Analysis and Modeling of Coplanar Strip Line

6. 1 Introduction

Coplanar Strip-lines (CPS) are used extensively in the MMIC and integrated optics applications. The CPS is a balanced transmission line and makes more efficient use of the wafer area than CPW. The CPS is used for the design of filters, antennas, mixers and opto-electronic devices. It has been adopted to the non-planar surfaces also. In this context, the elliptical coplanar strip lines (ECPS) and cylindrical coplanar strip lines (CCPS) have been examined. The quasi TEM parameters for ECPS and CCPS have been analyzed and analytical expressions for the losses of the planar symmetrical CPS are reported [48, 93- 94, 127, 143, 154]. The losses and dispersion of these non-planar CPS are not reported. The full wave methods like SDA [102], variational method [153] and mode matching [134] have been used to obtain the frequency dependent characteristic impedance and effective relative permittivity of the CPS line.

In this chapter, we present the modeling of conductor thickness and frequency dependent line parameters, dispersion and losses, of different configurations of CPS. All developed models are verified against available experimental results and the results obtained using the EM- simulators over wide range of parameters. The effect of strip asymmetry on the characteristics of CPS is also analyzed. All the developed models are further extended to the multilayered planar and non-planar (elliptical and circular cylindrical) CPS. Finally a circuit model, applicable to these structures, is also presented. It accounts for the effect of finite strip conductivity on the propagation constant and also on the characteristic impedance at lower range of microwave frequency.
6.2 Effect of Conductor Thickness on Characteristics of CPS

The symmetrical coplanar strip (CPS) line, shown in Fig.(6.1), consists of two finite widths $w$ metallic strips separated by a slot-gap of width $s$, substrate thickness $h$ and conductor thickness $t$ with relative permittivity $\varepsilon_r$. The conductor thickness independent static expressions to compute the effective relative permittivity $\varepsilon_{\text{eff}}$ and the characteristic impedance $Z_0$ of the CPS with finite substrate thickness are obtained, using the conformal mapping method, by Ghione and Naldi [45]. The expressions are summarized below:

$$\varepsilon_{\text{eff}} = 1 + \frac{(\varepsilon_r - 1) K(k_0) K(k_6')}{2 K(k_0) K(k_6')}$$ \hspace{1cm} (6.1)

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0)}{K(k_6')}$$ \hspace{1cm} (6.2)

where, modulus $k_0$ and $k_6$ along with their complementary modulus $k_0'$ and $k_6'$ are defined in equation-(3.12) and equation-(3.69) respectively. On modifying these expressions empirically to take into account the finite strip conductor thickness $t$ [73], the equivalent slot-width $s_{eq}$ is reduced and the equivalent strip-width $w_{eq}$ is enlarged.

$$s_{eq} = s - \Delta w \hspace{1cm} (i) \hspace{1cm} w_{eq} = w + \Delta w \hspace{1cm} (ii)$$

$$\Delta w = \frac{t}{2\pi \varepsilon_r} \left[ 1 + \ln \left( \frac{8\pi s}{t} \right) \right] \hspace{1cm} (iii)$$ \hspace{1cm} (6.3)
The modified aspect ratio \( k_{0,t} \) and \( k_{6,t} \) are given below:

\[
k_{0,t} = \frac{s_{eq}}{s_{eq} + 2w_{eq}} \quad (i) \quad k_{0,t}' = \sqrt{1 - k_{0,t}^2} \quad (ii) \quad (6.4)
\]

\[
k_{6,t} = \frac{\tanh\left[ \frac{\pi s_{eq}}{4h} \right]}{\tanh\left[ \frac{\pi}{4h} \left( s_{eq} + 2w_{eq} \right) \right]} \quad (i) \quad k_{6,t}' = \sqrt{1 - k_{6,t}^2} \quad (ii) \quad (6.5)
\]

The effective relative permittivity of the CPS with conductor thickness is computed from the following empirical relation [73]

\[
\varepsilon_{eff}(t) = \varepsilon_{eff}(t = 0) - \frac{1.4[\varepsilon_{eff}(t = 0) - 1]t/s}{[K(k_0)/K(k_0')]+1.4(t/s)} \quad (6.6)
\]

The characteristic impedance of the CPW with conductor thickness \( t \) is computed from the equation-(6.2), using conductor thickness dependent effective relative permittivity and modified aspect ratio:

\[
Z_0(t) = \frac{120\pi}{\sqrt{\varepsilon_{eff}(t) K(k_0)}} \quad (6.7)
\]

Both \( \varepsilon_{eff} \) and \( Z_0 \) decrease with increasing conductor thickness.

### 6.3 Dispersion in CPS

In this section, the dispersion expression for the CPS due to Frankel et al. [91] is used to include the finite conductivity and finite conductor thickness in propagation characteristics. Like CPW, we have incorporated skin-depth dependent factor \((S)\) with
static effective relative permittivity as well. This factor will account for the dispersion at low frequency due to the field penetration in the finite conductivity strip conductors.

6.3.1 Effective Relative Permittivity

The dispersion empirical expression for the CPS due to Frankel et. al. [91] is modified to include the conductor thickness up to mm-wave range, using the same methodology of formulation employed for CPW in Chapter-5. The modified closed-form expressions for CPS dispersion with skin-depth dependent factor \( S \) and finite strip thickness is summarized below:

\[
\varepsilon_{\text{eff}}(f, t, \delta) = \left[ \sqrt{S \times \varepsilon_{\text{eff}}(f = 0, t)} + \frac{\sqrt{\varepsilon_r} - \sqrt{S \times \varepsilon_{\text{eff}}(f = 0, t)}}{1 + m(f / f_{\text{TE}})^r} \right]^2 \tag{6.8}
\]

where

\[
S = \frac{K(k_{0, \delta}) K(k_{0, f})}{K(k_{0, \delta}) K(k_{0, f})} \tag{b}
\]

where \( r = 1.8 \). The cut-off frequency of the lowest order TE mode is given by equation-(5.11) and the parameter \( m \) is obtained from equation-(5.12).

Fig.(6.2a) shows characteristics of \( \varepsilon_{\text{eff}} \) in CPS on different substrates with \( \varepsilon_r = 2.5, 9.8, 12.9 \), and 37; \( s = 8 \mu m, w = 4 \mu m, h = 670 \mu m \), and \( t = 5 \mu m \). The dispersion expression for the CPS indicates that the dispersion is very small up to hundred GHz for the structure on GaAs \( (\varepsilon_r = 13.1) \) and Sapphire \( (\varepsilon_r = 10.5) \) substrate having thickness 500 \( \mu m \). When compared against the EM-simulators in the frequency range 0.1 GHz – 200 GHz, the model has 3.6% average deviation for \( \varepsilon_r = 37 \). At low frequency, 0.1 GHz, the results of CST are much higher than others for all the substrates. The model does account for the dispersion due to the skin-effect after incorporating the factor \( S \) and shows small peak at the lowest end of the frequency. Overall, the model closely follows the results of CST.
Fig.(6.2): Comparisons of $\varepsilon_{\text{eff}}(f,t)$ computed by closed-form model against EM-simulators and measurement data as a function of: (a) & (b) Frequency, (c) Conductor thickness, and (d) $s/(s+2w)$ ratio for CPS on various substrates.

The experimental results of Frankel et. al. [91] over frequency range 15 GHz -200 GHz for $\varepsilon_r = 10.5$ are also shown in Fig.(6.2a). Fig.(6.2b) shows comparison of the phase constant $\beta$ computed by the model and the EM-simulators against experimental results of Kiziloglu et. al. [74] over frequency range 2 GHz-18 GHz. The CPS is taken on 670 µm thick GaAs ($\varepsilon_r$=12.9) substrate with $t = 0.5$ µm, 3 µm and 9 µm; $w = 4.5$ µm and $s = 7$ µm. With an increase in conductor thickness, there is decrease in effective relative
permittivity which has resulted in subsequent decrease in phase constant. The results of the present model follow the experimental results, as well as the results of Sonnet and HFSS faithfully. The % average and % maximum deviation in the results obtained from the comparison against HFSS results are summarized in Table-6.1a. For $2.5 \leq \varepsilon_r \leq 37$, $0.25 \, \mu m \leq t \leq 9 \, \mu m$ and $0.2 \leq s/(s + 2w) \leq 0.6$, the closed-form model has % average and % maximum deviation of (2.9%, 5.4%), (2.5%, 6.5%) and (4.7%, 7.1%) against HFSS, Sonnet and CST respectively, as shown in Fig.(6.2c) and Fig.(6.2d).

6.3.2 Characteristic Impedance

Some authors have used the voltage-power definition for CPS [102], whereas others have used the current-power definition [153]. The voltage-current definition for the characteristic impedance of a CPS line monotonically decreases with increasing frequency. However on using power-current definition, it shows initial decrease and then increase in characteristic impedance with increasing frequency. The dispersion in the characteristic impedance of the CPS could be estimated from the following voltage-current definition based expression:

$$Z_0(f,t) = \frac{120\pi}{\sqrt{\varepsilon_{eff}(f,t)}} \times S \quad (a) \quad \text{where} \quad S = \left\{ \frac{K(k_{0,\delta})K'(k_{0,\delta})}{K'(k_{0,\delta})K(k_{0,\delta})} \right\} \quad (b) \quad (6.9)$$

The above expression accounts for the low frequency dispersion in characteristic impedance due to field penetration. It causes increase in characteristic impedance at low frequency due to increase in the internal inductance. Fig.(6.3a) shows the dispersion in $Z_0$ of CPS on different substrates with $\varepsilon_r = 2.5, 9.8, 12.9$ and 37; $s = 8 \, \mu m$, $w = 4 \, \mu m$, $h = 670 \, \mu m$ and $t = 5 \, \mu m$ in the frequency range 0.1GHz – 200 GHz.
Fig.(6.3): Comparisons of $Z_0(f,t)$ computed by the closed-form model, EM-simulators against the measurement data as a function of: (a) & (b) Frequency, (c) Conductor thickness, and (d) $s/(s+2w)$ ratio for CPS on various substrates.

The closed-form model is within 3.4% average deviation against the results obtained from EM-simulators till 70 GHz and beyond that the results of the model deviate from that of simulators with 15.4% average deviation. It is due to different definitions of the characteristic impedance adapted in the model and in the EM-simulators. Fig. (6.3b) shows the results on characteristic impedances of the planar CPS for conductor thicknesses: $t = 0.5\ \mu m$, $3\ \mu m$ and $9\ \mu m$ against experimental results of Kiziloglu et. al. [74]. The characteristic impedance decreases with increase in conductor thickness. There
are large deviations in the results of the model at the lower end of the frequency w.r.t. the experimental and EM-simulated results. This could be improved by the circuit model presented in section-6.8. The average and maximum % deviation of the present model and Sonnet are summarized in Table-6.1b. For $2.5 \leq \varepsilon_r \leq 37$, $0.25 \mu m \leq t \leq 9 \mu m$ and $0.2 \leq s/(s+2w) \leq 0.6$, the closed-form model has % average and % maximum deviation of (3.1%, 6.6%), (3.2%, 5.8%) and (2.2%, 5.3%) against HFSS, Sonnet and CST respectively, as shown in Fig.(6.3c) and Fig.(6.3d). The CPS line provides much higher value of $Z_0$ as compared to the CPW and with increase in aspect ratio ($s/(s+2w)$), $Z_0$ of the CPS increases while that of the CPW decreases.

**Table- 6.1:** % Average and maximum deviation of closed-form model, Sonnet and experimental results [74] against HFSS for CPS.

*[Data range: $t = 0.25 \mu m - 9 \mu m$; $Freq = 1 \text{ GHz - 60 GHz}$; $\varepsilon_r = 12.9$; $\tan \delta = 0.002$; $\sigma=4.1 \times 10^7 \text{ S/m}$; $h=670 \mu m$]*

*a) Effective relative permittivity*

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*b) Characteristic impedance*

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<td>3.6</td>
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In summary we can say that changes due to conductor thickness in the characteristic impedance, especially in the higher frequency range are more significant as compared to the changes in the effective relative permittivity. Thus for the variation in the normalized strip thickness from \( t/h = 0 \) to 0.1, \( \varepsilon_{\text{eff}} \) is changed by 6\%, whereas the \( Z_0 \) is changed by 9\% and 22\% at \( h/\lambda = 0.01 \) and \( h/\lambda = 0.1 \) respectively. The effect is more significant for the small strip-slot ratio CPS on the low permittivity substrate.

### 6.4 Computation of Losses in CPS

In this section, we present the improved closed-form models for computation of the dielectric loss and the conductor loss of the CPS, suitable for the microwave CAD purpose.

#### 6.4.1 Dielectric Loss

The following standard expression is used to compute the dielectric loss of the CPS structures [146]:

\[
\alpha_d = 27.29 \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{eff}}(f,t)}} \left[ \frac{\varepsilon_{\text{eff}}(f,t) - 1}{\varepsilon_r - 1} \right] \frac{\tan \delta}{\lambda_0} \quad \text{dB/unit length} \tag{6.10}
\]

where, \( \lambda_0 \), \( \tan \delta \) and \( \varepsilon_r \) are free-space wavelength, loss tangent and relative permittivity of the substrate respectively.

The frequency and conductor thickness dependent \( \varepsilon_{\text{eff}}(f,t) \) of CPS is computed using equation-(6.8). Fig.(6.4) compares the computation of \( \alpha_d \) by the closed-form model
6.4.2 Conductor Loss

The conductor loss of the CPS is computed using two closed-form models:

(i) Wheeler’s incremental inductance formulation

(ii) Improved Holloway and Kuester model (IHK)

The models are compared and validated against the results from experimental data and EM-simulators for wide range of data.
Wheeler’s Incremental Inductance Formulation

We have extended Wheeler’s incremental inductance formulation [13] to CPS for conductor loss computation. Therefore, the conductor loss of a CPS is computed from the following expression:

\[ \alpha_c = \frac{\pi}{\lambda_0} \sqrt{\frac{\varepsilon_{\text{eff}}(\varepsilon_r, w, s, h, f, t)}{\varepsilon_{\text{eff}}(\varepsilon_r = 1, w, s, h, f, t)}} \frac{\Delta Z(\varepsilon_r = 1, w, s, h, f, t, \delta_s)}{Z_0(w_{\text{eq}}, s_{\text{eq}}, h, f, t, \varepsilon_r = 1)} \] Np/m \quad (6.11)

where, \( \lambda_0 \) is free space wavelength. The expressions for the parameters \( \varepsilon_{\text{eff}}(\varepsilon_r, w, s, h, f, t) \) and \( Z_0(w_{\text{eq}}, s_{\text{eq}}, h, f, t, \varepsilon_r = 1) \) are obtained from equation-(6.8) and equation-(6.9) respectively.

Fig.(6.5): Application of Wheeler’s inductance rule to compute conductor loss in CPS.

Fig.(6.5a) shows that the slot-width of CPS is reduced due to the finite conductor thickness. The change in width due to strip conductor thickness is calculated by using equation-(6.3). Fig. (6.5b) shows the skin-depth increases the slot-width by \( \delta_s \) due to the field penetration all around the strip conductor. We note that due to the field penetration in the conductor by \( \delta_s/2 \) all around, the strip width has decreased by \( \delta_s \) and the slot width has increased by \( \delta_s \). Likewise, the substrate height also increases by \( \delta_s/2 \). We further note
that due to the skin-depth, the conductor thickness is also decreased by $\delta_s$. Thus the field penetration modifies equation – (6.3) as follows,

$$\Delta w' = \frac{t'}{2 \pi \epsilon_r} \left[ 1 + \ln \frac{8 \pi s}{t'} \right] \quad (a) \text{ where, } t' = t - \delta_s \quad (b) \quad (6.12)$$

The difference characteristic impedance, $\Delta Z$ of a CPS with and without field penetration is given by:

$$\Delta Z (\epsilon_r = 1, w, s, h, f, t, \delta_s) = Z_0 \left( w + \Delta w' - \delta_s, s - \Delta w' + \delta_s, h + \frac{\delta_s}{2}, f, t, \epsilon_r = 1 \right) - Z_0 \left( w_{eq}, s_{eq}, h, f, t, \epsilon_r = 1 \right) \quad (6.13)$$

The characteristic impedances for the parameters shown in equation- (6.13) are computed from the model discussed in previous sections.

- **Improved Holloway and Kuester model**

The second closed-form model, for computation of conductor loss of the CPS, is based on the standard perturbation method [38],[105] along with the concept of the stopping distance $\Delta$ [21], [25].

![Fig.(6.6): Infinitely thin CPS conductors with stopping distance ($\Delta$).](image)
The perturbation expression to compute the conductor loss of the planar shown in Fig.(6.6) is given as

$$\alpha_c = \frac{R_{sm}}{2Z_0(f,t)} \int \left( \frac{J}{I} \right)^2 dl$$

(6.14)

where, $Z_0(f,t)$ is the conductor thickness and frequency dependent characteristic impedance of the CPS structure, obtained from equation-(6.9). The surface resistance $R_{sm}$ of the strip conductor of finite thickness $t$ is computed from equation-(5.18). In CPS, the current density ($J$) over the strip conductors is defined by the following function [17]

$$J = \frac{A}{\sqrt{x^2 - \left(\frac{s}{2}\right)^2}} \left(\frac{w + \frac{s}{2}}{w + \frac{s}{2} - x}\right)^2$$

(a), where, $A = \frac{I}{K(k')} \left(\frac{w + \frac{s}{2}}{2}\right)$

(6.15)

The ratio of current density ($J$) to the longitudinal current ($I$) is integrated along whole of the strip as follows

$$\int \left( \frac{J}{I} \right)^2 dl = 2 \int_{\frac{s}{2} \pm \Delta}^{\left(\frac{w + \frac{s}{2}}{2}\right) - \Delta} \left( \frac{J}{I} \right)^2 dl$$

(6.16)

The stopping distance is computed using equations-(5.24)-(5.25), for the rectangular and trapezoidal strip cross-sections [24]. The final expression for the conductor loss of the CPS structure is

$$\alpha_c = \frac{R_{sm}(2w+s)^2}{16wZ_0(f,t)K^2(k'')(w+s)} \left\{ \frac{2}{s} \ln \left[ \frac{s}{\Delta} + 1 \right] \left( \frac{w - \Delta}{w + s - \Delta} \right) + \frac{2}{(2w+s)} \ln \left[ \frac{(2w+s)}{\Delta} - 1 \right] \left( \frac{w - \Delta}{w + s + \Delta} \right) \right\}$$

(6.17)
The total loss of the CPS structures can be computed by adding the conductor loss and dielectric loss together, \textit{i.e.}

\[
\alpha_T = \alpha_c + \alpha_d \quad dB / \text{unit length}
\]  

(6.18)

Fig.(6.7): Comparison of total loss in CPS against results of Kızıloglu et. al. [74].

Fig.(6.7) shows the comparison of the total loss computed by the closed-form model #1 (= Wheeler + \(\alpha_d\)), closed-form model #2 (= IHK + \(\alpha_d\)), two EM-simulators - Sonnet and HFSS against the experimental results [74]. The loss has been computed for strip conductor thickness \(t = 0.25 \, \mu m - 9 \, \mu m\). The CPS is on the substrate with \(\varepsilon_r = 12.9\), \(h = 670 \, \mu m\), \(w = 4.5 \, \mu m\), \(s = 7 \, \mu m\). The loss is computed over the frequency range 1 GHz - 60 GHz. The experimental results are available up to 18 GHz only. The closed-form model #1 being based on Wheeler’s incremental inductance model, is only applicable for the conductor thickness, \(t \geq 1.1\delta_s\). The closed-form model #2 comfortably computes for both thick (\(t \geq 1.1\delta_s\)) and thin (\(t < 1.1\delta_s\)) strip conductors. The computed conductor loss, using Wheeler’s model, is less for thicker conductor. Table-6.2 presents the %
average and maximum deviation in the results obtained from experiment, Sonnet and the closed-form models. The results of HFSS are taken as the reference.

**Table 6.2: % Average and maximum deviation of closed-form models, Sonnet and experimental results [74] against HFSS for CPS.**

[Data range: \( t = 0.25 \mu m - 9 \mu m;\) Freq = 1 GHz - 60 GHz; \( \varepsilon_r = 12.9;\) \( \tan \delta = 0.002;\) \( \sigma = 4.1 \times 10^7\) S/m; \( h = 670 \mu m\]}

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The effect of frequency on the characteristics of \( \alpha_T \) in CPS, on different substrates with \( \varepsilon_r = 3.78, 9.8 \) and 20; \( s = 8 \mu m, w = 4 \mu m, h = 670 \mu m,\) \( \tan \delta = 0.002 \) and \( t = 5 \mu m,\) is shown in Fig.(6.8a). The closed-form model #2 is used here for the computation of \( \alpha_T \) and compared against the EM-simulators with 4.8% average deviation till 60 GHz and beyond that it increases up to 18.9%. Fig.(6.8b) shows the comparison of the computed \( \alpha_T \) by the model against the EM-simulators as a function of \( s/(s+2w) \) ratio with 6.8% average deviation. Fig.(6.8c) shows the effect of frequency on the characteristics of \( Q_u \) in CPS on different substrates with \( \varepsilon_r = 3.78, 9.8 \) and 20; \( s = 8 \mu m, w = 4 \mu m, h = 670 \mu m,\) \( \tan \delta = 0.002 \) and \( t = 5 \mu m \) in the frequency range 0.1 GHz – 200 GHz with 6.9% average deviation till 125 GHz. Fig.(6.8d) compares computation of \( Q_u \) for \( \varepsilon_r = 3.78 \) and 20; \( f = 40 \) GHz as a function of \( s/(s+2w) \) ratio with % average and % maximum deviation of 8.4% and 25% respectively for the range \( 0.2 \leq s/(s+2w) \leq 0.6.\)
Fig.(6.8): Total loss, as a function of (a) Frequency and (b) $s/(s+2w)$ ratio and $Q$ factor, as function of (c) Frequency and (d) $s/(s+2w)$ ratio for CPS on various substrates.

### 6.5 Closed-form Dispersion and Loss Models for Multilayer CPS

The cross-sectional view of a CPS on multilayer dielectric substrates is shown in Fig.(6.9).
The conformal mapping technique proposed by Ghione et al. [45] to analyze CPS has poor accuracy for large inter-strip spacing and substrate thickness, which are often used in modern microwave circuits. The new corrected transformations are suggested by Gevorgian et al. [112] for analysis of multilayer substrate CPS, whose results are consistent with experiments and numerical simulations as well. The method employed is generalized enough to handle any number of lower or upper layers efficiently, but for the sake of convenience, we discuss and present results for three-layer CPS shown in Fig. (6.9). For multilayer CPS, $\varepsilon_{\text{eff}}$ and $Z_0$ are given by:

\[
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2} \frac{K(k_0)}{K(k_1')} + \frac{\varepsilon_{r2} - 1}{2} \frac{K(k_0)}{K(k_2')} + \frac{\varepsilon_{r3} - 1}{2} \frac{K(k_0)}{K(k_3')}
\]

\[
Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0)}{K(k_0')}
\]

where modulus $k_i$ ($i=1,2,3$) and $k_0$ along with their complements can be computed using equation-(3.10) and (3.12) respectively. The substrate thickness $h$ in equation – (3.10) will be replaced by $h_i$ ($i=1,2,3$) accordingly.

Then we have extended the conductor thickness and frequency based closed-form models for $\varepsilon_{\text{eff}}(f,t), Z_0(f,t), \alpha_c$ and $\alpha_d$ of single-layered CPS to multilayer CPS by first converting multilayer into single-layer of finite dielectric thickness using SLR:
• Effective relative permittivity

\[
\varepsilon_{\text{eff}}(f,t,\delta) = \left(\sqrt{S \times \varepsilon_{\text{eff}}(f = 0,t)} + \frac{\varepsilon_{\text{req}} - \sqrt{S \times \varepsilon_{\text{eff}}(f = 0,t)}}{1 + m(f/f_{\text{TE}})^r}\right)^2
\]

where Skin-effect factor, 

\[
S = \begin{pmatrix}
K(k_0,\delta) & K(k_0,t) \\
K'(k_0,\delta) & K'(k_0,t)
\end{pmatrix}
\]

• Characteristic impedance

\[
Z_0(f,t) = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}(f,t)}} \times S
\]

• Wheeler’s incremental inductance rule

\[
\alpha_c = \frac{27.29}{\lambda_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_{\text{req}},w,s,\varepsilon_{\text{req}},f,t)} \frac{\Delta Z(\varepsilon_{\text{req}} = 1,w,s,\varepsilon_{\text{eq}},f,t,\delta_s)}{Z_0(w_{\text{eq}},s_{\text{eq}},\varepsilon_{\text{eq}},f,t,\varepsilon_{\text{req}} = 1)} \text{ dB/m}
\]

• Improved Holloway and Kuster (IHK)

\[
\alpha_c = \frac{2 s \ln \left(\frac{2}{w+s} + 1\right)}{16 w Z_0(f,t) K^2(k_0')(w+s)} \left[\frac{2}{s} \ln \left(\frac{s+1}{w+s} \right) \left(\frac{1}{w+s+A}\right) \right] + \frac{2}{(2w+s)} \ln \left(\frac{2w+s}{A}\right) \left(\frac{w-A}{w+s+A}\right)
\]

• Dielectric loss

\[
\alpha_d = 27.29 \frac{\varepsilon_{\text{req}}}{\sqrt{\varepsilon_{\text{eff}}(f,t)}} \left[\frac{\varepsilon_{\text{eff}}(f,t)-1}{\varepsilon_{\text{req}}-1}\right] \tan \delta_{\text{eq}} \frac{\tan \delta_{\text{eq}}}{\lambda_0} \text{ dB/m}
\]

where \(h_{\text{eq}}\) is the total substrate thickness between strip conductors and bottom layer of the multilayer substrate. \(\varepsilon_{\text{req}}\) and \(\tan \delta_{\text{eq}}\) of the equivalent single-layer substrate CPS are obtained from equation-(4.28).
Fig. (6.10): Multilayer CPS: (a) Effective relative permittivity, (b) Characteristic impedance, (c) Conductor loss, and (d) Dielectric loss.

Fig. (6.10a) – (6.10d) shows comparison and validity of SLR-based computed line parameters of multilayer CPS against EM-simulators for frequency range 1 GHz – 60 GHz. The computed $\varepsilon_{\text{eff}}$ and $Z_0$ by the model has % average and % maximum deviation of (0.7%, 2.8%) and (2.2%, 6.5%) respectively against both the EM simulators, as shown in Fig.(6.10a) and (6.10b).
In Fig. (6.10c) and (6.10d), variation in $\alpha_c$ and $\alpha_d$ of multilayer CPS for $\varepsilon_{r1} = 10$ and 12.9. The % average and % maximum deviation in IHK and Wheeler are (4.4%, 6.8%) and (7.8%, 16.1%) respectively against HFSS and Sonnet. It is observed from Fig. (6.10c), that the conductor loss decreases with increasing $\varepsilon_{r1}$, which is explained by the fact that the field is “pulled out” from the lossy substrate into the high permittivity layer 1, where the losses are smaller [112]. The computed $\alpha_d$ by the model is in close agreement with HFSS and MoM based LINPAR, for frequency range 1 GHz – 60 GHz, with % average and % maximum deviation of 1.8% and 2.5% respectively, as shown in Fig. (6.10d).

### 6.6 Effect of Asymmetry in Characteristics of CPS

An asymmetric coplanar strip (ACPS) waveguide is flexible in practical device design. Along with small sizes, the impedance, effective dielectric constant and microwave refractive index can be adjusted by changing the width of one of the strips while keeping the width of the other strip and of the gap fixed. In Fig. (6.11), ACPS has substrate thickness $h$ with permittivity $\varepsilon_r$, $w_2$ is the width of the strip conductor, $s$ is the space between signal and ground conductors and $w_1$ is the width of ground conductor.

![Fig.(6.11): Structure of asymmetrical CPS (ACPS)](image)

In our study, we have used the conductor thickness independent static expressions for $\varepsilon_{eff}$ and $Z_0$ of the ACPS obtained from equations-(3.78) and (3.79) respectively:
Analysis and Modeling of CPS

\[ \varepsilon_{\text{eff}} = 1 + \frac{1}{2} (\varepsilon_r - 1) \frac{K(k^7)}{K(k^8)} \overline{K(k^7) K(k^8)} \]  
\[ (a) \]

\[ Z_0 = \frac{60\pi}{\sqrt{\varepsilon_{\text{eff}}} K(k^7)} \]  
\[ (b) \] (6.21)

where, modulus \( k_7 \) and \( k_8 \) along with their complementary modulus \( k'_7 \) and \( k'_8 \) are defined in equations-(3.75) and (3.80) respectively. However, these expressions are modified empirically to take into account the finite strip conductor thickness and the skin-depth penetration using equation-(6.8) and (6.9).

Fig. (6.12a) and (6.12b) compare \( \varepsilon_{\text{eff}} \) and \( Z_0 \) computed by the closed-form model on the substrate with \( \varepsilon_r = 12.9 \), \( t = 3 \mu m \), \( w_1 = 20 \mu m \) and \( h = 635 \mu m \) against HFSS and Sonnet for three cases of asymmetry i.e. \( w_1/w_2 = 1, 2 \) and 4 for frequency range 1 GHz - 60 GHz. The computation of \( \varepsilon_{\text{eff}} \) and \( Z_0 \) by the model has % average and % maximum deviations of (2.1%, 4.8%) and (2.4%, 5.6%) respectively against both the EM simulators.

The dielectric loss of ACPS is computed using equation-(6.10) in which the frequency and conductor thickness dependent \( \varepsilon_{\text{eff}}(f,t) \) is computed using equation-(6.8). On accounting asymmetry in Wheeler’s incremental inductance formulation given by equation-(6.11), the expression becomes:

\[ \alpha_c = \frac{\pi}{\Lambda_0} \sqrt{\varepsilon_{\text{eff}}(\varepsilon_r, w_1, w_2, s, h, f, t)} \frac{\Delta Z(\varepsilon_r = 1, w_1, w_2, s, h, f, t, \delta_s)}{Z_0(\text{w}_{\text{eq}1}, \text{w}_{\text{eq}2}, s_{\text{eq}}, h, f, t, \varepsilon_r = 1)} \text{ Np/m} \]  
\[ (6.22) \]

Holloway and Kuester have not accounted asymmetry in conductor loss computation of CPS [25]. The improved Holloway and Kuester (IHK) closed-form expression for computation of the conductor loss of ACPS is obtained by modifying equation-(6.17).
Due to asymmetry, the longitudinal current $I$ and the current density $J$ over the strip conductors from equation-(6.15), is defined by the following function:

\[
\begin{align*}
J &= \frac{A}{\sqrt{(x^2-a^2)(b_1^2-x^2)}} \\
\text{where, } A &= \frac{I}{4K(k)} \sqrt{(b_1+a)(b_2+a)}
\end{align*}
\]  

(6.23)

where $a = \frac{s}{2}$ ; $b_1 = w_1 + a$ ; $b_2 = w_2 + a$ ; $b = b_1 + b_2$.

The ratio of current density ($J$) to the longitudinal current ($I$) can be integrated along whole of the strip as follows:

\[
\oint \left( \frac{J}{I} \right)^2 \, dl = 2 \left( \frac{-a+\Delta}{-(b_2-\Delta)} \right)^2 \int_{\text{strip 1}} \left( \frac{J}{I} \right)^2 \, dl \left( \frac{b_1-\Delta}{a+\Delta} \right)^2 \int_{\text{strip 2}} \, dl
\]  

(6.24)
On integrating to the whole range, the final expression for the conductor loss of the ACPS structure is

\[
\alpha_c \approx \frac{R_{sm}}{32\varepsilon_0(f,t\kappa^2(k'))} \left\{ \left( \frac{b_1 + a}{b_2 - a} \right) \left( \frac{1}{a} - \ln \left( \frac{2a}{b_2 - a - d} \right) \right) + \left( \frac{b_2 + a}{b_1 - a} \right) \left( \frac{1}{a} - \ln \left( \frac{2a}{b_1 - a - d} \right) \right) \right\} (6.25)
\]

Fig.(6.13) compares the results for computed \( \alpha_T \) of ACPS by closed-form model #1, closed-form model #2, Sonnet and LINPAR against experimental results of Garg et.al. [93]. The ACPS is considered on a \( p \)-type substrate with substrate thickness \( h = 250 \mu m \). The Al metal layer of thickness 2 \( \mu m \) lie on top of SiO\(_2\) buffer layer of thickness 7.4 \( \mu m \) that is deposited on \( p \)-substrate. The strip conductor has conductivity \( 3.72 \times 10^7 \) S/m. The relative dielectric constant of SiO\(_2\) is 3.9 and \( \tan \delta = 0.0002 \). The operating frequency range is 0.1 GHz - 50 GHz. The effective relative permittivity for multilayer ACPS structure realized with the substrate stack has been calculated from equations-(6.19a) and (6.20a). Fig.(6.13a) - Fig.(6.13c) compare the results for three cases targeting approximately the same characteristic impedance \( i.e. Z_0=150\Omega \): (a) \( w_2 = 5 \mu m, s = 5 \mu m \), (b) \( w_2 = 10 \mu m, s = 10 \mu m \), and (c) \( w_2 = 30 \mu m, s = 30 \mu m \). In this case \( w_1 \) was fixed at twice that of \( w_2 \). The narrow line has more series resistance and hence more resistive loss is observed at low frequencies. At high frequencies, the wider line exhibits more dielectric loss due to increased spreading out of the electromagnetic field into the substrate. The closed-form models show better agreement with the results from experiment and Sonnet except at narrow strip conductor and narrow gap for lower frequency range. The LINPAR has more deviation from the experimental results all over the range. The line widths of around 10 \( \mu m \) give the optimum combination of low and high frequency loss over the bandwidth of the distributed mixer [93].
Fig.(6.13d) further shows the line characteristic impedance dependent $\alpha_T$ at 30 GHz as computed by both the closed-form models and HFSS. The loss increases with increase in characteristic impedance and line aspect ratio. Outcome of the overall comparison in terms of % average and % maximum deviation is summarized in Table-6.3.
Table – 6.3: % Average and Maximum deviation of models against Garg et. al. [93] 
[Data range: \( t = 2 \mu m; f = 0.1 \) GHz - 50 GHz; \( \varepsilon_r = 3.9; \tan\delta = 0.002; \sigma = 3.72 \times 10^7 \) S/m]

<table>
<thead>
<tr>
<th>% Deviation</th>
<th>Closed-form Model #1</th>
<th>Closed-form Model #2</th>
<th>HFSS</th>
<th>Sonnet</th>
<th>LINPAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
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<td>3.1</td>
<td>2.8</td>
<td>4.2</td>
<td>6.5</td>
</tr>
<tr>
<td>Maximum</td>
<td>7.5</td>
<td>6.3</td>
<td>4.3</td>
<td>6.8</td>
<td>10.3</td>
</tr>
</tbody>
</table>

6.7 Closed-form Dispersion and Loss Models for Non-Planar CPS

The elliptical coplanar strip lines (ECPS) can be used as adapters and slotlines as well as antennas [127-143]. The closed-form expressions for dispersion and losses presented in the previous sections for planar CPS could be adopted to the CPS with finite strip conductor thickness on the circular and elliptical cylindrical surfaces. In this section we will present the conductor thickness and frequency dependent closed-form models of line parameters for both single-layered and multilayer non-planar CPS, shown in Fig.(6.14) and (6.17).

6.7.1 Single-Layer Case

Fig.(6.14) presents the CPS on the elliptical and circular cylindrical surfaces with two different finite ground plane widths. The structural parameters of SC - transformed ECPS/ CCPS into the corresponding planar CPS are given by equation-(3.84):

\[
\begin{align*}
  s &= 2\Psi \quad (i) \\
  w &= \theta \quad (ii) \\
  h &= \ln\left(\frac{a_2 + b_2}{a_1 + b_1}\right) \quad (iii) \\
  t &= \ln\left(\frac{a_3 + b_3}{a_2 + b_2}\right) \quad (iv)
\end{align*}
\]

(6.26)

and the detailed definition of the above mentioned parameters are given in Chapter-3.
In our study, we have used the conductor thickness and frequency independent expressions for $\varepsilon_{\text{eff}}$ and $Z_0$ of the non-planar CPS obtained from equations-(3.82) and (3.83) respectively:

$$
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_0) \; K(k_1)}{K(k_0') \; K(k_1')}
$$

(a) \hspace{1cm}

$$
Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{\text{eff}}} \; K(k_0')}
$$

(b) \hspace{1cm}

(6.27)

where, modulus $k_0$ and $k_1$ along with their complementary modulus $k_0'$ and $k_1'$ for finite ground plane widths $(2\pi - 2\theta - 2\psi)$ and $(\pi - 2\theta - 2\psi)$, are obtained from equations-(3.42) - (3.44) and equations-(3.55)-(3.56) respectively after replacing $\theta$ with $\theta + \psi$.

**Fig.(6.14):** CPS on the curved surfaces: (a) Elliptical CPS (ECP S) , (b) Circular Cylindrical CPS (CCPS), (c) Semi-ellipsoidal CPS (SECPS) and (d) Semi- circular cylindrical CPS (SCCPS).
In order to account the strip conductor thickness for these line structures, we have to modify equation- (6.3) - (6.7) by using equation-(6.26). The equivalent strip width ($\theta_{eq}$) and equivalent slot width ($\psi_{eq}$) for the ECPS / CCPS are given as

$$\psi_{eq} = \psi - \Delta \theta \quad (i)$$

$$\theta_{eq} = \theta + \Delta \theta \quad (ii)$$

where, $\Delta \theta = \frac{1}{2 \pi \varepsilon_r} \left[ \ln \frac{a_2 + b_2}{a_2 + b_2} \right] + \ln \left( \frac{16 \pi \psi}{\ln \frac{a_1 + b_1}{a_2 + b_2}} \right) \quad (iii)$

(6.28)

and the modulus $k_0$ and $k_1$ will be modified into $k_{0j}$ and $k_{1j}$ along with their complementary modulus, in which $\psi$ and $\theta$ will be replaced by $\psi_{eq}$ and $\theta_{eq}$ respectively. The above equations - (6.28) is inserted in equations - (6.8) and (6.9) to compute the conductor thickness and frequency based $\varepsilon_{eff} (f,t)$ and $Z_0(f,t)$ of the ECPS and CCPS lines.

The computation of $\alpha_d$ is done using equation - (6.10) in which the frequency and conductor thickness dependent $\varepsilon_{eff} (f,t)$ of ECPS and CCPS is used. For $\alpha_c$, computation of non-planar CPS, firstly Wheeler’s incremental inductance formulation is modified using equation-(6.26):

$$\alpha_c = \frac{\pi}{\lambda_0} \sqrt{\varepsilon_{eff} \varepsilon_r \theta_{eq} \psi_{eq} \ln \frac{a_2 + b_2}{a_1 + b_1}, f, \ln \frac{a_3 + b_3}{a_2 + b_2}, \delta_{eff}} \left[ \frac{\Delta Z}{Z_0} \left[ \varepsilon_{eff}, \theta_{eq}, \psi_{eq}, \ln \frac{a_2 + b_2}{a_1 + b_1}, f, \ln \frac{a_3 + b_3}{a_2 + b_2}, \delta_{eff} \right] \right] \text{Np/m}$$

(6.29)
Then IHK model for ECPS/CCPS with ground plane width \((2\pi - 2\theta)\) is obtained by using equation – (6.26) with equation - (6.17):

\[
\alpha_c = \frac{R_{tot} \theta + \Psi^2}{4Z_{eff}(\theta + \Psi)} \left[ \frac{1}{\Psi} \ln \left( \frac{2\Psi + 1}{\theta + 2\Psi - \Delta} \right) + \frac{1}{\theta + \Psi} \ln \left( \frac{2\theta + \Psi}{\theta + 2\Psi + \Delta} \right) \right] \text{Np/m}
\]

(6.30)

When \(\pi\) is replaced by \(\pi/2\) in the above equation, IHK model for non-planar CPS with ground plane width \((\pi - 2\theta - 2\psi)\) is obtained. We have tested the accuracy of the closed-form models developed for propagation characteristics of non-planar CPS, for both \((2\pi - 2\theta - 2\psi)\) and \((\pi - 2\theta - 2\psi)\) ground plane width, against the results obtained from EM-simulators- HFSS and CST, as shown in Fig.(6.15) and Fig.(6.16).

Fig.(6.15) presents comparisons of performances of CPS on the circular and semi-circular cylindrical surfaces in respect of effective relative permittivity, characteristic impedance and losses. It also presents such comparisons for the CPS on the elliptical and semi-ellipsoidal cylindrical surfaces. The results are obtained over the frequency range, 1 GHz – 60 GHz. For simulation, we have taken the substrate with \(\varepsilon_r = 2.5\), \(\theta = 40^\circ\), \(\psi = 25^\circ\), \(h = 635 \mu\text{m}\) and \(t = 3 \mu\text{m}\). For ECPS, the ellipticity \(c = 0.7\) is considered. With increase in the ellipticity and decrease in the ground width, there is increase in both \(\varepsilon_{eff}\) and \(Z_0\). So by controlling ellipticity, \(\varepsilon_{eff}\) and \(Z_0\) of the line can be controlled as well. We have observed that in both HFSS and CST, increase in the value of effective relative permittivity starts after 100 GHz; whereas the closed-form model shows continuous increase after 10 GHz. The experimental results are not traceable to verify the nature of dispersion. However the model has an average deviation of 2.9% against the results of both HFSS and CST over whole frequency range.
Fig.(6.15): Comparison of different line parameters of non-planar CPS w.r.t. frequency.
Like CPW, the effective relative permittivity increases for the semi-circular and semi-ellipsoidal CPS as compared against the CPS on the circular and elliptical surfaces. The characteristic impedances of semi-circular and semi-ellipsoidal cases are also higher as compared to the CPS on the circular and elliptical surfaces. The total loss $\alpha_T$ of non-planar CPS is computed for $\varepsilon_r = 12.9$, $t = 3\, \mu m$, $\theta = 28^\circ$, $\psi = 25^\circ$, $c = 0.7$, $h = 635\, \mu m$, $\tan \delta = 0.0002$ and $\sigma = 4.1 \times 10^7\, S/m$ between frequency range 1 GHz – 60 GHz. On comparing, the closed-form model #1 and closed-form model #2 have % average and % maximum deviation of (6.5%, 16.7%) and (3%, 8%) respectively against both the EM-simulators, excluding results at 1 GHz due to large deviations. The closed-form model #2 is in close agreement with HFSS at higher frequencies. The losses are almost identical for both the cases. The nature of dispersion is identical for both cases. The results are summarized in Table- (6.4).

**Table - 6.4:** % Change in characteristics of models against simulators on non-planar CPS with different ground-plane widths

(a): % change in characteristics of semi-ellipsoidal CPS over elliptical CPS.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% av. change</td>
<td>% max. change</td>
<td>% av. change</td>
</tr>
<tr>
<td>Increase in Effective relative permittivity</td>
<td>3.6</td>
<td>3.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Increase in Characteristic impedance</td>
<td>2.4</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Decrease in Total loss</td>
<td>4.5</td>
<td>10.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>
(b): % change in characteristics of semi-circular CPS over circular cylindrical CPS.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Closed-form Model</th>
<th>HFSS</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td>% change</td>
<td>% max. change</td>
<td>% change</td>
<td>% max. change</td>
</tr>
<tr>
<td>Effective relative permittivity</td>
<td>4.4</td>
<td>4.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Characteristic impedance</td>
<td>2.4</td>
<td>2.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Total loss</td>
<td>5.5</td>
<td>15.5</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Fig.(6.16): Comparison of different line parameters of non-planar CPS w.r.t. structural parameters.
Analysis and Modeling of CPS

Fig. (6.16a) and (6.16b) show the effect of conductor thickness on $\varepsilon_{\text{eff}}$ and $Z_0$ of ECPS and CCPS with varying angular strip width $\theta/h$ for $\varepsilon_r=2.5$, $\psi=35^\circ$, $h=635\ \mu$m, $c=0.7$, and $f=10\ \text{GHz}$. The two conductor thicknesses $t=3\ \mu$m and $6\ \mu$m are used in the investigation. Both $\varepsilon_{\text{eff}}$ and $Z_0$ decrease with increase in the conductor thickness. Fig. (6.16c) further compares the total loss of the ECPS and CCPS for $\varepsilon_r=12.9$, $\theta=28^\circ$, $\psi=25^\circ$, $f=10\ \text{GHz}$, $h=635\ \mu$m, $\tan\delta=0.0002$ and $\sigma=4.1\times10^7\ \text{S/m}$ for conductor thickness range $0.25\mu$m - $10\mu$m. Both the models are in close agreement with EM-simulators. The closed-form model#1 fails to compute for $t < 1.1\delta_x$.

6.7.2 Multilayer Case

The mechanical strength and the average power handling of a CPS is increased by using another dielectric substrate below the fragile substrate. In this section, we have extended the improved closed-form models for the line parameters of single-layered, equations-(6.27)-(6.30), to multilayer non-planar CPS by applying SLR technique and using equation-(6.20).

The structural parameters of multilayer structure obtained from SC-transformation (discussed in Chapter-3) are:

$$s = 2\psi \quad \text{(i)} \quad w = \theta \quad \text{(ii)} \quad t = \ln\frac{a_4 + b_4}{a_3 + b_3} \quad \text{(iii)}$$

$$h_1 = \ln\frac{a_3 + b_3}{a_2 + b_2} \quad \text{(iv)} \quad h_2 = \ln\frac{a_3 + b_3}{a_1 + b_1} \quad \text{(v)} \quad h_3 = \ln\frac{a_5 + b_5}{a_3 + b_3} \quad \text{(vi)}$$

We have computed $\varepsilon_{\text{eff}}$ and $Z_0$ of multilayer CPS by using equation-(6.19). The modulus $k_0$ and $k_i (i=1,2,3)$ along with their complementary modulus for finite ground plane widths $(2\pi-2\theta-2\psi)$ and $(\pi-2\theta-2\psi)$, are obtained from equations- (3.42)-(3.44)
and equations-(3.55)-(3.56) respectively after replacing $\theta$ with $\theta + \psi$. The parameter $H$ in equation – (3.45) will be replaced by $h_i$ ($i=1,2,3$) accordingly.

Fig. (6.17): Multilayer CPS on the curved surfaces: (a) Elliptical CPS (MECPS), (b) Circular Cylindrical CPS (MCCPS), (c) Semi-ellipsoidal CPS (MSECPS) and (d) Semi-circular cylindrical (MSCCPS).

Fig. (6.17) shows comparison and validity of SLR-based computed line parameters of multilayer non-planar CPS with $\varepsilon_{r1}=12.9$, $\varepsilon_{r2}=10$, $\varepsilon_{r3}=10$, $\theta=6^\circ$ and $\psi=12^\circ$ against EM-simulators for frequency range 1 GHz – 60 GHz. Fig. (6.17a) shows that effective relative permittivities of semi-ellipsoidal CPS and semi-circular CPS are more than elliptical and circular CPS. Also effective relative permittivity of CPS on elliptical surface is higher than that of on circular surface. Fig. (6.17b) shows that characteristic impedance on these surfaces follows inverse rule. The computation of $\varepsilon_{eff}$ and $Z_0$ by the model has % average and % maximum deviation of (4.9%,7.6%) and (5.2%,11.9%) respectively against both the EM simulators, as shown in Fig.(6.17a) and (6.17b).
In Fig. (6.17c) variation in $T_{\alpha}$ of multilayer non-planar CPS for conductor thickness range 0.25 $\mu$m - 10 $\mu$m at $f = 25$ GHz is shown. The loss on semi-ellipsoidal CPS is the least and it is maximum on the circular CPS. The results are very close to each other. The $\%$ average and $\%$ maximum deviation of closed-form model #1 and closed-form model #2 against EM-simulators are (5.8%, 15.5%) and (4.4%, 11.4%) respectively.
6.8 Circuit Model of CPS

In this section, the accuracy of the circuit model of the CPS are compared against four softwares- HFSS, Sonnet, CST and MOM-based software LINPAR on the effective relative permittivity, real and imaginary characteristics impedances and total loss.

We compare accuracy of our parameters extraction against the extracted line parameters using EM-simulators- HFSS, Sonnet, CST and MOM-based software LINPAR. We have taken 0.01 GHz – 10 GHz frequency range and the CPS is considered on the substrate with \( \varepsilon_r = 12.9, s = 8\mu m, w = 4\mu m, h = 200\mu m \) and \( t = 0.25\mu m \). The extracted frequency dependent \( RLCG \) line parameters are shown and compared in Fig.(6.19).

The line resistance \( R \), presented in Fig.(6.19a) shows increase with frequency. As frequency increases, the skin-depth becomes smaller than the conductor thickness and surface resistance increases as the square-root of frequency. The results obtained from circuit model are closer to LINPAR and HFSS. The line inductance \( L \), presented in Fig.(6.19b), shows decrease in its value with increase in frequency from 0.01 GHz to 1 GHz. It is due to decrease in the internal inductance of the CPS. Above 1 GHz change in line inductance is insignificant. The results of the circuit model follow results of EM-simulators closely.

The line capacitance \( C \), presented in Fig.(6.19c), also shows decrease in its value with increase in frequency from 0.01 GHz to 10 GHz. The non-causal circuit model closely follows results of HFSS. It follows physical behavior of a capacitor. The results of circuit model and EM-simulators, presented in Fig.(6.19d), show increase in line conductance \( G \) with frequency. The circuit model closely follows EM-simulators.
Finally we obtain frequency dependent effective relative permittivity, characteristic impedance - real and imaginary parts, and total loss from the RLCG parameters. These results obtained from the circuit models are presented in Fig.(6.20) and compared against the results of EM-simulators. The results of circuit model follow results of EM-simulators; whereas results of our previous closed-form individual models do not follow simulators, especially at lower end of frequency inspite of our accounting for the internal inductance effect. It is true for all the parameters - effective relative permittivity, losses and characteristic impedance.
Table-6.5 consolidates the comparison of the models against HFSS for line parameters of CPS. The circuit model accurately predicts and improves the dispersive nature of the CPS at the lower frequency range for computation of \( \varepsilon_{\text{eff}}(f,t), \alpha_T(f,t) \), \( Re(Z_0(f,t)) \) and \( Im(Z_0(f,t)) \) with % average and % maximum deviation of (3.2%, 7.5%), (3.7%, 6.2%) and (2.6%, 5.5%) respectively against EM- simulators. The circuit model is applicable to all the line structures presented in this chapter. However for sake of brevity, we are not presenting the results.
Table - 6.5: % Deviation of models against HFSS
>Data range: t =0. 25 µm; f = 0.01 GHz - 10 GHz; ε_r=12.9; tanδ=0.0002; σ=4.1x 10^7 S/m

<table>
<thead>
<tr>
<th>Model</th>
<th>$\varepsilon_{eff}$</th>
<th>$Z_0$ (Ω)</th>
<th>$\alpha_T$ (Np/cm)</th>
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<tbody>
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<td>LINPAR</td>
<td>6.4</td>
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</tr>
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<td>Sonnet</td>
<td>1.8</td>
<td>4.5</td>
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<td>CST</td>
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<td>Closed-form Model</td>
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<td>63.3</td>
<td>19.7</td>
</tr>
<tr>
<td>Circuit Model</td>
<td>3.2</td>
<td>7.5</td>
<td>3.7</td>
</tr>
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