{F_d} \right)^{n_d} \right), \]  
(3.42)

\[ B_6 = -1. \]  
(3.43)

The steady-state fluctuations can simplistically be seen by transforming equations (3.29)-(3.30) into a time-stationary form given below in a commoving Galilean frame \( \rho = (\xi - \tau) \) as

\[ -\frac{\partial \Phi}{\partial \rho} + A_t \Phi \frac{\partial \Phi}{\partial \rho} + A_2 \frac{\partial \Phi}{\partial \rho} = A_t \Phi \frac{\partial \Psi}{\partial \rho} + A_2 \Psi \frac{\partial \Phi}{\partial \rho} + A_2 \Psi \frac{\partial \Psi}{\partial \rho}, \]  
(3.44)

\[ -\frac{\partial \Psi}{\partial \rho} + B_t \Psi \frac{\partial \Psi}{\partial \rho} + B_2 \frac{\partial \Psi}{\partial \rho} = B_t \Phi \frac{\partial \Phi}{\partial \rho} + B_2 \Psi \frac{\partial \Phi}{\partial \rho} + B_2 \Psi \frac{\partial \Psi}{\partial \rho}. \]  
(3.45)

It is clearly seen that equations (3.44)-(3.45) are analytically non-integrable and unsolvable due to involved nonlinear convective complications. Therefore, a numerical illustrative platform is constructed for analyzing their exact dynamical patterns as in the next section.

3.4 RESULTS AND DISCUSSIONS

The exact fluctuation patterns of the bimodal instability triggered by the long-range periodic gravito-electrostatic interplay (via equations (3.44)-(3.45)) are obtained with the help of a numerical illustrative analysis based on the fourth-order Runge-Kutta (RK-IV) method [94]. The so-obtained numerical results, which portray the diversified interstellar evolutionary wave spectra, are graphically displayed and interpreted in figures (3.1)-(3.4).
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Figure 3.1: Profile of the normalized (a) electrostatic potential ($\Phi_1$) varying with the normalized distance ($\xi$), and (b) potential gradient ($\Phi_{1,\xi}$) over the potential ($\Phi_1$). Various lines refer to (i) $q_{d0} = 100e$ (blue solid line), (ii) $q_{d0} = 150e$ (red dashed line) and (iii) $q_{d0} = 200e$ (black dotted line), respectively. The input details are given in the text.

In figure 3.1, we show the profile patterns of the normalized perturbed (a) electrostatic potential ($\Phi_1$) varying with the normalized distance ($\xi$), and (b) potential gradient ($\Phi_{1,\xi}$) over the potential ($\Phi_1$). Various lines correspond to different equilibrium dust charges defined as (i) $q_{d0} = 100e$ (blue solid line), (ii) $q_{d0} = 150e$ (red dashed line) and (iii) $q_{d0} = 200e$ (black dotted line), respectively. The initial values used in the analysis are $(\phi_0)_0 = 10^{-2}$, $(\phi_0)_1 = 10^{-6}$, $(\phi_{1,\xi})_0 = 10^{-3}$, $(\psi_0)_0 = 10^{-3}$, $(\psi_{1,\xi})_0 = 10^{-4}$ and $(\varphi_{1,\xi})_0 = 10^{-1}$. The other input parameters kept fixed are $n_0 = 1.20 \times 10^2$ m$^{-3}$, $n_0' = 4.95 \times 10^2$ m$^{-3}$, $n_{a0} = 2.35 \times 10^6$ m$^{-3}$, $n_{a0} = 4 \times 10^6$ m$^{-3}$, $m_d = 4 \times 10^{11}$ kg, $T_p = 1.00$ eV, $T_e = 1.00 \times 10^1$ eV, $B_0 = 1 \times 10^{-3} \mu T$, $v_{ec}/\omega_p = 0.1$, $v_{id}/\omega_p = 0.1$, $v_{dm}/\omega_p = 0.1$ and $v_{ad}/\omega_p = 0.1$. It is seen that the $\Phi_1$-fluctuations evolve as compressive monotonic aperiodic shock-like patterns with the wave amplitude decreasing with increase in $q_{d0}$, and vice versa (figure 3.1(a)). It is further seen that the dynamical phase trajectories of the $\Phi_1$-evolution appear as irregular non-homoclinic open curves in the phase-plane (figure 3.1(b)). Thus, the dynamical evolution of the dissipative shock-like patterns is numerically and pictorially confirmed by the open set of geometrical curves. The physical mechanism responsible behind is the complex gravito-electrostatic interplay sourcing to gravity-induced electrostatic polarization effects in the composite cloud. The decrement in the electrostatic wave amplitude with increment in the grain-charge is indeed due to the enhanced gravitational interaction (Newtonian)
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antagonistically correlated with the electrostatic counterpart (Coulombic) via the coupled polarization effects as briefed above.

\[
m_d = 3 \times 10^{-11} \text{ kg} \quad \text{(blue solid line)} \quad \text{and} \quad m_d = 5 \times 10^{-11} \text{ kg} \quad \text{(black dotted line)}, \text{ respectively.}
\]

Figure 3.2: Same as figure 3.1, but for the different \( m_d \)-values with a fixed \( q_{d0} \)=100e. Various lines link to (i) \( m_d = 3 \times 10^{-11} \text{ kg} \) (blue solid line), (ii) \( m_d = 4 \times 10^{-11} \text{ kg} \) (red dashed line) and (iii) \( m_d = 5 \times 10^{-11} \text{ kg} \) (black dotted line), respectively. It is seen that the \( \Phi_1 \)-amplitude decreases with enhancement in \( m_d \), and vice versa (figure 3.2(a)). The corresponding phase-plane pictographic exploration is offered for further eigenmode confirmation and validation (figure 3.2(b)). The basic physical mechanism here too, as in figure 3.1, lies in the complex gravity-induced electrostatic polarization effects in the cloud.

In figure 3.3, we portray the same as figure 3.1; but, now for the normalized (a) gravitational potential \( (\Psi_1) \) varying with the normalized distance \( (\zeta) \) and (b) potential gradient \( (\Psi_1') \) over the potential \( (\Psi_1) \). It is seen here that the \( \Psi_1' \)-fluctuations evolve as \textit{compressive non-monotonic oscillatory shock-like structures} with the wave amplitude decreasing with increase in \( q_{d0} \), and vice-versa. It happens physically due to the enhancement in the gravitoelectrostatic interplay sourced by the augmentation in the grain-charge, and consequently, in the grain-mass. A further confirmatory test for the eigenpatterns is provided with the construction of geometrical trajectories actively...
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evolving as a quasi-closed set of phase-plane curves as non-chaotic fixed point attractors (figure 3.3(b)).

![Figure 3.3: Same as figure 3.1, but for normalized (a) gravitational potential ($\Psi_1$) varying with normalized distance ($\zeta$) and (b) potential gradient ($\Psi_1'$) over potential ($\Psi_1$).](image)

Finally, as in figure 3.4, we portray the same as figure 3.2, but for the different $m_d$-values with a fixed $q_{d0} = 100 \, e$. It is found that the $\Psi_1$-amplitude increases with enhancement in $m_d$; and vice versa (figure 3.4(a)). The corresponding phase portraits confirming the $\Psi_1$-fluctuation geometry is presented to clarify the morphodynamics of the eigenpatterns (figure 4(b)). The $\Psi_1$-amplitude enhancement with the $m_d$-increment is attributable to the increment in the gravitational strength stemming from the mass-accretive grains.

In summary, it is evident that the nonlinear bimodal fluctuation dynamics governed by the derived $f$-KdV system of unique form is drastically affected by the component dust characteristic parameters in the considered turbulent astrocloud. The electrostatic phase portraits are of unique shape, unlike the existing scenarios, in the dynamical perspective. In comparison, the phase-space landscapes for the gravitational counterparts specifically are quite in corroborations with our earlier reports predicting atypical non-chaotic homoclinic fixed-point attractors in similar astrophysical eniron [97]. The dynamic role of the self-consistent nonlinear force terms in the derived $f$-KdV scheme is realizable in driving the nonlinear bimodal instability causing the local fluctuation amplitude to grow noticeably.
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Figure 3.4: Same as figure 3.3, but for the different $m_d$-values with a fixed $q_{d0} = 100e$. Various lines refer to (i) $m_d = 3 \times 10^{-11}$ kg (blue solid line), (ii) $m_d = 4 \times 10^{-11}$ kg (red dashed line) and (iii) $m_d = 5 \times 10^{-11}$ kg (black dotted line), respectively.

A specific comparison of our results (with turbulence) with the previous model predictions (without turbulence) available in the literature [41] may be put forward as follows. In the absence of turbulence, the electrostatic fluctuations have been shown by others to evolve as compressive solitary periodic spectral patterns governed by a driven KdV equation (cf. figures 1-3 in Ref. [41]). Moreover, the self-gravitational perturbations have been demonstrated to propagate as similar eigenmode structures; but, dictated by a usual KdV equation (cf. figures 4-5 in Ref. [41]). Thus, the inclusion of the multifluidic turbulence effects transforms the earlier turbulence-free KdV system [41] into the current turbulence-full $f$-KdV system. The excitation of electrostatic compressive monotonic aperiodic shock-like patterns and gravitational compressive non-monotonic oscillatory shock-like structures herein is triggered by the fluid turbulence effects in the turbo-magnetized astrocloud.

3.5 CONCLUSIONS
The nonlinear conjugational $f$-KdV dynamics dictating the evolutionary behaviours of the bimodal pulsational instability excitable in a complex self-gravitating turbo-magnetized collisional astrofluid in the multiscale analysis framework is theoretically investigated. We interestingly see that the nonlinear electrostatic eigenmodes evolving as compressive monotonic aperiodic shock-like patterns and gravitational eigenmodes germinate as compressive non-monotonic oscillatory shock-like structures. The evolutionary
eigenpatterns are further confirmed in a phase-plane landscape in detail. The $f$-KdV geometrical trajectories evolve as the electrostatic irregular non-homoclinic open trajectories and gravitational atypical non-chaotic homoclinic fixed-point attractors, respectively. A systematic comparison of our results in a concretized footing in the light of previous predictions existing literature is also provided to see the reliability of the presented calculation scheme. The main concluding points which can be drawn from our analysis are presented in the following order.

1. A theoretical pair $f$-KdV model is developed to study the nonlinear eigenmode structure formation of hybrid gravito-electrostatic source in a complex turbomagnetized collisional astrofluid in the presence of long-range self-gravity.

2. The electrostatic eigenmodes propagate as compressive monotonic aperiodic shock-like structures, and the gravitational ones as compressive non-monotonic oscillatory shock-like patterns conjointly confirmed with a set of phase-plane landscapes.

3. The dynamic role of the self-consistent nonlinear force terms in the derived $f$-KdV scheme is realizable in driving the nonlinear bimodal instability thereby causing the local fluctuation wave amplitude to grow considerably.

4. It is seen that the constitutive grain-charge (grain-mass) acts as electrostatic stabilizer (gravitational destabilizer) against the self-gravitational cloud collapse dynamics.

5. The proposed analysis can be extensively useful to understand astrofluid-material redistribution processes leading to bounded galactic unit formation.