Analysis and Modeling of Slotline

7.1 Introduction

The slotline structures frequently appear in the monolithic and hybrid microwave integrated circuits. There is growing interest in the slotline as evidenced by its recent applications in the design of microwave components; such as integrated balun broadband printed dipole [156], tunable and switchable band-pass filters [71], six-port network [88], coplanar- slotline cross [89], periodic patch loaded slotline [70] etc. These slotline based structures are analyzed by several full-wave methods [62, 105,118,120-122,124,133] and the commercial EM-simulators [15,115]. The EM-simulators could be used to extract dispersive line parameters, including dielectric and conductor losses and characteristic impedance of a slotline. However, these results are difficult to adopt to stand-alone software using the slotline. It is also difficult to use these results with the circuit simulators. Moreover, several slotline circuits can be analyzed and designed using much simpler design oriented equivalent circuit approach that needs closed-form expressions for the line parameters of a slotline [102,107]. Thus, there is an engineering need for a closed-form model of the lossy slotline that can compute accurately line parameters of a slotline. The model should also compute the losses for both the thin and thick strip conductors. The slotline could also be considered on the non-planar surfaces, such as circular, elliptical, semi-circular and semi-ellipsoidal cylindrical surfaces.

In this chapter, we report an accurate integrated closed-form model that computes the slotline parameters \( i.e. \, \varepsilon_{\text{eff}}, \, Z_0, \, \alpha_d \, \text{and} \, \alpha_c \). The circuit model for a slotline is also developed to account for low frequency features. The validity of the proposed integrated model is tested over wide range of parameters: \( 100\, \text{MHz} \leq f \leq 60\, \text{GHz} \),
1.5\mu m \leq t \leq 50\mu m, 9.7 \leq \varepsilon_r \leq 20 and 0.02 \leq w/h \leq 1.0 against the full-wave and simulated results. All the developed models are further extended to the multilayered planar and non-planar (elliptical and circular cylindrical) slotline.

7.2 Comparison of Existing Closed-form Dispersion Models

The physical parameters of a slotline are illustrated in Fig.(7.1). It has substrate of relative permittivity $\varepsilon_r$, thickness $h$, finite width $b$ and conductor thickness $t$. Three kinds of the closed-form dispersion models for a slotline are suggested by the investigators in the open literature [26,67,102,107]. However, none of these models takes into account the effect of finite conductor thickness on $\varepsilon_{\text{eff}}$ and $Z_0$ of a slotline. In this section we have compared accuracy of three available closed-form dispersion models of zero conductor thickness against the common SDA results.

![Fig.(7.1): Geometry of a slotline](image)

Garg and Gupta [107], based on the graphical data of Mariani et. al. [32], reported closed-form models to compute the normalized frequency dependent guided wavelength $\lambda_g/\lambda_0 = 1/\sqrt{\varepsilon_{\text{eff}}(f)}$ and the frequency dependent characteristic impedance $Z_0(f)$ of a
slotline. The results of Mariani et. al. are based on the equivalent waveguide model supporting the quasi-TE\textsubscript{10} mode. The models show the cut- off phenomenon at lower frequency end. However, the cut- off frequency phenomenon is not supported by the SDA results [62]. The models of Garg and Gupta work for \(9.7 \leq \varepsilon_r \leq 20\), \(0.02 \leq w/h \leq 1.0\) \((0.0002 \leq w/\lambda_0 \leq 0.25/\sqrt{(\varepsilon_r - 1)})\), \(0.01 \leq h/\lambda_0 \leq 0.25/\sqrt{(\varepsilon_r - 1)}\).

Acclaimed average accuracy of the models is 2% against the graphical results generated from the equivalent waveguide model. Janaswamy and Schaubert [102] extended the closed-form models to compute \(\varepsilon_{\text{eff}}(f)\) and \(Z_0(f)\) of a slotline on the low permittivity substrate in the range \(2.22 \leq \varepsilon_r \leq 9.8\), \(0.0015 \leq w/\lambda_0 \leq 1.0\), \(0.006 \leq h/\lambda_0 \leq 0.06\) by curve-fitting the SDA results. Their models also have average accuracy of 2% and maximum error about 5% against the results of SDA.

We treat the combined form of two sets of models as a closed-form model - I i.e. CF-I. This model works for \(2.22 \leq \varepsilon_r \leq 20\), \(0.02 \leq w/h \leq 1.0\) \((i.e.\, 0.0002 \leq w/\lambda_0 \leq 0.25/\sqrt{(\varepsilon_r - 1)})\), \(0.01 \leq h/\lambda_0 \leq 0.25/\sqrt{(\varepsilon_r - 1)}\) and has an average accuracy of 2%. The combined form of the closed-form model - I is mentioned below to compute the frequency dependent \(\varepsilon_{\text{eff}}\) and \(Z_0\) of zero conductor thickness slotline:

**CF-I:**

\[
\varepsilon_{\text{eff}}(f, t = 0) = \left(\frac{\lambda_0}{\lambda_g}\right)^2 \cdot \lambda_{\text{g}} = \lambda_0 (A_1 + B_1 \cdot f_1 + C_1 \cdot f_2 + D_1 \cdot f_3 + E_1 + F_1 + G_1) \quad (a)
\]

\[
Z_0(f, t = 0) = A_2 + B_2 \cdot g_1 + C_2 \cdot g_2 + D_2 \cdot g_3 + E_2 + F_2 \quad (b)
\]

The expressions for the parameters \(f_1, f_2\) and \(f_3\) and the coefficients \(A_1, B_1, C_1, D_1, E_1, F_1\) and \(G_1\) in relative permittivity ranges \(2.22 \leq \varepsilon_r \leq 3.8, 3.8 \leq \varepsilon_r \leq 9.8\) and \(9.7 \leq \varepsilon_r \leq 20\) for the normalized slot-width \(0.0015 \leq w/\lambda_0 \leq 0.075\), \(0.075 \leq w/\lambda_0 \leq 1.0\), \(0.02 \leq w/h \leq 0.2\) and \(0.2 \leq w/h \leq 1\) are summarized in the Appendix – B.1. Likewise,
the expressions for the parameters \( g_1, g_2 \) and \( g_3 \) and the coefficients \( A_2, B_2, C_2, D_2, E_2 \) and \( F_2 \) are summarized in the Appendix – B.2. These parameters and coefficients are obtained from the rearrangement of the closed-form expressions of Garg and Gupta [107] and Janaswamy and Schaubert [102].

Krowne [26] has reported another closed-form model – CF-II, based on curve-fitting the full-wave results. This model is tested against the equivalent waveguide model [32] for parameter \( 9.6 \leq \varepsilon_r \leq 20, \ 0.02 \leq w/h \leq 2.0, \ 0.015 \leq h/\lambda_0 \leq 0.08 \). It has an average accuracy of \( \pm 3.7\% \) for \( \varepsilon_{\text{eff}}(f) \). However, for \( Z_0(f) \) the CF-II has average accuracy \( 4\% \) and maximum error \( 14.5\% \). This model is given below

\[
\frac{\varepsilon_{\text{eff}}(f,t=0)}{\varepsilon_r} = f_1(\varepsilon_r) \left\{ f_2 \left( \frac{w}{H} \right) + f_3 \left( \frac{w}{H} \right) \left( \frac{H}{\lambda} \right)^{-4(w/h)} + f_5 \left( \frac{w}{H} \right) \right\} \quad (a) \tag{7.2}
\]

\[
Z_0(f,t=0) = g_1 \left( \varepsilon_r, \frac{w}{H}, \frac{H}{\lambda} \right) \cdot g_2 \left( \frac{w}{H}, \frac{H}{\lambda} \right) \quad (b)
\]

The expressions for the parameters \( f_1, f_2, f_3 \) and \( g_1, g_2 \) are obtained from Krowne [26].

Svačina [67] has further reported another closed-form model- CF-III based on the conformal mapping method that assumes the quasi-TEM mode on a slotline. The effective relative permittivity computed by this model is tested against the equivalent waveguide model [32] in the range: \( 2.22 \leq \varepsilon_r \leq 20, \ 0.02 \leq w/h \leq 1.0, \ 0.01 \leq h/\lambda_0 \leq 0.25/\sqrt{(\varepsilon_r-1)} \) with maximum error \( 2.2\% \) for \( \varepsilon_{\text{eff}}(f) \). However, accuracy of the CF-III is not tested by Svačina for the characteristic impedance \( Z_0(f) \). The model is summarized below,
CF-III:

\[
\varepsilon_{\text{eff}}(f,t = 0) = 1 + \frac{\varepsilon_r - 1}{2} \frac{\gamma}{K(k_1)} \frac{K(k_1)}{K(k_0)} \quad (a)
\]

\[
Z_0(f,t = 0) = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0)}{K(k_0)} \quad (b)
\]

where modulus pairs \((k_0, k_0')\) and \((k_1, k_1')\) in terms of the physical parameters of a slotline are obtained from the equations-(3.91)-(3.93).

**Table- 7.1:** Average and maximum deviation of models against Kitazawa [122]

<table>
<thead>
<tr>
<th>Closed-form Models</th>
<th>% Deviation in (\varepsilon_{\text{eff}})</th>
<th>% Deviation in (Z_0(\Omega))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Av.</td>
<td>Max.</td>
</tr>
<tr>
<td>CF-I</td>
<td>0.67</td>
<td>0.88</td>
</tr>
<tr>
<td>CF-II</td>
<td>3.29</td>
<td>7.20</td>
</tr>
<tr>
<td>CF-III</td>
<td>2.03</td>
<td>3.89</td>
</tr>
</tbody>
</table>

Fig.(7.2) compares the frequency dependent effective relative permittivity \((\sqrt{\varepsilon_{\text{eff}}(f)})\) and frequency dependent characteristic impedance \((Z_0(f))\) as computed by above mentioned three models against the spectral domain analysis (SDA) of Kitazawa [122]. Fig.(7.2a) compares three models of frequency dependent effective relative permittivity \((\sqrt{\varepsilon_{\text{eff}}(f)})\) against the SDA results of Kitazawa up to 10 GHz. However, results for three models are presented up to 20 GHz. The model CF-I shows better agreement against the SDA results up to 10 GHz. In the lower frequency range, the computed \((\sqrt{\varepsilon_{\text{eff}}(f)})\) by the model CF-II is higher than that of the SDA results. In the higher frequency range both the models
CF-I and CF-II have almost identical results. The model CF-III assumes the quasi-TEM mode for a slotline. Its results deviate from the SDA results both at the lower and higher frequency ranges.

![Comparison of three models against results of SDA](image)

**Fig.(7.2)**: Comparison of three closed-form models against results of SDA [122] for \( t=0, \varepsilon_r=20, w/h=0.5 \): (a) Effective relative permittivity, and (b) Characteristic impedance.

Fig.(7.2b) compares three models of the frequency dependent characteristic impedance of a slotline against the SDA results of Kitazawa up to frequency 14 GHz [122]. The results of models are shown up to 20 GHz. The **CF-I shows better agreement as compared to the results of CF-II**. The CF-II has more deviation in the lower frequency range. However, the nature of variation in \( Z_0(f) \) for both these models is same as that of the SDA results. The CF-I, CF-II and the SDA results show the flattened peak of the characteristic impedance in the middle frequency range. At the higher frequency, \( Z_0(f) \) declines to lower value. The quasi-static model CF-III has large deviation in the lower frequency range. Moreover, it does not follow the nature of variation in the characteristic impedance at high frequency range. Outcome of the comparison is summarized in Table-7.1. We select the combined model CF-I due to Garg-Gupta [107] and Janaswamy-Schaubert [102] for our further investigations.
7.3 Conductor Thickness Dependent Propagation Characteristics

The effect of the conductor thickness on the slotline parameters is examined through several models in the reported combined model [98]. However, none of the suggested models for the effective slot-width is accurate in the wide frequency range from 2 GHz to 60 GHz. Thus, we have developed integrated model to account for the effect of conductor thickness on the frequency dependent effective relative permittivity \( \varepsilon_{\text{eff}}(f) \) and the frequency dependent characteristic impedance \( Z_0(f) \) of a slotline. The integrated model is valid in the range,

\[
2.22 \leq \varepsilon_r \leq 20, \quad 0.02 \leq w/h \leq 1.0 \quad \text{and} \quad 0.01 \leq h/\lambda_0 \leq 0.25/\sqrt{\varepsilon_r - 1}.
\]

The integrated closed-form model [147] has four components - the combined model to compute effective relative permittivity and characteristic impedance, improved model for the frequency and conductor thickness dependent effective slot-width, model to compute the dielectric loss and the Wheeler’s or perturbation method to compute the conductor loss. The integrated closed-form model computes these line parameters separately without taking into account their mutual interaction. Thus, this model is not in position to compute the dispersion in a slotline operating below 1 GHz. It is also not able to provide information on the imaginary part of the characteristic impedance that is associated with a lossy slotline. In order to account for these effects, we have developed the circuit model for a slotline. The line constants of the circuit model are determined from the integrated model. In this section we summarize the integrated model to compute \( \varepsilon_{\text{eff}} \) and \( Z_0 \) of a slotline structure with finite thickness of the strip conductor and losses are considered in the next section.

7.3.1 Effective Relative Permittivity

The expression for the conductor thickness dependent effective relative permittivity is based on the combined model [98]. It is given below
### Analysis and Modeling of Slotline

\[
\varepsilon_{\text{eff}}(f, t = 0) = \begin{cases} 
\varepsilon_{r} - 1 & \frac{t}{h} \leq \frac{f}{\lambda_0} < 6.67 \times 10^{-4} \\
\varepsilon_{r} - 1 - \left( \frac{t}{h} \right)^2 & 6.67 \times 10^{-4} \leq \frac{f}{\lambda_0} < 3.3 \times 10^{-3} 
\end{cases} 
\]

(7.4)

where, \( p_0 = 0.0006 f^2 - 0.0369 f + 0.7714 \)

(7.4)

\[
\varepsilon_{\text{eff}}(f, t = 0) = \left( \frac{\lambda_0}{\lambda_g} \right)^2 
\]

(7.4)

where, \( \lambda_g = \lambda_0 \left( A_1 + B_1 \cdot f_1 + C_1 \cdot f_2 + D_1 \cdot f_3 + E_1 + F_1 + G_1 \right) \)

(7.4)

In the above expression, \( f \) is frequency in GHz; \( t, h, w, \lambda_0, \lambda_g, \varepsilon_r \) are conductor thickness, substrate thickness, slot-width, free-space wavelength, guided wavelength and relative permittivity of a substrate respectively. In this expression, \( \varepsilon_{\text{eff}}(f, t = 0) \) can be computed using equation-(7.1a).

Fig.(7.3a) shows frequency dependent \( \varepsilon_{\text{eff}} \) of slotline on different substrates with \( \varepsilon_r = 2.5, 9.8, 20 \) and 37; \( w/h = 0.5; h = 1 \text{ mm and } t = 10 \mu\text{m} \). The frequency range is 0.1 GHz – 200 GHz. For the parameters specified, the integrated model is valid till 40 GHz and is within 4.8% average deviation against the results obtained from SDA and EM-simulators. The slotline is more dispersive on high permittivity substrates. Above 50 GHz EM-field is confined in the dielectric slot-region and \( \varepsilon_{\text{eff}} \rightarrow \varepsilon_r \). Fig.(7.3b) variation in \( \varepsilon_{\text{eff}} \) against strip conductor thickness at 10 GHz. \( \varepsilon_{\text{eff}} \) decreases with strip conductor thickness and effect is more obvious for the slotline on high permittivity substrate. The results of the model is in between results results of HFSS and Sonnet, closer to the results of CST. Fig.(7.3c) futher compares results of the model to compute \( \varepsilon_{\text{eff}} \) against slot-width \( w/h \) at 30 GHz and 60 GHz. Again results of the model are closer to the results of CST.
Fig. (7.3): Comparisons of $\varepsilon_{\text{eff}}(f,t)$ computed by integrated model against EM-simulators and SDA as a function of: (a) & (b) Frequency, (c) Conductor thickness, and (d) w/h ratio for slotline on various substrates.

Finally compared against each EM simulator for $2.5 \leq \varepsilon_{r} \leq 20$, $0.25 \mu m \leq t \leq 9 \mu m$ and $0.2 \leq w/h \leq 1$, the model has average and maximum deviation of (1.5%, 7.1%), (1.9%, 6.5%) and (1.4%, 6.2%) against HFSS, Sonnet and CST respectively. The slotline structure is realized in EM-simulators as a limiting case of the CPW which provides the consistent results as compared to the HFSS suggested wave-port excitation of a slotline [147]. The % deviation in $\varepsilon_{\text{eff}}$, as obtained from the integrated model and EM-simulators against SDA based results, is summarized in Table-7.2. The range of frequency and
substrates are also shown in the table. The average and maximum deviations of the results of Sonnet, HFSS and integrated model, against the full-wave results, are (2.3 %, 6.2%), (1.8%, 5%), and (0.96%, 4.4%) respectively. If we treat the results of the HFSS as the reference then the integrated model, Sonnet and SDA of Kitazawa [120]-[122] have average and maximum deviations as (1.85%, 3.72%), (4.12%, 7.15%) and (1.16%, 4.19%) respectively. We conclude that accuracy of the model is comparable that of the EM-simulators. However, model does not account for increase in \( \varepsilon_{\text{eff}} \) at lower frequency due to conductor losses. It is accounted through the circuit model.

**Table-7.2: % Deviation of \( \varepsilon_{\text{eff}}(f,t) \) & \( Z_0(f,t) \) computed by models and EM-simulators against results of SDA.**

<table>
<thead>
<tr>
<th>Models</th>
<th>( \varepsilon_{\text{eff}}(t) )</th>
<th>( Z_0(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%Av. error</td>
<td>%Max. error</td>
</tr>
<tr>
<td>Kitazawa [122] : ( \varepsilon_r = 20 ), ( w/h = 0.5 ), ( t = 0, 10, 20 ) and 50 ( \mu )m, 2-20 GHz</td>
<td>integrated model</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>HFSS</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>Sonnet</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>circuit model</td>
<td>0.98</td>
</tr>
<tr>
<td>Kitazawa [121] : ( \varepsilon_r = 9.8 ), ( h = 0.635 ) mm, ( t = 6 \mu)m, ( f = 2, 18 ) and 30 GHz</td>
<td>integrated model</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>HFSS</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>Sonnet</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>circuit model</td>
<td>0.32</td>
</tr>
<tr>
<td>Bornemann [82] : ( \varepsilon_r = 10.2 ), ( h = 0.1 ) mm, ( t = 3 \mu)m, ( f = 6-25 ) GHz</td>
<td>integrated model</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>HFSS</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>Sonnet</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>circuit model</td>
<td>0.89</td>
</tr>
<tr>
<td>Kitazawa [120] : ( \varepsilon_r = 12.8 ), ( w/h = 0.2-1.0 ), ( t = 3 \mu)m, ( f = 60 ) GHz</td>
<td>integrated model</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>HFSS</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>Sonnet</td>
<td>3.25</td>
</tr>
<tr>
<td></td>
<td>circuit model</td>
<td>1.45</td>
</tr>
<tr>
<td>overall performance</td>
<td>integrated model</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>HFSS</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>Sonnet</td>
<td>2.26</td>
</tr>
<tr>
<td></td>
<td>circuit model</td>
<td>0.73</td>
</tr>
</tbody>
</table>

### 7.3.2 Characteristic Impedance

The effect of a conductor thickness \( t \) on the frequency dependent characteristic impedance, from equation-(7.1b), is accounted by replacing the physical slot-width \( w \) in the functional parameters \( A_2 - F_2 \) and \( g_1 - g_3 \) by the equivalent slot-width, \( w_{\text{eq}} = w - \Delta w \).
Analysis and Modeling of Slotline

An empirical expression for the incremental width, $\Delta w$, is obtained by empirically modifying some expressions primarily applicable to a microstrip [98]. The data on the parameter $q_0$ have been obtained from the results of the SDA [120] - [122]. These are curve-fitted to obtain the following empirical expression:

$$\Delta w = \begin{cases} 
\frac{t}{2\pi} \left[ 1 + \frac{1}{\varepsilon_r} \right] \times \ln \left( \frac{10.872}{(t/\lambda_0)^{q_0} + \left( \frac{1}{\varepsilon_r} \right)\lambda_0^2 (w/h) + 1.1} \right), & \text{for } 4 \times 10^{-5} \leq \frac{t}{\lambda_0} < 3.6 \times 10^{-4} \\
\frac{t}{2\pi} \left[ 1 + \frac{1}{\cosh(\sqrt{\varepsilon_r}) - 1} \right] \times \ln \left( \frac{10.872}{(t/\lambda_0)^{q_0} + \frac{1}{\varepsilon_r} \lambda_0^2 (w/h) / \sqrt{6.517}} \right), & \text{for } 3.6 \times 10^{-4} \leq \frac{t}{\lambda_0} < 6 \times 10^{-4} \\
\frac{t}{\pi} \left[ 1 + \ln(4) - 0.5 \ln \left( \frac{t}{\lambda_0} \right)^{q_0} + \left( \frac{t}{\pi w} \right)^2 \right], & \text{for } 6 \times 10^{-4} \leq \frac{t}{\lambda_0} \leq 3.33 \times 10^{-3}
\end{cases}$$

where, $q_0 = \begin{cases} 2, & \text{for } f < 18\text{GHz} \\
0.092 \left( f^{0.4184} \right), & \text{for } f \geq 18\text{GHz}
\end{cases}$

(7.5)

In the above expression, frequency $f$ is in GHz. The validity of the integrated model to compute the characteristic impedance of a slotline, over the range $2.22 \leq \varepsilon_r \leq 20$, $0 \leq t \leq 50 \mu m$, $0.02 \leq w/h \leq 1.0$ and $2\text{GHz} \leq f \leq 60\text{GHz}$, has been tested against the full-wave results and the results obtained from HFSS and Sonnet. Fig.(7.4a) shows the dispersion in $Z_0$ of slotline on different substrates with $\varepsilon_r = 2.5, 9.8, 20$ and 37; $w/h = 0.5$; $h = 1$ mm and $t = 10 \mu m$ in the frequency range $0.1 \text{ GHz} - 200 \text{ GHz}$. The integrated model is within 4.3% average deviation against the EM-simulators in the frequency range $2 \text{ GHz} - 40 \text{ GHz}$. We have noted further that initially there is an increase in $Z_0$ but after a certain frequency point, the value starts decreasing and gradually saturates at higher frequencies.
The frequency for saturation decreases with increase in relative permittivity of substrates. All the three softwares support this trend in variation of $Z_0$ with frequency and are in close agreement with each other. Fig.(7.4b) and Fig.(7.4c) show variation characteristic impedance of a slotline with respect to conductor thickness, at 10 GHz and slot-width, at 30 GHz and 60 GHz, respectively. The results of model shows close agreement with results of EM-simulators. For $2.5 \leq \varepsilon_r \leq 20$, $0.25 \mu m \leq t \leq 9 \mu m$ and $0.2 \leq w/h \leq 1$, the
model has average and maximum deviation of (2.8%, 9.4%), (3.3%, 8.3%) and (1.8%, 6.1%) against HFSS, Sonnet and CST respectively.

The results of the integrated model follow the results of HFSS and the SDA closely over the significant range of frequency, slot-width and strip conductor thickness. The detailed comparisons are also shown in Table-7.2. The average and maximum deviations in the integrated model, HFSS and Sonnet are (1.67%, 4.34%), (2.52%, 5.64%) and (2.98%, 5.14%) respectively against the results of the SDA. If we take the HFSS as our reference, then % average and %maximum deviations in the integrated model, Sonnet and SDA of Kitazawa [120] - [122] are (2.53%, 4.47%), (0.25%, 1.41%) and (3.98%, 6.2%) respectively. In this case the integrated model is closer to the EM-simulators as compared to the results of the SDA.

### 7.4 Computation of Losses in Slotline

In this section, we present the closed-form models to compute the dielectric and conductor losses of a slotline.

#### 7.4.1 Dielectric Loss

Once we have computed the conductor thickness and frequency dependent effective relative permittivity of a slotline, its dielectric loss is computed from the following expression [12],

\[
\alpha_d = \frac{\pi}{\lambda_0} \frac{\varepsilon_r \left( \varepsilon_{eff} (f, t) - 1 \right)}{\sqrt{\varepsilon_{eff} (f, t) \cdot (\varepsilon_r - 1)}} \tan \delta \quad \text{Np/m} \quad (7.6)
\]

where, \(\lambda_0\), \(\tan \delta\), \(\varepsilon_r\) and \(\varepsilon_{eff} (f, t)\) are free-space wavelength, loss tangent, relative permittivity of the substrate and its effective relative permittivity respectively.
Fig. 7.5: Dielectric loss of slotline as a function of (a) Frequency, (b) Slot width, and (c) Conductor thickness.

Fig. (7.5) compares the computed dielectric losses by the integrated model against the full-wave results [121,124] and EM-simulators. Fig. (7.5a) presents the frequency dependent $\alpha_d$ of a slotline between 2 GHz- 30 GHz for the slot-width 0.25 mm, 0.5 mm and 0.7 mm. With increasing slot-width and frequency, $\alpha_d$ of a slotline increases. However, Fig. (7.5b) shows that $\alpha_d$ is not significantly influenced by the slot-width. At 2 GHz, $\alpha_d$ slightly lowers with increasing slot-width; whereas at 30 GHz it slightly increases with the slot-width. In Fig. (7.5c) results obtained from the computation of $\alpha_d$
by the model and the EM-simulators are compared for the conductor thickness range $0.25 \, \mu m \leq t \leq 0.9 \, \mu m$, against the full-wave results of Kitazawa et al. [121] and Rozzi et al. [124] respectively. The dielectric losses of a slotline obtained from the integrated model are less than those given by Rozzi et. al. [124]. However, dielectric losses computed by the model show closer agreement with the SDA results of Kitazawa et. al. [121]. The average and maximum deviations of the integrated model are 0.011 Np/m and 0.041 Np/m respectively. The maximum deviation occurs only at 30 GHz for 1 mm wide slot-width that decreases to 0.022 Np/m for the narrow slot-width 0.1 mm at 30 GHz. The EM-simulators are within 8% average deviation amongst themselves.

7.4.2 Conductor Loss

The conductor loss of the slotline is computed using two closed-form models:

(i) Wheeler’s incremental inductance formulation

(ii) Perturbation method

The models are compared and validated against the results from experimental data and EM-simulators for wide range of data.

• Wheeler’s Incremental Inductance Formulation

A practical slotline with finite width two-conductor structure supports the quasi-TEM type mode [26] that has no cut-off frequency [62]. The cut-off frequency shown in literature is due process of modeling of the slotline using the quasi-TE mode supporting equivalent waveguide model for a slotline [67]. Therefore, Wheeler’s incremental inductance formulation; that is suitable for $(t \geq 1.1 \delta_s)$ [6], is used below to compute the conductor losses of a slotline. Wheeler’s incremental inductance formulation is based on
the incremental change in the characteristic impedance due to the EM – field penetration [6]. In usual formulation of Wheeler’s model, the characteristic impedance with finitely thick strip conductors and the difference characteristic impedance $\Delta Z_0$ are taken on the air-substrate. However, combined model of slotline is not applicable for the relative permittivity less than 2.22. Thus we have to take the characteristic impedance with finitely thick strip conductors and the difference characteristic impedance $\Delta Z_0$ on dielectric substrate.

![Diagram](image)

**Fig.(7.6): Application of Wheeler’s inductance rule to compute conductor loss in slotline.**

The conductor loss of a slotline on the dielectric substrate is computed from the following expression

$$\alpha_c = \frac{\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_r, w, h, f, t)} \cdot \frac{\Delta Z_0(\varepsilon_r = 1, w, h, f, t, \delta_s)}{Z_0(\varepsilon_r = 1, w - \Delta w, h, f, t)} - \frac{\pi}{\lambda_0} \frac{\Delta Z_0(\varepsilon_r = 1, w, h, f, t)}{Z_0(\varepsilon_r, w - \Delta w, h, f, t)} \quad \text{Np/m (7.7)}$$

where, $\lambda_0$ is free-space wavelength. The dispersive effective relative permittivity, $\varepsilon_{\text{eff}}(\varepsilon_r, w, h, f, t)$, of slotline with conductor thickness is calculated from the equation- (7.4). The frequency dependent characteristic impedance $Z_0(\varepsilon_r, w - \Delta w, h, f, t)$ of a slotline with the conductor thickness, is computed from the equation- (7.1b). Fig.(7.6a) shows reduction in the slot- width of a slotline due to the finite conductor thickness. The
change in slot-width $\Delta w$ due to conductor thickness is computed by using equation-(7.5). Fig. (7.6b) shows that the skin-depth increases the slot-width by $\delta_s$ due to the field penetration all around the strip conductor. Likewise, the substrate height also increases by $\delta_s/2$. In case of a slotline with very thick substrate, influence of the substrate could be ignored. We further note that due to the skin-depth, the conductor thickness is also decreased by $\delta_s$. The slot-width $\Delta w'$ that accounts for the skin-depth also, is also computed by equation – (7.5). However, in this case the conductor thickness $t$ is replaced by $t' = t - \delta_s$.

The difference characteristic impedance, $\Delta Z_0$ of a slotline on the dielectric substrate, with and without field penetration is given by:

$$\Delta Z_0(\varepsilon_r, 1, w, h, f, t, \delta_s) = Z_0(\varepsilon_r, f, w-\Delta w+\delta_s, h+\frac{\delta_s}{2}, t)\sqrt{\varepsilon_{\text{eff}}(\varepsilon_r, f, w-\Delta w+\delta_s, h+\frac{\delta_s}{2}, t)}$$

$$- Z_0(\varepsilon_r, f, w-\Delta w, h, t)\sqrt{\varepsilon_{\text{eff}}(\varepsilon_r, f, w-\Delta w, h, t)}$$

The characteristic impedances and effective relative permittivity for the parameters shown above are computed from the combined model.

- **Perturbation method**

We consider the slotline as a limiting case of the CPW with large ground plane. The central strip conductor of the CPW is reduced to zero. We apply the stopping distance based perturbation method by Holloway and Kuester [21] to slotline. For CPW, it is discussed and improved in Chapter-5. The closed-form expression to compute the conductor loss of a slotline, with respect to Fig.(7.7) is given below:
Analysis and Modeling of Slotline

Fig.(7.7): The slotline with stopping distance.

\[ \alpha_c = \frac{R_{sm}}{16Z_0(f,t)K^2(k)w} \ln \left( \frac{2w}{\Delta} + 1 \left( \frac{w + \Delta}{w - \Delta} \right) \right) \text{ Np/m} \]  \hspace{1cm} (7.9)

where, \( w \) is the slot-width and \( \Delta \) is the stopping distance. The characteristic impedance \( Z_0 \) of a slotline with conductor thickness is calculated from equation-(7.5). The surface resistance \( R_{sm} \) of the strip conductor of finite thickness \( t \) is computed from equation-(5.18). The elliptic integral of the first kind \( K(k) \) is evaluated using the closed-form expressions [38]. Svačina has provided the following expression for the modulus \( k \) of a slotline [67]

\[ k = 2 \cdot \frac{a_0}{1 + a_0}, \hspace{0.5cm} k' = 1 - k^2 \text{ where } a_0 = \tanh \frac{\pi w}{2h} \] \hspace{1cm} (7.10)

The stopping distance (\( \Delta \)) for a slotline is shown in Fig.(7.7). We have developed closed-form expression for stopping distance applicable to a slotline. The data for the stopping distance is extracted from the full-wave results of Rozzi et al. [124] in the frequency range 2 GHz to 30 GHz on conductor loss of slotline [147]. The process of extraction of stopping distance is mentioned Appendix-A. The following expression for the stopping distance \( y \equiv (t/\Delta) \) with respect to the variable \( x \equiv (t/2\delta_s) \) over the range \( 0.816 < x < 7.3 \) is obtained on curve-fitting the extracted data:
Analysis and Modeling of Slotline

\( y = T_1 e^{T_2 x} \quad (a) \)
where, \( T_1 = 12.093 e^{(-25.101w)} \quad (b) \)
\( T_2 = -50w^4 + 30w^3 + 30w^2 - 0.691w + 1.0109 \quad (c) \) \hspace{1cm} (7.11)

where, the slot-width \( w \) is in mm.

Fig. (7.8) compares the normalized stopping distance \( t/\Delta \) of an isolated strip conductor computed by Holloway and Kuester [24] and the extracted stopping distance of a slotline. The nature of the stopping distance for a slotline is very much different from that of a microstrip, CPW etc. For a thin conductor \( i.e. \) for \( t/2\delta_i < 4 \), \( y \equiv (t/\Delta) \) is almost constant, approximately 0.2 - 0.3. For the thick conductor, it increases exponentially. The expression of the stopping distance is used to compute the conductor loss of a slotline with the conductor thickness, both less than and more than the skin-depth.

![Comparison of the stopping distance of slot line and isolated strip conductor.](image)

The validity of the perturbation method, a component of the integrated model, to compute the conductor loss of a slotline is tested against the full-wave results provided by Heinrich [133], Kitazawa [121] and Rozzi et. al. [124]. We have also validated the integrated model against the results of HFSS and Sonnet. In order to compare the results,
we have used the data over wide range of parameters, \( 2.22 \leq \varepsilon_r \leq 20, \ 0.1 \leq w/h \leq 1.0, \ 0 \leq t \leq 50 \mu m, \ 2 \text{GHz} \leq f \leq 60 \text{GHz}. \)

Fig.(7.9) presents some of the results of the comparisons. The results from the Wheeler’s incremental rules are also shown. The results of Kitazawa deviate much from other results. The integrated model (perturbation method) gives higher loss only at narrow width \( w = 0.1 \text{mm} \). The rest of the results are close to each other. The integrated model closely follows results of the EM-simulators- HFSS. It is observed that the inaccuracy in the Wheeler’s model increases in the mm-wave ranges and it does not work for the slot-width \( w < 0.25 \text{mm} \) and \( w > 0.6 \text{mm} \).

Table-7.3 compares deviation in the conductor loss computed by the present integrated model and Wheeler’s model against 5 sources. The integrated model \( i.e. \) the perturbation method has better accuracy than Wheeler’s model against the SDA and EM-simulators, even in the range where Wheeler’s model is functional. We have also taken the results of HFSS as reference and compared other results against them. Thus, the integrated model, results of Rozzi \( et. \ al. \), Kitazawa and Sonnet have average and maximum deviations
(4.91%, 7.37%), (5.76%, 8.97%), (15.1%, 27.1%), and (2.07%, 4.49%) respectively against the HFSS. These results show that the integrated model for computation of the conductor loss of a slotline is as effective as the full-wave methods and numerically it is much faster. The conductor loss and dielectric loss from integrated model together are used for the computation of total loss of the slotline structures, i.e.

\[ \alpha_T = \alpha_c + \alpha_d \quad dB / unit \ length \]  

(7.12)

Fig.(7.10) shows comparisons of total loss \( \alpha_T \) of slotline and its \( Q \)-factor as computed by the integrated model and EM-simulator. Fig.(7.10a) show \( \alpha_T \) on different substrates with \( \varepsilon_r = 3.78, 9.8 \) and 12.9; \( w/h = 0.5; h = 1 \) mm and \( t = 10 \) \( \mu \)m up to 200 GHz. For this slotline structure, the model is operational only up to 50 GHz. The integrated model, within its workable range, has 5.7% average deviation against the EM-simulators. Fig.(7.10b) shows the comparison of computed \( \alpha_T \) by the model against the simulators as a function of \( w/h \) ratio with % average and % maximum deviation of 6.1% and 14.2% respectively. The results are at 30 GHz and 60 GHz.

Fig.(7.10c) shows the variation of \( Q_u \) in slotline on different substrates with \( \varepsilon_r = 3.78 \) and 20; \( w/h = 0.5; h = 1 \) mm and \( t = 10 \) \( \mu \)m in the frequency range 0.1 GHz – 200 GHz. Again for this slotline structure, the model is operational only up to 50 GHz. Fig.(7.10d) compares computation of \( Q_u \) for \( \varepsilon_r = 9.8 \) and 12.8; as a function of \( w/h \) ratio at at 30 GHz and 60 GHz. Overall average and maximum deviations of the integrated model, within its workable range, against EM-simulators are 5.9% and 16.6% respectively.
Table 7.3: Deviation of conductor loss computed by integrated model and Wheeler’s Model against 5 sources

[Data range: t = 0 – 50 µm; εr = 2.22 - 20; w/h = 0.2 – 1; Freq=2 GHz-60 GHz]

<table>
<thead>
<tr>
<th>Closed-form models</th>
<th>Rozzi [124] (Np/m)</th>
<th>Kitazawa [121] (Np/m)</th>
<th>Heinrich [133] (Np/m)</th>
<th>HFSS (Np/m)</th>
<th>Sonnet (Np/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheeler’s model</td>
<td>0.057</td>
<td>0.42</td>
<td>0.39</td>
<td>0.72</td>
<td>0.11</td>
</tr>
<tr>
<td>Integrated model</td>
<td>0.046</td>
<td>0.19</td>
<td>0.13</td>
<td>0.69</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Fig. (7.10): Total loss, as a function of (a) Frequency and (b) w/h ratio and Q factor, as a function of (c) Frequency and (d) w/h ratio for slotline on various substrates.
7.5 Closed-form Dispersion and Loss Models for Multilayer Slotline

A serious handicap in slotline circuit analysis and design is that the various methods of analysis and closed-form models for single-layered slotline do not lead to computation and analysis of line parameters of multilayered slotline. Only Svačina [67] has applied conformal mapping technique to obtain dispersion characteristics of multilayer slotline. The utilization of multilayered structures in slotline is desirable in order to introduce flexibility in the design of such structures, e.g. in the values of characteristic impedance, effective relative permittivity. Consequently, in multilayered transmission structures, further improvements can be achieved, such as the reduction of high-speed signal distortion [67]. The cross-sectional view of a slotline on multilayer dielectric substrates, slotline on composite substrate, is shown in Fig.(7.11).

![Multilayered Slotline: composite substrate.](image)

For slotline on composite substrate, \( \varepsilon_{\text{eff}} \) and \( Z_0 \) are given by:

\[
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2} \left[ \frac{K(k_1') K(k_0')}{K(k_1') K(k_0')} \right] + \frac{\varepsilon_{r2} - 1}{2} \left[ \frac{K(k_2') K(k_0')}{K(k_2') K(k_0')} \right] \quad (a) \\
Z_0 = \frac{6\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0)}{K(k_0')} \quad (b)
\]

where modulus \( k_0 \) and \( k_i \) (i=1,2) along with their complements can be computed using equation-(3.92) and (3.99) respectively.
We have extended the conductor thickness and frequency based closed-form models for $\varepsilon_{\text{eff}} (f,t), Z_0(f,t), \alpha_c$ and $\alpha_d$ of single-layered slotline to multilayer slotline by first converting multilayer into single-layer of finite dielectric thickness using SLR discussed in Chapter-4:

- **Effective relative permittivity**

$$
\varepsilon_{\text{eff}} (f,t) = \begin{cases} 
\varepsilon_{\text{req}} - \frac{t}{h_{\text{eq}}} & \text{for } 4 \times 10^{-5} \leq \frac{t}{\lambda_0} < 6.67 \times 10^{-4} \\
\varepsilon_{\text{req}} - \frac{t}{h_{\text{eq}}} - \left( \frac{t}{\lambda_0} \right)^2 & \text{for } 6.67 \times 10^{-4} \leq \frac{t}{\lambda_0} < 3.3 \times 10^{-3} 
\end{cases} \quad (a)
$$

where, $p_0 = 0.0006 f^2 - 0.0369 f + 0.7714$

- **Characteristic impedance**

$$
\Delta w = \begin{cases} 
\frac{t}{2\pi} \left[ 1 + \frac{1}{\varepsilon_{\text{req}}} \right] \ln \left( \frac{10.872}{(t/\lambda_0)^q_0 + (1/\pi) + 1} \right) & \text{for } 4 \times 10^{-5} \leq \frac{t}{\lambda_0} < 3.6 \times 10^{-4} \\
\frac{t}{2\pi} \left[ 1 + \frac{1}{\cosh \left( \varepsilon_{\text{req}} - 1 \right) \sqrt{6.517 (w/h_{\text{eq}})} \right] \ln \left( \frac{t}{\lambda_0} \right) \coth^{-1} \left( \frac{1}{\sqrt{6.517 (w/h_{\text{eq}})}} \right) & \text{for } 3.6 \times 10^{-4} \leq \frac{t}{\lambda_0} < 6 \times 10^{-4} \\
\frac{t}{\pi} \left[ 1 + \ln(4) - 0.5 \ln \left( \frac{t}{\lambda_0} \right) \left[ \frac{t}{\lambda_0} \right] \right. & \text{for } 6 \times 10^{-4} \leq \frac{t}{\lambda_0} < 3.33 \times 10^{-3} 
\end{cases}
$$

where, $q_0 = \begin{cases} 
2, & \text{for } f < 18 \text{GHz} \\
0.092 \left( f/0.4184 \right), & \text{for } f \geq 18 \text{GHz} 
\end{cases} \quad (b)$
• Conductor loss using Wheeler’s incremental inductance rule

\[
\alpha_c = \frac{\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_{\text{req}}, w, h_{\text{eq}}, f, t)} \cdot \frac{\Delta Z(\varepsilon_{\text{req}} = 1, w, h_{\text{eq}}, f, t, \delta_s)}{Z_0(\varepsilon_{\text{req}} = 1, w - \Delta w, h_{\text{eq}}, f, t)} = \frac{\pi}{\lambda_0} \frac{\Delta Z(\varepsilon_{\text{req}} = 1, w, h_{\text{eq}}, f, t, \delta_s)}{Z_0(\varepsilon_{\text{req}} = 1, w - \Delta w, h_{\text{eq}}, f, t)} \text{ Np/m}
\]

where

\[
\Delta Z(\varepsilon_{\text{req}} = 1, w, h, f, t, \delta_s) = Z_0(\varepsilon_{\text{req}}, f, w - \Delta w', h + \frac{\delta_s}{2}, t) \sqrt{\varepsilon_{\text{eff}}(\varepsilon_{\text{req}}, f, w - \Delta w', h + \frac{\delta_s}{2}, t)} - Z_0(\varepsilon_{\text{req}}, f, w, h, t) \sqrt{\varepsilon_{\text{eff}}(\varepsilon_{\text{req}}, f, w - \Delta w, h, t)}
\]

(c)

• Conductor loss using Perturbation method

\[
\alpha_c = \frac{R_{sm}}{16 Z_0(f, t) K^2(k) w} \ln \left( \frac{2w}{\Delta} + 1 \right) \left( \frac{w + \Delta}{w - \Delta} \right) \text{ Np/m}
\]

(d)

• Dielectric loss

\[
\alpha_d = \frac{\pi}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(f, t)} \frac{\varepsilon_{\text{req}}(f, t) - 1}{\tan \delta_{eq}} \text{ Np/m}
\]

(7.14)

where \( h_{eq} \) is the total substrate thickness between strip conductors and bottom layer of the multilayer substrate. \( \varepsilon_{\text{req}} \) and \( \tan \delta_{eq} \) of the equivalent single-layer substrate slotline are obtained from equation-(4.28).

Fig.(7.12a) – (7.12d) shows comparison and validity of SLR-based computed line parameters of slotline on composite substrate against EM-simulators for two different structures in the frequency range 1 GHz – 60 GHz. The computed \( \varepsilon_{\text{eff}}, \) Fig.(7.12a), and
Zo, Fig.(7.12b), by the model, has average and maximum deviation of (3.6%, 8.8%) and (4.2%, 7.5%) respectively against both the EM simulators. In Fig.(7.12c) and (7.12d), show variation in $\alpha_c$ and $\alpha_d$ of composite substrate slotline with respect to frequency. Overall, average and maximum deviation in Wheeler and integrated model (perturbation method) against EM-simulators are (15.3%, 35.8%) and (5.8%, 12.6%) respectively. The computed $\alpha_d$ by the model is in close agreement with both the EM simulators for frequency range 1 GHz – 60 GHz, with average deviation of 3.4%. Thus, the integrated model results are in close agreement with the EM-simulators for all the line parameters.

![Graphs showing analysis results](image)

Fig.(7.12): Multilayer slotline: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.
7.6 Closed-form Dispersion and Loss Models for Non-Planar Slotline

The closed-form expressions for dispersion and losses presented in the previous sections for planar slotline could be adopted to the slotline with finite strip conductor thickness on the circular and elliptical cylindrical surfaces. In this section, we present the conductor thickness and frequency dependent closed-form models of line parameters for both single-layer and multilayer non-planar slotline, shown in Fig.(7.13) and (7.15).

7.6.1 Single-Layer Case

Fig.(7.13) presents the slotline on the elliptical and circular cylindrical surfaces with two different finite ground plane widths. The structural parameters of SC - transformed ES/CS into the corresponding planar slotline are given by equation-(3.96):

\[
\begin{align*}
  w &= 2\psi \quad (i) \\
  h &= \ln \frac{c_2}{c_1} = \ln \frac{a_2 + b_2}{a_1 + b_1} \\
  t &= \ln \frac{c_3}{c_2} = \ln \frac{a_3 + b_3}{a_2 + b_2} \\
\end{align*}
\]  

(7.15)

The detailed definition of the above mentioned parameters are given in Chapter-3. In our study, we have used the conductor thickness independent expressions for \( \varepsilon_{\text{eff}} \) and \( Z_0 \) of the planar slotline by Svačina [67] and applied them to the non-planar slotline for computation of line parameters:

\[
\begin{align*}
  \varepsilon_{\text{eff}} &= 1 + \frac{\varepsilon - 1}{2} \frac{K(k_1')}{K(k_1)} \frac{K(k_0')}{K(k_0)} \quad (a) \\
  Z_0 &= \frac{60\pi}{\varepsilon_{\text{eff}}} \frac{K(k_0')}{K(k_0')} \quad (b) \\
\end{align*}
\]  

(7.16)

where, modulus \( k_0 \) and \( k_1 \) along with their complementary modulus \( k_1' \) and \( k_0' \) for finite ground plane widths \( (2\pi - 2\psi) \) and \( (\pi - 2\psi) \), are obtained from equations-(3.42) - (3.44) and equations-(3.55)-(3.56) respectively.
Analysis and Modeling of Slotline

Fig. (7.13): Slotline on the curved surfaces: (a) Elliptical Slotline (ES), (b) Circular Cylindrical Slotline (CS), (c) Semi-Elliptical Slotline (SES) and (d) Semi-Circular Cylindrical Slotline (SCS).

In order to compute the conductor thickness and frequency based $\varepsilon_{\text{eff}}(f,t)$ and $Z_0(f,t)$ of the ES and CS lines, we have to modify equations-(7.4) and (7.5) by using equation-(7.15) to get an equivalent slot-width ($\psi_{\text{eq}}$) for the ES / CS:

$$\psi_{\text{eq}} = \psi - \Delta\psi$$  \hspace{1cm} (7.17)

where $\Delta\psi$ is obtained from equation-(7.5) after substituting equation-(7.15) in it. The modulus $k_0$ and $k_1$ will be modified into $k_{0,t}$ and $k_{1,t}$ along with their complementary modulus, in which $\psi$ will be replaced by $\psi_{\text{eq}}$. 
Using equation-(7.6) along with the frequency and conductor thickness dependent $\varepsilon_{\text{eff}}(f,t)$, $\alpha_d$ of ES and CS is computed. For $\alpha_c$ computation of non-planar slotline, firstly Wheeler’s incremental inductance formulation is modified using equation-(7.15):

$$\alpha_c = \frac{\pi}{\lambda_0} \left[ \varepsilon_{\text{eq}}, \ln \frac{a_2 + b_2}{a_1 + b_1}, f, \ln \frac{a_3 + b_3}{a_2 + b_2}, \delta_s \right] \text{ Np/m } \quad (7.18)$$

Then integrated model for conductor loss of ES/CS with ground plane width $(2\pi - 2\psi)$ is obtained by using equation – (7.15) with equation – (7.9):

$$\alpha_c = \frac{R_{sm}}{32Z_0(f,t)K^2} \frac{\ln \left( \frac{4\psi + A}{4\psi - A} \right)}{\psi^2} \text{ Np/m } \quad (7.19)$$

When $\pi$ is replaced by $\pi/2$, integrated model for non-planar slotline with ground plane width $(\pi - 2\psi)$ is obtained. We have tested the accuracy of the closed-form models developed for propagation characteristics of non-planar slotline for both $(2\pi - 2\psi)$ and $(\pi - 2\psi)$ ground plane width, against the results obtained from EM-simulators-HFSS and CST, as shown in Fig.(7.14).

Fig. (7.14a) and (7.14b) give $\varepsilon_{\text{eff}}$ and $Z_0$ of ES and CS structures over the conductor thickness range 0.25 µm – 9 µm. For simulation, we have taken the substrate with $\varepsilon_s=20$, $\psi = 40^\circ$, $h = 1$ mm and $f = 30$GHz. For ES, the ellipticity $c = 0.7$ is considered. With increase in the ellipticity and decrease in the ground width, there is increase in both $\varepsilon_{\text{eff}}$ and $Z_0$. The closed-form model follows closely the results of both HFSS and CST.
with % average and % maximum deviation of 2.5% and 7.9 % respectively. Both $\varepsilon_{eff}$ and $Z_0$ decreases with increase in the conductor thickness.

![Graphs showing comparison of effective relative permittivity, characteristic impedance, and total loss](image)

**Fig.(7.14):** Comparison of: (a) Effective relative permittivity, (b) Characteristic impedance, and (c) Total loss of non-planar slotline.

Fig. (7.14c) further compares the total loss of the ES and CS for $\varepsilon_r=20$, $\psi = 40^\circ$, $f = 10$GHz, $h = 1$ mm, $\tan \delta = 0.002$ and $\sigma = 4.1 \times 10^7$ S/m for conductor thickness range 0.25μm - 10μm. The closed-form model # 1 (= Wheeler + $\alpha_{eq}$) fails to compute for $t < 1.1\delta_x$. The closed-form model # 2 (integrated model) is in close agreement with
HFSS at higher frequencies. The % average and % maximum deviation of model #1 and model # 2 against results of HFSS are (17.8%, 39.6%) and (5.38%, 24.4%) respectively.

7.6.2 Multilayer Case

In this section, we have extended the improved closed-form models for the line parameters of single-layered, equations-(7.16)-(7.19), to multilayer non-planar slotline by applying SLR technique and using equation-(7.14). The structural parameters of multilayer structure obtained from SC-transformation (discussed in Chapter-3) are:

\[ w = 2\Psi \quad (i) \quad t = \ln \frac{r_4}{r_3} \quad (ii) \quad h_1 = \ln \frac{r_3}{r_2} \quad (iii) \quad h_2 = \ln \frac{r_2}{r_1} \quad (iv) \]

(7.20)
We have computed $\varepsilon_{eff}$ and $Z_0$ of multilayer slotline by using equation-(7.13). The modulus $k_0$ and $k_i$ ($i=1,2$) along with their complementary modulus are obtained from equations- (3.42) - (3.44) and equations-(3.55) - (3.56) respectively. The parameter $H$ in equation – (3.45) will be replaced by $h_i$ ($i=1,2$) accordingly.

Fig. (7.16): Multilayer non-planar slotline: (a) Effective relative permittivity, (b) Characteristic impedance, and (c) Total loss.

Fig. (7.16) shows comparison and validity of SLR-based computed line parameters of multilayer non-planar slotline with $\varepsilon_r1=10, \varepsilon_r2=3.78$ and $\psi=45^\circ$ against EM-simulators.
Analysis and Modeling of Slotline

for frequency range 1 GHz – 60 GHz. The computation of $\varepsilon_{eff}$ and $Z_0$ by the model has % average and % maximum deviation of (4.3%, 9.6%) and (4.8%, 7.9%) respectively against both the EM simulators, as shown in Fig.(7.16a) and (7.16b).

In Fig.(7.16c) variation in $\alpha_T$ of multilayer non-planar slotline for conductor thickness range 0.25µm - 10µm at $f = 30$ GHz is shown. The model #2 is closer to simulated results as compared to the results of model #1. The % average and % maximum deviation of closed-form model #1 and closed-form model #2 against EM-simulators are (15.8%, 35.5%) and (5.4%, 15.4%) respectively. It is observed that losses in non-planar slotline are higher than losses in planar slotline.

7.7 Circuit Model of Slotline

In this section we present the circuit model of a slotline that account for its low frequency features. We note that all the line parameters are frequency and conductor thickness dependent. The accuracy of the circuit model of a slotline is tested against the EM-simulators- HFSS and Sonnet. First we obtain the $RLCG$ line parameters, both from the integrated model and EM-simulators, HFSS and Sonnet. The comparisons are shown in Fig.(7.17). For line resistance, the circuit model shows deviation between 2 GHz and 7 GHz. The circuit model shows good agreement with the results for EM-simulators, for line inductance even at low frequency. However, line capacitance computation, at low frequency, shows deviation; even though nature of variation is identical to the one obtained from the EM-simulator. Again computation of line conductance shows very good agreement with the results of EM-simulators.

Fig.(7.18) shows characteristics of slotline on the substrate with $\varepsilon_r = 12.9$, $w/h = 0.5$, $h = 1$ mm and $t = 6$µm. The frequency range is 0.01 GHz – 10 GHz. The effects of the finite conductivity of the strip conductor on the line parameters are visible from 1 GHz
downwards. Fig.(7.18a), using the circuit model and EM-simulator, shows that $\varepsilon_{eff}$ of a lossy slotline increases with decreasing frequency. The integrated model is not able to consider such effect. Fig.(7.18b), using the circuit model and EM-simulator, shows that its attenuation decreases with decrease in frequency. Again the results of integrated model show much deviation. We can summarize that the results of the circuit model are between the results of two EM-simulators. The results of the integrated model are not acceptable below 1 GHz. We also note that the circuit model significantly improves the computation of the conductor loss.

**Fig.(7.17):** Extraction of RLCG parameters of lossy planar slotline using circuit model and EM-simulators.
Fig.(7.18c), using the circuit model and EM-simulator, shows that the real part of \( Z_0 \) of a slotline also increases significantly with a decrease in frequency below 1 GHz. No such increase is shown by the integrated model. Finally Fig.(7.18d) shows computation of imaginary of characteristic impedance that is not possible with integrated model. The increase is shown by the integrated model. Finally Fig.(7.18d) shows computation of imaginary of characteristic impedance increases with frequency and its nature also changes. The results of circuit model and simulators are in close agreement.

![Graphs showing line parameters comparison](image)

**Fig.(7.18):** Comparison of line parameters of lossy planar slotline: (a) \( E_{\text{eff}}(f,t) \), (b) \( \alpha_f(f,t) \), (c) \( \Re(Z_0^*(f,t)) \) and (d) \( \Im(Z_0^*(f,t)) \).
Table-7.4 consolidates the comparison of the models and full-wave results against HFSS for line parameters of slotline on the alumina substrate. We note that both EM-simulators have high deviation \( w.r.t. \) \( \varepsilon_{\text{eff}}(f,t) \)– average deviation of 4.12% and maximum deviation of 7.15%. The full-wave results of both Kitazawa [121] and Rozzi et. al. [124] have high deviations. In this respect integrated model has acceptable average and maximum accuracy for \( \varepsilon_{\text{eff}}(f,t), Z_0(f,t) \) and loss- (1.85%, 3.72%), (2.53%, 4.47%) and (4.91%, 7.37%) respectively. The accuracy, for computation of \( \varepsilon_{\text{eff}}(f,t), Z_0(f,t) \) and loss, further improves in the circuit model to (1.62%, 3.02%), (2.11%, 3.45%) and (2.15%, 6.92%) respectively.

**Table-7.4:** % Deviations of models against HFSS  
[Data range: \( \varepsilon_{r} = 9.8, t = 6 \mu m, \sigma = 4.1 \times 10^7 \text{ S/m, } w/h = 0.5 \)]

<table>
<thead>
<tr>
<th>Model</th>
<th>( \varepsilon_{\text{eff}} )</th>
<th>( Z_0(\Omega) )</th>
<th>( \alpha_f ) (Np/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated Model</td>
<td>1.85</td>
<td>3.72</td>
<td>2.53</td>
</tr>
<tr>
<td>Kitazawa [120-122]</td>
<td>1.16</td>
<td>4.19</td>
<td>3.98</td>
</tr>
<tr>
<td>Sonnet</td>
<td>4.12</td>
<td>7.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Rozzi [124]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Circuit Model</td>
<td>1.62</td>
<td>3.02</td>
<td>2.11</td>
</tr>
</tbody>
</table>