Chapter 1

Introduction

1.1 Hypernuclear Physics - A General Description

An event recorded, about more than five decades ago [1] i.e. in 1953, in a stack of photographic emulsions exposed to cosmic rays around 26 km above the ground, has been shown in figure 1.1. On the basis of detailed analysis and interpretation of the emulsion record, Danysz and Pniowski [1] have suggested a new name "hypernucleus" to the event which turned out to be a short lived bound state of a hyperon and nucleons. The hypernucleus was found to have a charge of about 5. Thus a new field 'Hypernuclear physics' was born. In general, a \( \Lambda \)-hypernucleus of total baryon number \( A \), symbolically is written as \( ^A_\Lambda Z \): where \( Z \) is the charge of the core nucleus (bound or unbound) containing \( (A - 1-Z) \) neutrons and a \( \Lambda \) hyperon. For example, a hypernucleus of carbon is written as \( ^{12}_\Lambda C \). Here the symbol C of the parent nucleus is retained and a \( \Lambda \) prefixed, indicating replacement of a neutron by a \( \Lambda \) particle. \( ^{12}_\Lambda C \) implies a nuclear system of 12 baryons; 6 protons, 5 neutrons and one \( \Lambda \) particle.

The concept of \( \Lambda \) hypernucleus has been extended and generalized to include hypernuclei containing hyperon(s) \( (\Sigma, \Xi, \Omega \) or a combination of these; \( \Lambda \Sigma, \Lambda \Xi, \Lambda \Omega \), etc.) or charm, beauty particles. The experimentally one \( \Sigma \)-hypernucleus [2] has been identified. The first double \( \Lambda \) hypernucleus of berrilium was discovered in Warsaw, in 1962 [3], in a nuclear emulsion exposed to a beam of \( K^- \) mesons at CERN. In 1991, a second \( \Lambda \Lambda \) hypernucleus of boron [4] was discovered using hybrid technique of nuclear emulsions and detectors at KEK (High Energy Accelerator Research Organisation) Proton Synchrotron, Japan. In the last decade, KEK hybrid emulsion experiment [5] E373 has unambiguously identified the \( ^6_{\Lambda \Lambda} \)He, named as NAGARA event. The other event, from Brookhaven alternating-gradient synchrotron (AGS) experiment [6] E906 conjectured as \( ^4_{\Lambda \Lambda} \)H, has been reinterpreted as most likely the decay [7] of \( ^3_{\Lambda \Lambda} \)He. At present, only three \( \Lambda \Lambda \) hypernuclei are unambiguously identified and their binding energies measured. With the advancement in the technology, now there is a possibility of producing neutron rich hypernuclei [8] e.g. \( ^6_{\Lambda} \)H at Nuclotron in Dubna (Russia).

After the discovery of first hyperfragment, beginning from the end of the sixties, hypernuclei
Figure 1.1: The first hypernuclear event observed in a nuclear emulsion [1] (Figure taken from Ref. [13]).

[9] were produced from $K^-$ beam interacting with photographic emulsions and bubble chambers (filled with He or heavy liquids) via $K^- + n \rightarrow \Lambda + \pi^-$ called as ‘strangeness exchange reaction’. Further efforts from the beginning of seventies, continued with $K^-$ beam of selected momenta so as to produce hypernuclei through ‘recoilless’ and ‘quasi free’ transitions [10] using the aforesaid reaction on suitable targets. The production and observation of well defined excited states of hypernuclei which began at CERN and continued at Brookhaven (USA). The desire to enlarge the list of hypernuclei by adding new species with the measured ground and excited states energies, a new technique [11] of an ‘associated production reaction’ ($\pi^+ + n \rightarrow \Lambda + K^+$) was employed at the BNL-AGS in the eighties. The reaction [12] was first used to produce $^{12}\text{C}$ via reaction $^{12}\text{C}(\pi^+, K^+)^{12}\text{C}$. The technique mentioned just above (see also Ref. [13]) was extensively used at the KEK where Superconducting Kaon Spectrometer
recording the hypernuclear spectra with the 2-2.3 MeV (FWHM) energy resolution [14] for mass numbers $^{10}_{\Lambda}$B, $^{12}_{\Lambda}$C, $^{28}_{\Lambda}$Si, $^{80}_{\Lambda}$Y, $^{130}_{\Lambda}$La, and $^{208}_{\Lambda}$Pb covering almost whole of the periodic table. The hypernuclear mass spectrum [14] of $^{80}_{\Lambda}$Y is shown in figure 1.2 with bump structures corresponding to the bound major shell orbits $s, p, d,$ and $f$, a beautiful manifestation of single particle levels of $\Lambda$. A similar pattern is observed in the mass spectrum of $^{208}_{\Lambda}$Pb. Thus we can say that a weakly interacting $\Lambda$ particle, not Pauli blocked, forms the deeply bound sharp hypernuclear states which are easily observed in the suitably designed experiments. On the contrary, it is difficult task to study the deeply bound single particle nucleonic hole states because these become too broad to be essentially unobservable. Hypernuclei have also been produced using electro- and photo-production [15].

The binding energy $B_{\Lambda}$ of a single $\Lambda$ particle in the hypernucleus is defined as:

$$B_{\Lambda} = M_{\text{core}} + M_{\Lambda} - M_{\text{hyp}},$$

where $M_{\text{core}}$ is the mass of the nucleus $^{A-1}_{Z}$, $M_{\Lambda}$, the mass of the $\Lambda$ particle and $M_{\text{hyp}}$, the mass of the hypernucleus $^{A}_{Z}$. Similarly, one can define $B_{\Lambda\Lambda}$ for double $\Lambda$ hypernucleus. The
incremental binding energy $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$, in the double $\Lambda$ hypernucleus, is a source of information about $\Lambda\Lambda$ potential. Likewise, we can also define binding and incremental binding energies for the hypernuclei of other hyperons. As of now, a large number of $\Lambda$ hypernuclei have been produced and their binding energies measured. So far hypernuclei (see Refs. [1] to [14]) that have been experimentally produced are: 40 single-$\Lambda$ hypernuclei, 3 double-$\Lambda$ hypernuclei, 8 $\Xi^-$ hypernuclei with ambiguous identity and inaccurate $B_{\Xi}$ data from emulsion experiments and $\Sigma$ hypernucleus $\frac{5}{2}^+_\text{He}$. In the “Segrè table” (see figure 1.3) more than 30 $\Lambda$-hypernuclei [13] with baryons number $A = 3$ to 209 are depicted. We have also reproduced hypernuclear chart [16, 17] where $\Lambda$ and $\Lambda\Lambda$ hypernuclei in figure 1.4 are shown in blocked shapes; second layer of $\Lambda$ hypernuclei blocks is imposed on the first layer of nuclear species and the top layer contains a few $\Lambda\Lambda$ hypernuclei.

1.1.1 Why do we study Hypernuclei?

The primary objective of nuclear physics is to find the nature of the interactions among the octet of baryons ($N, \Lambda, \Sigma, \Xi$) in a unified way. The strange and multi-strange hypernuclei, where strange particle(s) is(are) made to stick to ordinary nuclei stable or unstable, help in realizing such an objective.

The existence of hypernuclei with impurities of hyperon(s) embedded in nuclei represents a new state of nuclear matter that may exhibit new symmetries, selection rules, etc. Furthermore, the presence of hyperon(s) in a nucleus may modify the properties such as: moment of inertia
of a deformed nucleus, the rotational and vibrational states structure of nuclei, the quadrupole moment etc. Therefore, a hypernucleus serves as a unique laboratory to explore the basic properties of hyperons in the nuclear medium and strange exotic systems. Further, the presence of strange baryon(s) in nuclei may act as a sensitive probe in extracting the information about the nuclear structure. The study of hypernuclei is important from another point of view. The non-mesonic decay modes provide a means of exploring the four fermions, strangeness non-conserving weak interaction on $\Lambda N \rightarrow NN$.

At present, it is believed that hyperons matter that forms the inner cores of the neutron stars, would have a substantial impact on their properties. This is borne out from the study [18] that the nature of hyperon-nucleon $YN$ ($Y=\Lambda, \Sigma$) and hyperon-hyperon $YY$ interactions will have a direct bearing on the mass-radius relationship of neutron stars. Therefore, for
investigating the properties of the neutron stars, the determination of $YN$ and $YY$ potentials is extremely important. The equation of state [18] for nuclear matter also gets modified in the presence of hyperons. Thus hypernuclear physics is likely to play an important role in the study of the properties of neutron stars and equation of state of nuclear matter. Figure 1.5, taken from Ref. [16], shows the interdisciplinary nature of hypernuclear physics that links fields of particle, nuclear, many-body and astro physics.

![Diagram](image)

Figure 1.5: Hypernuclei and their link to other fields of physics (Figure taken from Ref. [16]).

1.1.2 Soft Core Baryon-Baryon Interactions

In this subsection, we intend to provide a very brief survey of the approaches used in deriving/constructing the baryon-baryon ($BB$) potentials for use in the structure calculations of nuclei and hypernuclei. The $BB$ interaction has been constructed following the three procedures [19, 20, 21, 22, 23, 24, 25, 26, 27]: first one is ab initio method where field theoretic boson-exchange models [24, 25, 26] have been extensively employed in deriving the potential, the other is phenomenological approach [19, 20, 21, 22, 23] where large number of terms involving operators, satisfying invariance principles, are used. The third approach [27], still in its infancy, the quenched quantum chromodynamics, a fundamental theory of strong interaction, has been applied in calculating the $BB$ potentials. The $BB$ potentials from the latter approach are in the initial stage of development and still far from being used in the investigation of the properties of nuclei and hypernuclei. In view of the above, we will confine our brief review on the $BB$ potentials, inspired by the former two approaches. These potentials are being continuously modified and refined, based on the constraints imposed by the growing
Figure 1.6: $\Lambda N$ total cross section data [29] for $P_{lab}$ in the range from 0 to 1000 MeV/c compared with predictions from the Nijmegen soft-core potential model.

input $BB$ scattering data base. At present, $BB$ data base [28] consists of about 4,000 very accurate and rich scattering data points for $NN$ below 350 MeV and $YN$ scattering data [29] are less than 600 events below 300 MeV/c and less than 250 events between 300 and 1500 MeV/c but these are sparse and not so precise as $NN$ data. The other input data are the properties of deuteron. In $\Lambda N$ case, we have imprecise total differential cross section and forward-to-backward ratio data and no data related to spin-dependence of $YN$ system. There is no scattering data on $YY$, $\Xi N$, $\Xi Y$, and $\Xi \Xi$.

In the first approach, the Nijmegen group has developed many versions [24, 25, 30] of boson-exchange $BB$ potentials and these are being continuously improved from the feedback of analyzes of the relevant $BB$ scattering ($NN$ and $YN$) and hypernuclear binding energy data. One of the popular versions of $BB$ potentials constrained by the aforesaid data is the soft core NSC97 [24, 25] with models a to f for strangeness $S = 0$ and $-1$ to $-4$. In this model, one-boson-exchange (OBE) generates the potential. The coupling constants of many mesons hitherto not known, fixed from SU(3) symmetry arguments, are the main ingredients that are constrained from a fit to the $BB$ scattering data and the properties of deuteron. One of the weakness of this version is that the calculated incremental $\Delta B_{\Lambda\Lambda}$ does not agree with the value 1.01 MeV obtained from the NAGARA event [5]. We may remark that simulated three-range Gaussian $\Lambda\Lambda$ and $\Lambda\Xi$ potentials from these models have been recently used in the structure calculation of hypernuclei. The recent $BB$ potentials termed as extended soft core versions ESC04 and ESC06 [30], which are being still developed are improvement over NSC97. In these models, apart from OBE as considered in NSC97, the contributions from (i) two-meson-exchanges, and (ii) meson-pair-exchanges, are also included. Special features of these models are (i) the inclusion of the axial-vector meson potentials, and (ii) a zero in the scalar-meson form-factors. Rijken and Yamamoto [30] claim that these features endow the
Figure 1.7: Cross section for $\Lambda p$ elastic scattering. The curves A, B and F are for different effective range parameters shown in table 1.1. (Figure taken from Ref. [31]).

model a rather flexible dynamical framework. For example, it was found feasible to keep the parameters of the model rather close to the predictions of the $^3P_0$ quark-pair-creation model. This turned out to be true in the case of meson-baryon coupling constants and the magnetic vector $F/(F + D)$-ratios. The basic assumption in these models is that $\text{SU}_F(3)$-symmetry allows all the mesons coupling constants to be determined from a fit to data described above (see also figure 1.6). The assumption of $\text{SU}_F(3)$-symmetry is used to extend the potential model to strangeness $S = -2$ to $-4$ sector. The NSC97 models are the first of its kind to give $BB$ potentials for $S < -1$.

The Bonn-Jülich group [26] has also developed a OBE potential determined from the data on $BB$ scattering in the $S = 0$ and $-1$ channels. This potential has been extended for $S = -2$ sector to describe the masses of known $\Lambda\Lambda$ hypernuclei. In this analysis, the role of long-range nuclear correlations i.e. random phase approximation (RPA) has been investigated in detail and was found to be important in explaining the $B_{\Lambda\Lambda}$ of the three known hypernuclei.

Baryon-baryon interaction of Nijmegen group is the only interaction that covers all the channels from $S = -1$ to $-4$ i.e. $YN$, $YY$, $\Xi N$, $\Xi \Lambda$ etc. In chapter 2, we have used potentials simulated from NSC97 model to study and predict the ground state energies of strange and multi-strange $\alpha$ cluster hypernuclei.

Notwithstanding the efforts that have gone into constructing the $BB$ potentials of Nijmegen group from the field theoretic meson-exchange models, the existing data provide only a limited
Table 1.1: Effective range parameters [31] corresponding to the curves A, B and F shown in figure 1.7.

<table>
<thead>
<tr>
<th>Fit</th>
<th>$a_s$ (fm)</th>
<th>$r_{0s}$ (fm)</th>
<th>$a_t$ (fm)</th>
<th>$r_{0t}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-2.0</td>
<td>5.0</td>
<td>-2.2</td>
<td>3.5</td>
</tr>
<tr>
<td>B</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.4</td>
<td>3.0</td>
</tr>
<tr>
<td>F</td>
<td>-8.0</td>
<td>1.5</td>
<td>-0.6</td>
<td>5.0</td>
</tr>
</tbody>
</table>

constraint on various meson-coupling constants, even for potentials that are fitted to both the $NN$ and $YN$ data. Further, extension of the model to $S = -2$ to -4 sector in the absence of the data on free-space scattering of $YY$, $YN$, $\Xi N$ and $\Xi \Xi$ may have enhanced the uncertainties in the parameters of the potentials. Moreover, treatment for short-range behavior of interaction is still in the phenomenological stage. Therefore, a systematic and detailed measurement of spectroscopic data of $S < -1$ hypernuclei with single and multiple hyperons in future experimental efforts offer the possibility of constraining the parameters of the model. Although available $\Lambda p$-scattering data from bubble chambers have been used extensively in the development of meson exchange $BB$ potentials discussed above, we may point that even effective range parameters in the singlet and triplet states can not be determined uniquely [31] as is evident from figure 1.7 and table 1.1. The source of ambiguity in the effective range parameters is due to large statistical errors in the $\Lambda p$-scattering data at lower energies, which are responsible for determining the effective range parameters. Therefore, the precise measurement for the spin-dependent parts of the $\Lambda p$ interaction is still an open problem. An effort in this direction (see e.g. Machner and Siudak [31]) is being made using polarized beam of protons in the reaction $p + p \rightarrow K^+ + \Lambda + p$ to extract the information about effective range parameters from $\Lambda p$ final state interaction. Several such possibilities [29] e.g. reaction $K^- + 2H \rightarrow \pi^- + \Lambda + p$ at J-PARC, reaction $\gamma + ^2H \rightarrow K^0 + \Lambda + p$ at JLab and capture of $K^-$ in the reaction $K^- + 2H \rightarrow \gamma + \Lambda + n$ have been proposed to enhance the $\Lambda N$ scattering data base.

In the phenomenological approach, the local operators terms in the $NN$ potential are constructed by the requirement of the $BB$ data and the invariance principles. The operator structure of each term in the potential has been chosen to simplify many-body calculations. Several such well known potentials e.g. Reid soft core [19], Paris [20], Urbana $v_{14}$ [21] and Argonne $v_{14}$ [22] have been constructed and are being extensively used in the nuclear structure calculations. These potentials have a strong short-range repulsion and intermediate range attraction followed by long-range one-pion-exchange (OPE) part. The strong repulsive part
of $v_{14}$ $NN$ potential at short distance, has been chosen to have Woods-Saxon shape, and intermediate range parts are assumed to arise from the two-pion exchange (TPE) processes which are dominated by tensor interaction. The long-range OPE has well known radial shape. The operator structure of Argonne $v_{14}$ is identical to that in the Urbana $v_{14}$ and has the same number of operators terms as the parameterized Paris potential. A new high-quality nucleon-nucleon potential, an improved version of Argonne $v_{14}$, is Argonne $v_{18}$ [23] with explicit charge-dependence and charge-asymmetry breaking operators. These potentials are called realistic as these give very good fit to the $NN$ scattering data for laboratory energy $\leq 350$ MeV and the ground state properties of deuteron. At present, it is not possible to construct such potentials for $YN$ in view of the availability of a few and not very accurate data for $\Lambda p$-scattering from bubble chambers experiments [32, 33] i.e. total scattering cross section and forward-to-backward ratio in the center of mass (c.m.) energy range $\leq 20$ MeV. Despite these limitations, Bodmer et al. [34] have constructed a phenomenological central charge-independent, spin-dependent, and space-exchange two-body $\Lambda N$ potential inspired by Urbana $NN$ potential from a fit to the $\Lambda p$-scattering data that is meager in statistics and have large uncertainties in the incident energies of $\Lambda$ and cross-section data. We briefly discuss the Urbana-type potential for $\Lambda N$ in section 1.3 and this we have been used to analyze the $p$-shell $\alpha$ cluster hypernuclei as a partial $\Lambda$-body systems in chapters 3 and 4.

1.2 Experimental Facilities for Hypernuclear Physics Program

In the last few years, different laboratories (see Refs. [35, 36, 37]) around the world have started/are planning to start a hypernuclear physics program to produce and identify hypernuclei with strangeness content $S = -1$ to $-3$. From the advancement in state-of-the-art experimental facilities, it has now become possible to explore $\Xi^-$ and $\Omega^-$ hypernuclei which hitherto were inaccessible. We, in the following, present a very brief overview of the present scenario of a few leading experimental facilities in the area of strange physics although the answers to many other fundamental problems in nuclear physics are also being explored.

- **TJNAF** (Thomas Jefferson National Accelerator Facility) at Newport News in USA: The **CEBAF** (Continuous Electron Beam Accelerator Facility) at **TJNAF** is a facility where hypernuclear spectroscopy with an energy resolution of approximately 600 keV (FWHM) will be explored through the electro-production reaction $e + p \rightarrow e' + K^+ + \Lambda$. The $(e, e' K^+)$ reaction produces states which are complement to the n-hole-$\Lambda$-particle states created in the reactions $(\pi^+, K^+)$ and $(K^-, \pi^-)$. Hence a new field of mirror hypernuclei for theoretical investigations is likely to become available. A electron beam of 1.7 to 1.8 GeV is bombarded at the target to produce hypernuclei. The above reaction was successfully employed, for the first time, for the preliminary spectroscopy of $^{12}_{\Lambda}$B and $^{28}_{\Lambda}$Al.
• **MAMI (MAinz MIcrotron)-C**: The MAMI facility will be used to study the structure of hadrons and nuclei with electrons energy of 1.5 GeV. In near future, electro-production experiments \( (e, e'K^+) \) will add more detailed information on the structure of single \( \Lambda \) hypernuclear states.

• **FINUDA (FIsica NUcleare a DAΦNE)**: A special accelerator, DAΦNE (Double Annular ring For Nice Experiments) has been designed at INFN (Istituto Nazionale di Fisica Nucleare) where electron and the positron beams each of 510 MeV collide head-on to produce the \( \Phi \) particle (mass=1020 MeV, mean life time \( \tau = 20 \times 10^{-23} \) s) which subsequently decays mostly into charged \( (K^+, K^-) \) (mass=493.7 MeV, \( S = +1 \) and \( -1 \)) and neutral \( (K^0, \bar{K}^0) \) mesons. Presently, DAΦNE is producing about 150 \( \Phi \)/s, half of which decay into a pair of opposite charged kaons with very low momentum, 127 MeV/c that is almost monochromatic. The copious production of \( \Phi \) (1020) particles will produce beam of \( K^- \) mesons of extremely high intensity and precise energy that is expected to insert "strangeness" inside nuclei to produce hypernuclei as: \( K_{\text{Stop}}^- + {}^A Z \rightarrow \pi^- + {}^A \Lambda \).

• **Nuclotron in Dubna** (Russia): Hypernuclear program at Nuclotron started in 1988 and at present a program to produce relativistic hypernuclei to measure their life times is being implemented. Beam of \( ^3 \text{He}, ^4 \text{He}, ^6 \text{Li}, ^7 \text{Li} \) with GeV/nucleon in the range 5.14 to 3.0 will be directed on appropriate target to produce neutron rich hypernuclei; e.g. \( ^7 \text{Li} + C \rightarrow {}^4 \text{H} + p + \ldots \rightarrow {}^4 \text{He} + \pi^- + p, A = 6, 4, 3 \).

• **J-PARC** (Japan Proton Accelerator Research Complex) at KEK: Already a rich data related to both the spectroscopy and decay of hypernuclei at KEK have been measured. In the near future, a program [38] for production and unambiguous identification of \( \Xi \)-hypernuclei i.e. \( {}^{12}_{\Xi \text{-Be}}, {}^{11}_{\Xi \text{-Li}} \) etc. is being planned as a first day experiment.

• **\( \bar{P}\text{ANDA} \)** (\( \bar{P} \) ANnihilations at DArmstadt): In this experiment, an antiproton \( \bar{p} \) beam hits primary target to produce \( (\Xi, \bar{\Xi}) \) pair through the reactions: \( \bar{p} + p \rightarrow \Xi^+ + \Xi^- \) and \( \bar{p} + n \rightarrow \bar{\Xi}^0 + \Xi^- \). The stopping and absorption of double strange baryons \( \Xi^- \) in a nucleus in the secondary target produce double \( \Lambda \) hypernuclei. A pictorial explanation of the two-step production process of double hypernuclei [16, 39] is shown in figure 1.8. The measurement of the properties and extending the list of these \( \Lambda \Lambda \) hypernuclei is one of the major goal of this laboratory. There is also a programme [16] to produce \( S = -3 \) \( \Omega^- \) hypernuclei at \( \bar{P}\text{ANDA} \).

It is expected that the precise data about the existence of \( \Xi \)-hypernuclei will be available from **J-PARC** in the near future. We hope to have data on \( S = -4 \) multi-strange hypernuclei as well. The latest update in hypernuclear physics field, can be obtained from the website of hypernuclear physics conference [40] held this year.
Figure 1.8: Various steps of the double hypernuclei production in \( \bar{P} \)ANDA (Figure taken from Ref. [16]).

1.3 A Survey of Earlier Theoretical Studies

Since the discovery of first hypernucleus, a large number of analyzes using shell model, cluster model, self consistent Hartree-Fock and variational methods have been made to study the properties of hypernuclei. The details of the chronological development in hypernuclear physics prior to 1995 can be traced from the review articles by Gal [41] and Gibson and Hungerford III [42]. A persistent problem [43, 44, 45] before 1985, was that central, charge-independent and spin-dependent two-body \( \Lambda N \) potential consistent with \( \Lambda p \)-scattering data overbinds \( ^9\text{He} \) by about 3 MeV but explained \( B_\Lambda \) of hypernuclei \( ^2\text{H} \) and \( ^3\text{H} \). This was the well-known overbinding problem.

Several attempts either by incorporating the three-body \( \Lambda NN \) potentials [45] or tensor
\( NN \) and \( \Lambda N \) forces \([46]\) could not resolve the overbinding problem of \( ^5 \Lambda \) He. Bodmer \textit{et al.} \([34, 47, 48]\) again investigated this problem based on the simple two-body \( NN \), \( \Lambda N \) and the three-body \( \Delta NN \) forces. The central, charge-independent, spin-dependent two-body \( \Lambda N \) potential of Urbana-type was chosen to have the form;

\[
V_{\Delta N} = (1 - \epsilon + \epsilon P_\pi) \tilde{V}_{\Delta N}^0,
\]

(1.1)

where

\[
\tilde{V}_{\Delta N}^0 = V_{2\pi} = W(r) - \left[ \tilde{V} - \frac{1}{4} V_\sigma(\sigma_\Lambda \cdot \sigma_N) \right] T_\pi^0(r),
\]

(1.2)

with spin-averaged \( \tilde{V} = \frac{1}{4} V_s + \frac{3}{4} V_t \) and spin-dependent \( V_\sigma = V_s - V_t \) parameters related to the singlet \( (V_s) \) and triplet \( (V_t) \) strengths of \( \Delta N \) potential, \( P_\pi \) is the Majorana space-exchange operator for \( \Lambda \) and nucleon with strength parameter \( \epsilon \). \( W(r) \) is a Woods-Saxon repulsive core which is given as

\[
W(r) = W_0 \left[ 1 + \exp \left( \frac{r - R}{d} \right) \right]^{-1},
\]

(1.3)

where \( W_0 = 2137 \ \text{MeV} \), \( R = 0.5 \ \text{fm} \), \( d = 0.2 \ \text{fm} \) and \( T_\pi(r) \) is the OPE tensor potential shape modified with a cut-off,

\[
T_\pi(r) = \left[ 1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right] \frac{\exp(-\mu r)}{\mu r} [1 - \exp(-\mu^2 r^2)]^2,
\]

(1.4)

with \( \mu = 0.7 \ \text{fm}^{-1} \) and the cut-off parameter \( c = 2.0 \ \text{fm}^{-2} \). The strength \( \tilde{V} \) of \( \Delta N \) potential, which is consistent with \( \Lambda p \)-scattering, determined accurately \([48]\) and has a value \( 6.15 \pm 0.05 \ \text{MeV} \). \( V_\sigma \) lies in the range \( 0 \) to \( 0.5 \ \text{MeV} \), very uncertain, which is also reflected from the effective range parameters in table 1.1. The exchange parameter \( (\epsilon) \) is quite poorly determined from the \( \Lambda p \) forward-to-backward scattering data and is found to be \( \approx 0.1 \) to \( 0.38 \). Here we may point out that for hypernuclei with zero-spin core \( S_C = 0 \) (e.g., \( ^5 \Lambda \) He, \( ^6 \Lambda \) Be), the contribution to potential energy arises from the spin-average \( (\tilde{V}) \) while the spin component \( (V_\sigma) \) effectively contributes nothing.

For the \( NN \) pair, Bodmer \textit{et al.} \([34]\) have used the central, spin-isospin independent Malfliet-Tjon (MT) potential \([49]\) that gives reasonable ground state energies and root mean square (rms) radii for charge symmetric pair \((^3 \Lambda \text{He}, ^3 \text{He})\) and \(^4 \text{He}\) within variational Monte Carlo (VMC) approach as we notice from table 1.2. This potential has the form;
Table 1.2: VMC results [34] for Malfliet-Tjon $NN$ potential for the binding energies in column two and rms radii in the last column for charge symmetric pair ($^3$H, $^3$He) and $^4$He in the first column.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Binding energy (MeV)</th>
<th>rms radius (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$H, $^3$He</td>
<td>8.25 ± 0.01</td>
<td>1.66</td>
</tr>
<tr>
<td>$^4$He</td>
<td>31.20 ± 0.03</td>
<td>1.42</td>
</tr>
</tbody>
</table>

$$V_{NN}(r) = \frac{\hbar c}{r} \left[ 7.39 \exp(-3.11r) - 2.93 \exp(-1.55r) \right].$$ (1.5)

For the spin $S_C = 1$ and $I = 0$ for $^2$H which is a core nucleus for $^3$H, the coefficient of the attractive term in Eq. (1.5) is slightly increased to 3.201 from 2.93 to reproduce the binding 2.23 MeV close to the experimental value.

The phenomenological dispersive three-body $\Lambda NN$ potential used by Bodmer et al. [34] is a representation of the suppression of the TPE $\Lambda N$ potential arising from modification ("dispersion") of the intermediate $\Sigma$, $N^*$, $\Delta$, ..., components by the medium (a "second" nucleon $N_2$) as shown in figure 1.9. The phenomenological dispersive $\Lambda NN$ potential has been constructed primarily to resolve the overbinding of $^5\Lambda$He and $\Lambda$-nuclear matter. This has the form:

$$Spin - independent: V_{\Lambda NN} = W_d T_\pi^2(r_{A1}) T_\pi^2(r_{A2}),$$ (1.6)

where the strength parameter $W_d > 0$ makes dispersive energy repulsive. The radial form of $T_\pi(r)$ is given in Eq. (1.4).

In the VMC analysis, Bodmer et al. [34] analyzed hypernuclei with $A = 3, 5$ as $A$-body systems and studied a few hypernuclei of closed core nuclei $A = 5, 9, 13, \infty$ using the effective interaction method. $\Lambda$-binding energy in infinite nuclear matter was calculated with the Fermi hypernetted chain (FHNC) variational method. This study demonstrated for the first time the important role played by the dispersive three-body $\Lambda NN$ interaction, in particular, in resolving the overbinding problem of hypernuclei with $A \geq 5$. The repulsive contribution of dispersive $\Lambda NN$ force in $^3\Lambda$Be in $\Lambda\alpha\alpha$ model from 16 pairs of nucleons interacting with a $\Lambda$, where each $\alpha$ is contributing a nucleon, is about 1.0 MeV. The similar is the situation
for $^{13}$C. These hypernuclei are found to be grossly overbound without three-body dispersive $\Lambda NN$ force. Subsequently, Bodmer and Usmani [48] again performed the VMC calculations for $s$-shell hypernuclei ($A \leq 5$) including spin-flip excited states of $A = 4$ and applied FHNC technique for $\Lambda$ well depth using the phenomenological central two-body $\Lambda N$ potential and $2\pi$-exchange three-body $\Lambda NN$ force and strongly repulsive spin-independent and spin-dependent dispersive $\Lambda NN$ interactions with a view to extract the spin-dependence $V_0$ from the spin-flip excited states of $A = 4$ systems. The spin-dependent dispersive $\Lambda NN$ force has the form:

$$
Spin - dependent: V_{\Lambda NN}^{DS} = V_{\Lambda NN}^{D}[1 + \frac{1}{6}\bar{\sigma}_\Lambda \cdot (\bar{\sigma}_1 + \bar{\sigma}_2)].
$$

(1.7)

The $V_{\Lambda NN}^{DS}$ potential is obtained by assuming that dispersive (suppressive) modification acts only for triplet $\Lambda N_1$ state (figure 1.9), and then symmetrizing between $N_1$ and $N_2$. The Eq. (1.7) can be written as

$$
V_{\Lambda NN}^{DS} = V_{\Lambda NN}^{D}(1 + \frac{1}{3}\bar{\sigma}_\Lambda \cdot \bar{S}_{12}),
$$

(1.8)

where spin operator $\bar{S}_{12} = (\bar{\sigma}_1 + \bar{\sigma}_2)/2$ for the operators $\bar{\sigma}_1$ and $\bar{\sigma}_2$ of two participating nucleons. The expectation value of the operator $\bar{S}_{ij}$ for the core nuclei of spin $S_C$ is written as

$$
\sum_{i<j}^{A-1}(1 + \frac{1}{3}\bar{\sigma}_\Lambda \cdot \bar{S}_{ij}) = \frac{1}{2}(A - 1)(A - 2) + \frac{1}{3}(A - 2)\bar{\sigma}_\Lambda \cdot \bar{S}_C.
$$

(1.9)
For spinless core \((S_C=0)\) of \(^{\Lambda}\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!
\[ + Y(r_{\Lambda 2}) T^2(r_{\Lambda 2}) [ T(r_{\Lambda 1}) (3 \cos^2 \theta_{2\Lambda 1} - 1) - 1] \]
\[ \times \frac{1}{9} \bar{\tau}_i \cdot \bar{\tau}_j (\bar{\sigma}_i \cdot \bar{\sigma}_j + \bar{\sigma}_{\Lambda} \cdot \bar{S}_{ij}) \]  
(1.11)

where
\[ T(r) = \left[ 1 + \frac{3}{\mu r} + \left( \frac{3}{\mu r} \right)^2 \right] \left[ 1 - e^{(-c' r^2)} \right] \]  
(1.12)

and
\[ Y(r) = \frac{e^{(-\mu r)}}{\mu r} \left[ 1 - e^{(-c' r^2)} \right] \]  
(1.13)

modified by cut-off radius \( c' \). The expectation values for the spin-isospin factor
\[ \frac{1}{9} \sum_{i<j}^{A-1} \bar{\tau}_i \cdot \bar{\tau}_j (\bar{\sigma}_i \cdot \bar{\sigma}_j + \bar{\sigma}_{\Lambda} \cdot \bar{S}_{ij}) \]  
(1.14)

for \( s \)-shell hypernuclei have been evaluated by Gal [51].

Shoeb et al. [52] have re-examined the problem of overbinding of \( ^{\Lambda}_{\Lambda+1} \)He using new form of dispersive \( \Lambda NN \) force Eq. (1.11) in the VMC framework. A detailed analysis of \( s \)-shell hypernuclei shows that the above dispersive force for appropriate correlations makes a small attractive contribution in the case of \( ^{\Lambda}_{\Lambda+1} \)H while it is repulsive for \( ^{\Lambda+1}_{\Lambda+1} \)H, \( ^{\Lambda+1}_{\Lambda+1} \)He, and \( ^{\Lambda+1}_{\Lambda+1} \)He*. The analysis of Shoeb et al. [52] confirms the results of Ref. [48] that \( 0^{+}-1^{+} \) spin doublet splitting is partly due to the two-body \( \Lambda NN \) spin-dependence \( V_{\sigma} \) and the dispersive \( \Lambda NN \) force. Thus the investigations of Refs. [48] and [52] reinforce the statement of Gibson [53] that the separation of levels in \( A = 4 \) \( \Lambda \)-hypernuclei is not a measure of two-body \( \Lambda NN \) spin-dependence. Not too long ago, Akaishi et al. [54] have also solved overbinding problem by introducing, in \( Y \)-nucleus potential, the \( \Lambda-\Sigma \) coupling that can be separated into the coherent and incoherent processes. In the incoherent \( \Lambda-\Sigma \) coupling, a nucleon after the interaction changes to an excited state that gives suppression effect in the nuclear medium while in the other process, the nucleon remains in the ground state giving rise to an attractive effect. The suppression of incoherent \( \Lambda N-\Sigma N \) coupling solves the overbinding problem in \( ^{\Lambda+1}_{\Lambda+1} \)He. The coherent \( \Lambda N-\Sigma N \) coupling, the main part of which is equivalent to the \( \Lambda NN \) force, contributes dominantly in the \( 0^{+} \) ground states of \( ^{\Lambda+1}_{\Lambda+1} \)H and \( ^{\Lambda+1}_{\Lambda+1} \)He. The two types of \( Y N \) potentials D2 and SC97f(S) which solved the overbinding problem are the simplified forms of Nijmegen group realistic \( YN \) interactions. The D2 which is equivalent to the Nijmegen D interaction, has the contributions from central parts of both the \( \Lambda N \) and \( \Sigma N \) channels while SC97f(S) that includes the central and tensor parts of both the channels, is obtained from Nijmegen soft core SC97f.
interaction [25]. Thus, solution of overbinding problem of $^5_{\Lambda}\text{He}$, either using the dispersive $\Lambda NN$ forces [34, 48, 52] or introducing in $Y$-nucleus potential, the $\Lambda-\Sigma$ coupling [54], seems to be almost equivalent descriptions. In another VMC calculation, Shoeb et al. [55] have analyzed the $s$-shell $\Lambda$-hypernuclei treating these as $A(=\text{baryon number})$-body systems and $p$-shell hypernucleus $^9_{\Lambda}\text{Be}$ as a partial nine-body problem in the $\Lambda\alpha\alpha$ model and found that space-exchange $\Lambda N$ and dispersive three-body $\Lambda NN$ forces can not be determined uniquely as one can mask the presence of other. Here we may remark that Shoeb et al. [55] have ignored the space-exchange correlations, a priori without any convincing physical arguments. However, this approximation turned out to be justified in a recent VMC analysis [56] where the space-exchange $\Lambda N$ correlations in the trial wave function were explicitly included along with the important two- and three-body ones. The inclusion of the effect of the space-exchange correlations on the $B_\Lambda$ of $^5_{\Lambda}\text{He}$ based on realistic Argonne $v_{18}$ $NN$ and Urbana-type space-exchange $\Lambda N$ potentials turned out to be insignificant.

The finding of Brookhaven AGS experiment [6] E906, conjectured earlier as $^4_{\Lambda}\Lambda\text{H}$, has been reinterpreted as most likely the decay [7] of $^5_{\Lambda}\text{He}$. The importance of exploring experimentally and theoretically the system $^4_{\Lambda}\Lambda\text{H}$ which will mark the onset of double $\Lambda$ hypernuclei is that it would play as fundamental role for finding the $\Lambda\Lambda$ force as $^3_{\Lambda}\text{H}$ and $^2\text{H}$ have played for exploring the $\Lambda N$ and $NN$ forces in the hypernuclear and conventional nuclear physics. Therefore, theoretical investigations for its stability and the experimental search for its existence assume special significance. In the recent past, four theoretical studies [57, 58, 59, 60] on the issue of the stability of the lightest $S = -2$ hypernucleus $^4_{\Lambda}\Lambda\text{H}$ have been reported. The first three analyzes have employed the same $NN$, $\Lambda N$ and $\Lambda\Lambda$ potentials in the four-body $\Lambda\Lambda np$ model. The Faddeev-Yakubovsky (FY) method [57] does not bind $^4_{\Lambda}\Lambda\text{H}$ but the stochastic variational method (SVM) [58] and variational Monte Carlo (VMC) calculation [59] predict a particle stable system. The $\Lambda\Lambda$ binding energy for the two independent approaches SVM and VMC is found to be the same. Using the Faddeev method [61] and VMC approach [59], the three-body $\Lambda\Lambda d$ ($d=\text{deuteron}$) cluster model calculation also favors a stable $^4_{\Lambda}\Lambda\text{H}$. A coupled channel calculation [60] using SVM also predicts a very weakly bound $^4_{\Lambda}\Lambda\text{H}$.

The measured $B_{\Lambda\Lambda}$ value $7.25 \pm 0.19_{-0.11}^{+0.18}$ MeV of the NAGARA event [5] identified as $^6_{\Lambda\Lambda}\text{He}$ suggests a relative weak $\Lambda\Lambda$ interaction compared to the value $10.92 \pm 0.6$ MeV given by Prowse [62], the authenticity of whose is now considered doubtful. The NAGARA and the other double $\Lambda$ events [6, 7] have revived the interest in extracting the nature of $\Lambda\Lambda$ potential and therefore, giving rise to the hope of constraining the parameters of $\Lambda\Lambda$ boson-exchange interaction models. Consequently, in the recent past many analyzes [57, 58, 59, 61, 63, 64] of $s$-shell hypernuclei treating these as $A$-body systems and that of $s$- and $p$-shell hypernuclei in the $S = -1$ and $-2$ sector in the cluster model [61, 65, 66, 67, 68, 69, 70, 71, 72] have been performed. The finding of these two types of approaches will help in incorporating modifications leading to the improvement in the potentials models and wave functions of the hypernuclear systems in the cluster model.

The energies of the ground and excited states of $\Lambda$ and $\Lambda\Lambda$ hypernuclei have been extensively
analyzed in the cluster model in Refs. [48, 61, 68, 69, 70, 71] using a variety of methods: VMC framework, variational method, Gaussian-basis coupled-rearrangement channel method and Faddeev and Faddeev-Yakubovsky methods. The input $\Delta \Delta$, $\Delta \alpha$, and $\alpha \alpha$ potentials with soft repulsive cores and reasonable shapes, constrained by the data relevant to the each interacting pair, have been used. In the past decade, in a series of papers [61, 65, 66, 67, 68, 69, 70], the $s$- and $p$-shell $\Delta$ and $\Delta \Delta$ hypernuclei have been analyzed in the cluster model. The Faddeev method for the three-particle clusters and for the four-particle clusters, the FY framework [61, 65, 66, 67] have been used. The $B_{\Delta \Delta}$ of $^6_{\Delta \Delta}$He, $^5_{\Delta \Delta}$He and $^5_{\Delta \Delta}$H is calculated for a wide range of simulated $\Delta \Delta$ interaction models of Nijmegen group and the latter two systems are predicted to be particle stable. The results of the variational calculation of Myint et al. [68] agree with the Faddeev results [66] for $A = 5$ and $6$ $\Delta \Delta$ hypernuclei. In the recent work Shoeb and coworkers [69, 70] have employed the VMC method in analyzing the $s$-shell $\Delta \Delta$ hypernuclei in the cluster model. Shoeb et al. [69] have made extensive calculations of the $\Delta \Delta$ binding energies of the $s$- and $p$-shell hypernuclei in the cluster model. In this analysis [69], a variety of the phenomenological $\Lambda$-core potentials consistent with the $\Lambda$-core energies and a wide range of simulated $s$-state $\Delta \Delta$ potentials including those used in Refs. [61, 68] are taken as input. The binding energy $B_{\Delta \Delta}$ of $^6_{\Delta \Delta}$He has been explained and the systems $^5_{\Delta \Delta}$He and $^5_{\Delta \Delta}$H found to be particle stable in the $\Lambda$-$\Lambda$-core model. The results for the $s$-shell $\Delta \Delta$ hypernuclei are in good agreement with the non VMC methods [61, 68]. All the analyzes [61, 68, 69] mentioned just above predict particle stable $^5_{\Delta \Delta}$He and $^5_{\Delta \Delta}$H.

Filikhin and Gal [61] have also analyzed the $B_{\Lambda}$ of $^9_{\Lambda}$Be in the $\Lambda + 2\alpha$ model in the Faddeev approach using the $s$-state Isle $\Lambda \alpha$ potential. The surprising result is that the $B_{\Lambda}$ is explained without either normalizing the $\Lambda \alpha$ potential or the inclusion of dispersive three-body force. However, subsequent investigations by Filikhin and coworkers [65, 67] showed it to be the result of ignoring the contributions from higher partial waves ($l > 0$). The use of weakened $p$-state $\Lambda \alpha$ interaction, obtained by Myint et al. [68] from NSC97e potential model, brings out significant reduction in the calculated $B_{\Lambda}$ of $^9_{\Lambda}$Be compared to the $s$-state model [65, 67]. Moreover, the $\Lambda \alpha$ potential which fits $B_{\Lambda}$ of $^9_{\Lambda}$Be in Ref. [61] gives binding for $^{10}_{\Lambda \Lambda}$Be in $\Delta \Lambda + \alpha \alpha$ model close to the experimental value for a very strong $\Delta \Lambda$ potential model ND. This is again a reflection of restricting the $\Lambda \alpha$ potential to $s$-wave only, as we have pointed out just above for $^9_{\Lambda}$Be. Hiyama et al. [71] have made variational calculation for the energy of $^{10}_{\Lambda \Lambda}$Be, apart from many other systems, in the cluster model using the Jacobian-coordinate Gaussian-basis functions. The $\Lambda N$ interaction in $l = 1$ state, obtained from $G$-matrix in the QTM prescription, is used to calculate the $\Lambda \alpha$ potential from folding procedure. The $\Lambda \alpha$ potential so obtained does not fit the $B_{\Lambda}$ of $^9_{\Lambda}$Be in the $\Lambda \alpha \alpha$ model. Therefore, odd state $\Lambda N$ interaction is modified to fit the $B_{\Lambda}$ of $^9_{\Lambda}$Be and consequently, microscopically calculated $\Lambda \alpha$ potential becomes phenomenological one. The same $\Lambda \alpha$ potential yields the calculated $B_{\Delta \Lambda}$ of $^{10}_{\Lambda \Lambda}$Be in the $\Delta \Lambda \alpha \alpha$ model about 15% less than the currently accepted experimental [3] value $17.6 \pm 0.4$ MeV. Shoeb and coworkers [69, 70] have also analyzed energies of the ground and excited states (see tables 2.5 and 2.6) of the $p$-shell $\alpha$ cluster hypernuclei $^9_{\Lambda}$Be and $^{10}_{\Lambda \Lambda}$Be using
the VMC method. In these analyzes [69, 70], a $V_{\Lambda\alpha\alpha}$ phenomenological dispersive three-body $\Lambda\alpha\alpha$ force of Yukawa shape, following the analogy with the one suggested in explaining the spectra of $^{12}$C using the three-cluster $\alpha\alpha\alpha$ potential [73, 74, 75], has been proposed. The need for introducing $V_{\Lambda\alpha\alpha}$ was felt, in the light of results from microscopic calculations [34, 55, 76] that $(V_{\Lambda\alpha\alpha})$, the contribution of dispersive $\Lambda NN$ force from the two-alpha where a nucleon from each $\alpha$ is participating in the sixteen $\Lambda NN$ triads, is about 1 MeV, and thus makes it an important part of energy without which the $^9_{\Lambda}\text{Be}$ is overbound. A similar situation has been found in the case of $^{10}_{\Lambda\Lambda}\text{Be}$. Therefore, in the cluster model analyzes of these hypernuclei a $V_{\Lambda\alpha\alpha}$ force of Yukawa shape along with the appropriate $\Lambda\alpha$ and $\alpha\alpha$ potentials explains the ground and excited states energies of $^{9}_{\Lambda}\text{Be}$. However, for the system $^{10}_{\Lambda\Lambda}\text{Be}$, calculated ground state energy for $V_{\Lambda\alpha\alpha}$ force and a $\Lambda\Lambda$ potential that fits $B_{\Lambda\Lambda}$ of $^{9}_{\Lambda}\text{He}$, comes out to be about 15% higher than the currently accepted observed value [3]. Similar is the situation in the work of Hiyama et al. [71] where odd state $\Lambda N$ interaction has been modified to explain the ground state energies of $^{9}_{\Lambda}\text{Be}$ and $^{10}_{\Lambda\Lambda}\text{Be}$. This strengthens the speculation that in the measurement of $B_{\Lambda\Lambda}$, a $\gamma$-ray [77] of 3.04 MeV must have escaped undetected from the decay products of $^{10}_{\Lambda\Lambda}\text{Be}$ in the emulsion. The analysis of Shoeb [70] also explains the Demachi-Yanagi event [78] at energy 12.33$^{+0.30}_{-0.27}$ MeV as the $2^+$ excited state of $^{10}_{\Lambda\Lambda}\text{Be}$ which supports the finding of Hiyama et al. [71]. Thus cluster model calculations of Shoeb [70] and Hiyama et al. [71] for the energy of the $2^+$ excited state of $^{10}_{\Lambda\Lambda}\text{Be}$ incidentally agree with the $B_{\Lambda\Lambda}$ of Demachi-Yanagi event [78]. Thus, the Demachi-Yanagi event, based on the calculation of the cluster model, is identified with the $2^+$ excited state of $^{10}_{\Lambda\Lambda}\text{Be}$. Shoeb and coworkers [69, 70] have also calculated the rms radii of $\Lambda\alpha$, $\alpha\alpha$ and $\Lambda\Lambda$ pairs to gain further insight into the structure of the hypernuclei $^8_{\Lambda}\text{Be}$ and $^{10}_{\Lambda\Lambda}\text{Be}$. The calculation for quadrupole moment for $^9_{\Lambda}\text{Be}$ in the excited state by Shoeb [70] confirms the finding of Cravo et al. [79]. Here we remark that experimental $B_{\Lambda\Lambda}$ of $^9_{\Lambda}\text{He}$ and $^{10}_{\Lambda\Lambda}\text{Be}$ are not compatible as is evident from the work of Shoeb et al. [69] and Hiyama et al. [71].

From the analyzes mentioned in the preceding paragraph, we note that the model of Hiyama et al. [71] where the role of odd-state $\Lambda N$ interaction in the calculation of $\Lambda\alpha$ potential or of Shoeb [70] for dispersive cluster $\Lambda\alpha\alpha$ force, in explaining the energies of $^8_{\Lambda}\text{Be}$ and $^{10}_{\Lambda\Lambda}\text{Be}$, are equally satisfactory. Therefore, we can say that either the dispersive $\Lambda NN$ force is simulating the effect of odd state $l$-dependent $\Lambda N$ interaction or vice versa. It is possible that in future, a combination of two prescriptions may turn out to be a more appropriate option for analyzing the $p$-shell cluster hypernuclei as is evident from the microscopic analysis of shoeb et al. [55]. Here it is noteworthy to point out that the oldest calculation for the excited state of $^9_{\Lambda}\text{Be}$ is by Ali et al. [80].

The Bonn-Jülich [26] $\Lambda\Lambda$ potential for $S = -2$ sector has been used to describe the masses of hypernuclei $^6_{\Lambda}\text{He}$, $^{10}_{\Lambda\Lambda}\text{Be}$, $^{13}_{\Lambda\Lambda}\text{B}$, $^{12}_{\Lambda}\text{Ca}$, $^{92}_{\Lambda}\text{Zr}$ and $^{210}_{\Lambda\Lambda}\text{Pb}$ in the three-body $\Lambda\Lambda$-core model using Jastrow correlation for $\Lambda\Lambda$ pair. In this analysis, the role of long-range nuclear correlations (RPA) and $\phi$-exchange between two $\Lambda$s have been investigated in detail on the $B_{\Lambda\Lambda}$ of hypernuclei. The main conclusion of this analysis is that the calculated energies of the
systems $^6_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be are compatible with the experimental values which is contrary to the finding of Refs. [69, 71]. The variational energy of $^{13}_{\Lambda\Lambda}$B is also consistent with the observed value. The ground state energy for the remaining three heavier hypernuclei is predicted. Future measurement for the ground state energy of $^{10}_{\Lambda\Lambda}$Be may resolve the issue of the compatibility or incompatibility and thus the issue of suitability of the model for the calculation of $B_{\Lambda\Lambda}$ of $\Lambda\Lambda$ hypernuclei will emerge.

The many $BB$ states for the two-body systems are given in figure 1.11. The channels $\Lambda N \to \Sigma N$ and $\Lambda\Lambda \to \Xi N$ with a difference of about 80 MeV and 25 MeV, respectively, are likely to make the significant contributions in hypernuclei. Apart from these, there are other channels $\Sigma\Sigma$ and $\Lambda\Sigma$ for the case of $\Lambda\Lambda$. However, $NN \to \Delta N$ does not have that important role in nuclei as played by the aforesaid channels in hypernuclei because $\Delta N$ lies roughly 295 MeV above $NN$. In all the above mentioned analyzes [61, 65, 66, 67, 68, 69, 70, 71, 72] of hypernuclei, the aforementioned channels must have been simulated in an effective way. In this context, we discuss the main features of a full coupled channel SVM calculation of Nemura et al. [60] for the $s$-shell $\Lambda\Lambda$ hypernuclei $^4_{\Lambda\Lambda}$H, $^5_{\Lambda\Lambda}$H, $^7_{\Lambda\Lambda}$He, $^{10}_{\Lambda\Lambda}$He treating these as $A$-body systems. The trial wave functions include $\Lambda\Lambda$, $\Lambda\Sigma$, $N\Xi$ and $\Sigma\Sigma$ channels.
The $NN$ potential which describes reasonably well the energy of core nuclei $^2\text{H}$, $^3\text{H}$, $^3\text{He}$ and $^4\text{He}$, has been chosen. The Nijmegen group potentials D2' for $YN$ and simulated mND$_S$ and NF$_S$ for $YY$ interaction are used. SVM is used to calculate the energy. The energy of $^6\Lambda\Lambda$He is satisfactorily explained and other systems are predicted to be particle stable. In this calculation $\Lambda\Lambda$ component forms the dominant part of the wave function while $\Sigma\Sigma$ is very small part due to large mass difference ($m_{2\Sigma} - m_{\Lambda\Lambda} \approx 155 \text{ MeV}$) between the $\Lambda\Lambda$ and $\Sigma\Sigma$ channels and other channels $\Xi N$ and $\Lambda\Sigma$ make a small but significant contributions. The hypernucleus $^4\Lambda\Lambda$H is found to be particle stable just by 2 keV. Gal [81] has argued that giving the uncertainty in the theoretical input and in the calculation, $^4\Lambda\Lambda$H could may turn out unstable. Therefore, we feel a very sophisticated four-body calculation using realistic $BB$ interactions is desirable to decide the issue of stability till such a time when the information from the measurement about the existence of $^4\Lambda\Lambda$H also becomes available.

In nuclear physics, the spin-dependent parts of the $BB$ interaction are of major interest as these related to the modelling of the short-range part of the interaction which is yet to be understood unambiguously. From the recent measurement (figure 1.12) of $\gamma$ rays emitted in the nuclear transitions in $^9\Lambda\text{Be}$ in the BNL-AGS E930 experiment [82] the splitting of the $5/2^+$-3/2$^+$ levels was measured to be $31 \pm 2 \text{ keV}$. Thus for the $\Lambda$ particle, the spin-orbit interaction is about an order of magnitude smaller than for the nucleon. The calculation by Hiyama et al. [83] using meson exchange models for the spin-orbit splitting $5/2^+-3/2^+$ doublet in $^9\Lambda\text{Be}$ gave a value 80—200 keV, depending on the interaction used. On the other side, calculation based on quark model which naturally accounts for the short-range part of the interaction, predicts a much smaller values of 35—40 keV compared to the meson exchange model. In fact, the predicted features for the antisymmetric part of the spin-orbit force are different for the two types of models. However, we feel that quark models have not yet reached at a stage where it could provide a satisfactory description of the $YN$ interaction.

In yet another measurement of the spin-orbit splitting of the $3/2^--1/2^-$ in $^{13}\Lambda\text{C}$ at KEK, a value $152 \pm 54\,\text{(stat)} \pm 36\,\text{(syst)} \text{ keV}$ of doublet has been measured through with a coarse resolution on $\gamma$ rays [84]. In this case, the meson exchange models predict a splitting 390—960 keV, depending on the interaction used while quark based models [83] give 150—200 keV. Thus measurements [82, 84] related to the strength of the $\Lambda$ spin-orbit interaction have yet no satisfactory explanation as we noticed above. Here we may point out that the measured spectroscopic data (figure 1.2) for $^{89}\Lambda\text{Y}$, particularly at large value of $f_\Lambda$ suggest a larger spin-orbit splitting of 1.7 MeV. However a careful shell model analysis by Motoba et al. [85] shows that only 200 keV out of 1.7 MeV is due to the spin-orbit splitting, the remaining part is due to the mixing of different $\Lambda N^{-1}$ particle-hole excitations.

Apart from the aforesaid theoretical attempts in understanding the origin of spin-orbit force in hypernuclei, we mention the most recent explanation [86] for the very strong spin-orbit force in the case of nuclei and weak one in hypernuclei as a result of balancing mechanism between short- and long-range components of the force. According to Kaiser and Weise [86], the origin of spin-orbit interaction has three sources: (i) a short-range component arising
Figure 1.12: $\gamma$-ray energy spectrum from the deexcitation of the $^9$Be hypernucleus [82]. The energy difference ($31 \pm 2$ keV) between the two E2 ($5/2^+ \rightarrow 1/2^+$ and $3/2^+ \rightarrow 1/2^+$) transitions gives information on the strength of the $\Lambda$ spin-orbit interaction.

due to exchange of a scalar $\sigma$ boson and a vector $\omega$ boson between nucleons, (ii) a "wrong sign" spin-orbit term generated by the iterated OPE (the second-order tensor force), and (iii) a contribution from three-body spin-orbit interaction of Fujita-Miyazawa type generated by TPE with excitation of virtual $\Delta(1232)$-isobars. The contributions from the last two add up to almost negligibly small, leaving the contribution of the short-range origin, responsible for the strong spin-orbit mechanisms of nucleons in nuclei. In contrast, TPE three-body contribution for $\Lambda$ hyperons in hypernuclei is absent leading to an almost complete cancellation between the first two contributions and therefore, giving rise to a very weak $\Lambda$ spin-orbit splitting.

Very recently, Filikhin et al. [87] have performed the binding energy calculation for exploring the particle stability of strange and multi-strange $S = -2, -3$, and $-4$ $s$- and $p$-shell hypernuclei in the $\alpha$ cluster model using the Faddeev and FY methods. For simplicity, the four-body cluster model calculations were restricted to the $s$-wave approximation and the cluster reduction method was employed to solve the set of differential equations. Filikhin et al. have used two types of $\Xi^0\alpha$ potentials: the Isle [87] potential and the Woods-Saxon (WS) shape [88] potential. The system $^6\Lambda_{\Xi=0}$He has a negative energy for both types of $\Xi^0\alpha$ potentials but for $^7\Lambda_{\Lambda=0}$He hypernucleus, the binding energy is very strongly dependent on the radial shapes of $\Xi^0\alpha$ potential. For the Isle $\Xi^0\alpha$ potential, the system $^7\Lambda_{\Lambda=0}$He is unstable and on the other hand, for the WS shape it is strongly bound. This amounts to claiming that the higher momentum components which probe short-range behavior of the $\Xi^0\alpha$ potential are excited predominantly in the wave function of $^7\Lambda_{\Lambda=0}$He for Isle potential. Given the energy
scale involved in these hypernuclei, this result is very unlikely and raises the question: are the gross properties e.g. binding energy data capable of discriminating between the radial shapes of the central two-body $BB$ potentials? We may remark that from earlier analyzes [69, 70] of the $s$- and $p$-shell $\Lambda\Lambda$ hypernuclei, it has shown that the results for binding energies of $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be are approximately within 3% whether a soft core repulsive Ise [61] or purely attractive [68] $\Lambda\alpha$ potential is used. Therefore, we have examined the question raised just above, in chapter 2. Filikhin et al. [87] have also studied the $^{9}_{\Lambda\Sigma}$Be system and predicted to be particle stable for the two shapes of $\Xi^{0}_\Sigma$ potential. Earlier analysis [89] for $^{6}_{\Lambda\Sigma}$He in the cluster model using Faddeev method could not draw any definitive conclusion about its stability.

The nucleus $^{12}_C$ forms the nuclear core of many hypernuclei included in the present work. But the theoretical study of the properties of nucleus $^{12}_C$ in the $\alpha$ cluster model is considered to be an unsolved problem, as stated in Ref. [73], owing to the difficulty in constructing correct asymptotic wave function of the unbound two-body $\alpha\alpha$ system. Although the Faddeev method [73, 90] has been applied to explain the experimental binding energy (7.26 MeV) and root mean square (rms) radius (2.47 fm) of $^{12}_C$ for appropriate two-body $\alpha\alpha$ potential using strongly attractive 3-body $\alpha\alpha\alpha$ cluster force. The calculated binding energy [87, 90] of $^{13}_\Lambda$C in FY framework is consistent with the experimental value. This seems to be the result of choosing the purely attractive $\Lambda\alpha$ potential [87] as against the more realistic Ise one that has been extensively adopted in the analyzes of $\alpha$ cluster hypernuclei. There are other analyzes [34, 91] that have calculated the binding energy of $^{13}_\Lambda$C. We note that the binding energy of $^{13}_\Lambda$C has been found to have a wide variation depending on the choice of potentials and prescription adopted for calculating it. In the variational calculation with Jacobian-coordinate Gaussian-basis function Hiyama et al. [92] suggested a repulsive 3-body $\alpha\alpha\alpha$ cluster force coupled with a strongly attractive two-body $\alpha\alpha$ potential to explain the ground state properties of $^{12}_C$. The $\Lambda$-binding of $^{13}_\Lambda$C along with the other $\alpha$ cluster hypernuclei is reproduced by adjusting the odd state part of $YN$ $G$-matrix ($YNG$) interactions. In Ref. [87], $^{13}_\Xi$C system is also found to be stable for the very strongly attractive Woods-Saxon shape $\Xi^{0}_\Sigma$ potential.

The production of many $\Xi$-hypernuclei that are to be produced as day one experiment at J-PARC is likely to put a constraint of $\Xi^{-}N$ interaction of Nijmegen group. Therefore, recently few-body variational calculations for the prediction of the binding energies of the light $\Xi$-hypernuclei with cluster units $\alpha\alpha\Xi^{-}(^{12}_{\Xi}Be)$, $\alpha\alpha\Xi^{-}(^{11}_{\Xi}Li)$, $\alpha\alpha\Xi^{-}(^{16}_{\Xi}He)$, $\alpha\alpha\Xi^{-}(^{13}_{\Xi}H)$ were reported by Hiyama et al. [93] in the Gaussian expansion method. The $\Xi^{-}\alpha(t)$ interactions are obtained from folding of $\Xi^{-}N$ $G$-matrix interaction, derived for ESC04d and ND model of Nijmegen group for values of Fermi momentum corresponding to the average density of $\alpha(t)$, into the density of the systems. The interactions among $\alpha\alpha\alpha$, $\alpha\alpha\Xi$, $\alpha\Xi\Xi$, $\alpha\Xi\Xi$ and $\alpha\Xi\Xi$ are either constrained by the scattering phase shifts or by the bound state energies of the relevant cluster units. The systems $^{10}_{\Xi}Li$ and $^{7}_{\Xi}H$ are found to be particle stable but energy and width depend strongly on the Nijmegen model chosen for $\Xi^{-}N$ interaction. One may question the quantitative prediction of these calculations in view of the QTQ prescription.
and the approximation made in the calculation of $\Xi^{-}-\alpha(t)$ potentials.

In all the above-mentioned analyzes [48, 61, 68, 69, 70, 71, 73, 90], $\alpha$ clusters are treated
as rigid entities devoid of structure and thus the effects of the role played by the dynamical
correlations among the baryons are not explicitly manifested in the energy calculation. There-
fore, such studies are deficient in accommodating the realities of the internal structure of $\alpha$
and seem to be therefore, far from satisfactory. Earlier, Shoeb et al. [55] have calculated the
ground state energy of $^{\Lambda}_{3}\text{Be}$, treating it as a partial nine-body system in the $\Lambda + 2\alpha$ model.
In the analysis, a single potential parameters set of $C_p$, the strength and $\hat{c}$, the cut-off radius
of TPE $\Lambda NN$ force was used. A simple central two-body $\Lambda N$ and the three-body dispersive
and TPE $\Lambda NN$ forces constrained by the $\Lambda p$-scattering data and the observed energy of $^{\Lambda}_{3}\text{He}$
were employed in this analysis [55]. This work presupposes the existence of the $\alpha$ structure.
The two-body $NN$ correlations within $\alpha$ were explicitly incorporated. However, the effect of
$NN$ correlations between the nucleons in two $\alpha$ is simulated through $\alpha\alpha$ correlation. The
$NN$ antisymmetrization between two $\alpha$ as that has been ignored is taken care of through soft
repulsive core in the $\alpha\alpha$ potential [94]. The three-body $\Lambda NN$ correlations were included in the
trial wave function but $\Lambda N$ space-exchange correlations were ignored. From this study it was
concluded that the dispersive three-body $\Lambda NN$ force and space-exchange $\Lambda N$ potential can
not be determined uniquely. The presence of one masks the determination of other. However,
$B_\Lambda$ of $^{\Lambda}_{3}\text{Be}$ is satisfactorily explained.

The three-body $\Lambda\alpha\alpha$ cluster model for $^{\Lambda}_{3}\text{Be}$ has remarkable success [69, 70, 71] to its
credit. The simplicity offered by the cluster model makes it seemingly a serious alternative to
a partial nine-body model in explaining the observed energy of the ground and excited states
of $^{\Lambda}_{3}\text{Be}$. Moreover, application of an $\alpha$ cluster model to the excited state is expected to give
a better description than for the ground state, because the $\Lambda$ particle induces less distortion
in the core due to the extended $\alpha\alpha$ separation in $l = 2$ than for $l = 0$. Notwithstanding
these characteristics, the success of the cluster model may be assigned to the microscopic
calculation [34] which led to the inclusion of the phenomenological dispersive three-body $\Lambda\alpha\alpha$
force [69, 70] or to the improvement of the $\Lambda\alpha$ potential due to the modification of the odd
state $\Lambda N$ interaction [71].

Very recently, Zhou et al. [95] have made a detailed study of the ground state energies of
the complex open-shell non-spherical hypernuclei based on the deformed Shell Model Hartree-
Fock framework for the core nuclei covering baryon number $A_c$ in the range 7 to 207. The
rms radii of the cores in the hypernuclei in general decrease or remain constant as the number
of $\Lambda$ hyperon increases from one to two. About three decades old, Hartree-Fock calculation
of Žofka [96] for the $\Lambda$ hypernuclei containing deformed $N = Z$ even-even $A \leq 40$ nuclei
shows selfconsistent mutual deformation of both nuclear core and hypernuclear orbits. In
the light of the remark made by Žofka [96], one may question the application of Hartree-Fock
selfconsistent scheme for $p$-shell hypernuclei and thus reliability of the results for these systems
is questionable.

In the last, we would like to mention that a large number of analyzes [97, 98, 99, 100] have
been made within shell model framework for the $p$-shell hypernuclei. In Refs. [97, 100], Talmi type matrix elements of the two- and three-body forces have been investigated and are being modified and refined continuously from the input of the ground and excited states of $p$-shell hypernuclei. In these analyzes, the energy of a hypernucleus is expressed in terms of the four parameters; $\Delta$, $S_A$, $S_N$ and $T$ which are matrix elements of the spin-spin term $V_\sigma(r)$, the $\Lambda$-spin-dependent spin-orbit term $V_\Lambda(r)$, the nucleon-spin-dependent spin-orbit term $V_N(r)$ and the tensor term $V_T(r)$, respectively, for the $s_{\Lambda pN}$ wave function. In the recent analysis [100], it is found that all the level spacings of doublets (hypernuclear structure) observed so far ($\frac{3}{2}$Li($3/2^+, 1/2^+$), $\frac{3}{2}$Li($7/2^+, 5/2^+$), $\frac{5}{2}$Be($3/2^+, 5/2^+$), $\frac{5}{2}$B($7/2^+, 5/2^+$), $\frac{16}{10}$O($1-, 0^-$), $\frac{16}{10}$O($2^-, 1^-$)) can be well reproduced with a common set of parameters $\Delta=0.33$ (for $A=7$) or 0.43 (for $A=11, 16$) MeV, $S_A=-0.01$ MeV, $S_N=-0.4$ MeV, $T=0.02$ MeV. This parameter set [101] cannot explain energy spectra for $A=10, 11, 12$ hypernuclei. While in Refs. [98, 99], matrix elements for definite choice of the $\Lambda N$ forces were expressed in terms of the oscillator size parameters of lambda and nucleons which vary over the hypernuclei of $p$-shell. These authors could explain the ground state energies of selected $p$-shell hypernuclei including $\frac{5}{2}$He.

1.4 Objective of the Present Work

In the last few years, theoretical investigations for the study of the properties of hypernuclei have made tremendous progress using the data obtained from state-of-the-art experimental facilities. Now it has become possible to explore those areas of strange particles hypernuclei which hitherto were inaccessible. An extensive programme to produce and identify light hypernuclei with strangeness content $S=-1$ and $-2$ is being planned in the very near future at J-PARC and in a few years from now for $S=-1$ at DANCE and for $S=-3$ hypernuclei at PANDA. In future, we hope to have data on $S=-3$ and $-4$ multi-strange hypernuclei as well. The data on these systems are likely to constraint the parameters of $YN$, $YY$, $\Lambda \Xi$ and $\Xi \Sigma$ etc. interaction models [30, 102]. Therefore, in view of the above, it is interesting to theoretically investigate for the stability of the strange and multi-strange light $s$- and $p$-shell $\alpha$ cluster hypernuclei with the strangeness content referred to above using simulated $\Xi N$ potential of Nijmegen group, $Y \alpha$ and $\alpha \alpha$ phenomenological potentials constrained by the relevant data. Many analyzes [87, 88, 89, 93, 103, 104] on this theme have appeared in the literature. Such studies are likely to have implications on future experimental efforts in producing, identifying, and measuring the properties of strange and multi-strange systems. At present, there are no reliable experimental observations for the binding energy data of hypernuclei involving the $\Xi$-hyperon except those poorly measured from the emulsion experiments and tabulated in Ref. [105]. Nonetheless, these data may serve as a useful guide in theoretical analyzes. For example, $\Xi^+ C$ with a experimental $B_{\Xi^+}=18.1 \pm 3.2$ MeV is a good candidate for the quali-
1.4 OBJECTIVE OF THE PRESENT WORK

tative comparison of $B_{\Xi}$ of neighboring system obtained here. The $(\Xi^- + ^{12}\text{N})$ system, after subtracting coulomb energy $\approx 5.0$ MeV [87], has a $(B_{\Xi})_{\text{nuc}}$, nuclear component of binding, in the range 10.0 to 16.0 MeV. Therefore, $\frac{4}{3}$ \text{C} considered here, is expected to have $B_{\Xi}$ in the range just mentioned.

In the present work we have looked for the answer of the question raised (bold face sentence) in the preceding section and re-analyzed the issue of the existence [87] of strange and multi-strange $\alpha$ cluster hypernuclei already predicted and further enlarge the list of such systems by adding two more species. We shall make use of VMC method since it proved to be most successful by being able to be easily extended beyond four particles system compared to other methods [57, 58, 61, 71]. Furthermore, the secret of efficiency of the Monte Carlo method lies in use of an importance sampling technique which samples the most relevant part of the configuration space. We have, therefore, performed a VMC calculation of the binding energies of $6_{\Lambda=0}\text{He}$ and $7_{\Lambda=0}\text{He}$ and to check the sensitivity of the energy of $7_{\Lambda=0}\text{He}$ on the radial shapes of $\Xi^0\alpha$ potential. The two types of $\Xi^0\alpha$ potentials are considered; one is the modified Isle [87] potential and other has a WS shape [88], both give $B_{\Xi=0}$, the $\Xi^0$ binding in $\frac{4}{3}$ \text{He} equal to 2.09 MeV. However, these potentials differ in the low energy scattering parameters $(a_{\Xi=0}, r_{\Xi=0})$ (in fm) which for WS and Isle potentials are (4.85, 2.19) and (12.56, 2.93), respectively, and $R_{\Xi=0\alpha}$, $\Xi^0\alpha$ rms radii for the two are 3.36 fm and 3.45 fm. On physical ground it is expected that the modified Isle potential due to inner repulsive core has large scattering length and marginally larger $R_{\Xi=0\alpha}$ compared to the WS shape. Woods-Saxon $\Xi^0\alpha$ potential with attractive central depths 24.0 and 14.0 MeV is considered in the present work. The systems are analyzed for NSC97 $\Lambda\Xi^0$ potential and for a variety of three-range Gaussian $\Lambda\Lambda$ potentials simulated from models of Nijmegen group. Further, we have studied $\frac{9}{5}\text{Be}$ along with the two new hypothetical species $10_{\Lambda=0}\text{Be}$ and $11_{\Lambda=0}\text{Be}$. In view of the strong conversion $\Xi N \rightarrow \Lambda\Lambda$, we have looked afresh at the stability of $S = -3$ systems and investigate the stability of $S = -4$ hypernuclei consisting of $\Lambda\Lambda\Xi^0+(\text{nuclear core})$ in the light of suggestion [106] of the Pauli blocking of two $\Lambda$s in the decay of $\Xi N$. We have performed the cluster model VMC calculation to look for the $4^+$ rotational state of $10_{\Lambda\Lambda}\text{Be}$ to find out whether it is particle stable or not.

For the first time, the VMC method is applied to explain the ground state binding energy and rms radius of $^{12}\text{C}$, in the $\alpha$ cluster model because this forms the subsystem of hypernuclei $^{13}_{\Lambda}\text{C}$ and $^{13}_{\Xi}\text{C}$ included here. The attractive three-body $\alpha\alpha\alpha$ force is also required to fit the ground state energy and rms radius of $^{12}\text{C}$ for an $\alpha\alpha$ potential, similar to earlier works. The cluster model VMC calculation for the $^{13}_{\Lambda}\text{C}$ system is performed for Isle $\Lambda\alpha$ potential including a dispersive type three-body $\Lambda\alpha\alpha$ force as suggested by Shoeb et al. [69]. We would like to compare VMC results for $^{13}_{\Lambda}\text{C}$ with the experimental value and the earlier analyzes [34, 87, 90, 91, 92]. A symbol $\Xi$ appearing in this thesis implies $\Xi^0$ except otherwise stated. The value of energy for hypernuclei containing a $\Xi^-$ can be inferred after making coulomb corrections [87] in the value of the energy of hypernuclei of $\Xi^0$ and therefore, we have excluded these systems in the present work. A thorough study of the systems discussed here and in the
previous paragraph has been presented in chapter 2.

The satisfactory application of a partial nine-body problem [55] within the \( \Lambda + 2\alpha \) model in explaining the \( B_\Lambda \) of \( ^9_\Lambda \text{Be} \), though for a single set of potential parameters, motivated us to apply it to analyze the experimental [77] binding energy \( B_\Lambda = 3.67 \) MeV of the degenerate doublet (3+ / 2, 5+ / 2) of \(^9_\Lambda \text{Be} \). We may point out that antisymmetrization which has been ignored among the nucleons of two well separated \( \alpha \)s is being simulated through the soft repulsive core in the \( \alpha \alpha \) potential [73, 94]. Following the spirit of the earlier work [55], for the simplicity of the calculation, we have chosen simple \( BB \) potentials along with the corresponding simple correlation functions. We have also calculated the magnetic and quadrupole moments to gain further insight into the structure of the hypernucleus \(^9_\Lambda \text{Be} \). To our knowledge, this is the first application of VMC method for the calculation of the excited state and this study forms the subject matter of chapter 3. Preliminary results on the excited degenerate doublet of \(^9_\Lambda \text{Be} \) were presented earlier [107].

The success of a partial nine-body model [55] of \(^9_\Lambda \text{Be} \) in explaining the difference of the energy of the ground and excited states (chapter 3) has encouraged us to extend the model for the case of \(^{10}_{\Lambda \Lambda} \text{Be} \) and to investigate how far it succeeds in explaining the energy of the ground and 2+ excited states. The addition of a \( \Lambda \) to \(^9_\Lambda \text{Be} \) increases the \( B_{\Lambda\Lambda} \) of the resultant system, thus physically one expects a significant reduction in the \( \alpha \alpha \) separation in comparison to what has been found in \(^9_\Lambda \text{Be} \). The excited 2+ state of \(^{10}_{\Lambda \Lambda} \text{Be} \), assumed to be built on the \( \alpha \alpha \) relative \( l = 2 \), should have an extended structure compared to the ground state because of the centrifugal barrier. The \(^{10}_{\Lambda \Lambda} \text{Be} \) is treated as a partial ten-body problem in the \( \Lambda \Lambda + \alpha \alpha \) model where nucleonic degrees of freedom within \( \alpha \)s are taken into consideration ignoring the antisymmetrization between two \( \alpha \)s. Here we may further emphasized that central two-body \( \Lambda N \) and \( \Lambda \Lambda \) and the three-body dispersive and TPE \( \Lambda NN \) forces, constrained by the \( \Lambda p \)-scattering data and the observed ground state energies of \(^6_\Lambda \text{He} \) and \(^6_{\Lambda \Lambda} \text{He} \), are employed. Preliminary results for the ground state energy of \(^{10}_{\Lambda \Lambda} \text{Be} \) were reported earlier [108]. Quadrupole moment of the hypernucleus \(^{10}_{\Lambda \Lambda} \text{Be} \) in the 2+ state has also been predicted to know more about its structure. To our knowledge, this is the first time that \(^{10}_{\Lambda \Lambda} \text{Be} \) is being studied as a partial ten-body problem in VMC approach. The detail of this analysis forms chapter 4 of the present work.

For the calculations of energy of \(^9_\Lambda \text{Be} \) and \(^{10}_{\Lambda \Lambda} \text{Be} \), now we have two competing models: classical cluster and partial \( \Lambda (\pm 9, 10) \)-body models. In the former, internal structure is ignored while the latter takes into account the relevant correlations consistent with the philosophy of the model. Here it will be interesting to make a comparative discussion of the two approaches and we have reserved a chapter 5 for this purpose. Finally, in the last chapter, we have concluded the work presented in our thesis.