Chapter 1

Introduction

In development of modern physics, symmetry principles have been proved to be the most invaluable tools. Gauge field theories which are based on the local gauge invariance of the Lagrangian density of the theories have found enormous importance in describing all the fundamental interactions of nature and play the key role in understanding the particle physics phenomenon. The standard model of particle physics which describes strong, weak and electromagnetic interactions in the unified manner is regarded as the most successful theory because of its ability to explain the varieties of experimental results. The standard model is a non-Abelian gauge theory which serves as a paradigm example of quantum field theory. It illustrates wide range of physics such as spontaneous symmetry breaking, study of anomalies, non-perturbative behavior etc. Recently it has found applications in many other fields such as nuclear physics, astrophysics, cosmology etc.

In 1954, C.N. Yang and R. Mills [1] proposed a theory of the strong interactions between protons and neutrons, which is based on the $SU(3)$ algebra known as non-Abelian gauge theory. Non-Abelian theories are fundamental building blocks for the construction of physical theories. However, one faces various problems to develop the quantum version of such theories with local gauge invariance consistently. In path-integral quantization of these theories the vacuum-vacuum transition amplitude or generating functional is ill defined for such theories. This problem arises due to over counting of physically equivalent gauge configurations grouped together in different gauge orbits. To solve this problem the
method of gauge-fixing was used. This helped in removing infinite factor in path integral measure by choosing one gauge field from each orbit. The gauge-fixing was achieved by adding an extra term consisting of arbitrary function of gauge field and arbitrary parameter in the action. The addition of gauge fixing term solved the problem of over counting but introduced other problems like, the physical theory became dependent on arbitrary function of gauge field and/or an arbitrary parameter which is not desirable. To tackle these problems Faddeev-Popov (FP) [2] proposed an effective action by introducing ghost fields. Ghost fields are scalars in nature but behaves like Grassmanians and hence do not follow spin-statistics theorem. These unphysical fields compensate for the effect of arbitrary gauge-fixing function hence preserve the unitarity of the theory. But the total action is no longer gauge invariant which leads to various difficulties in the theory. For example the choice of counter terms in renormalization of the theory is no more restricted to gauge invariant terms as the gauge invariance is broken for the theory itself. This leads to the difficulties in renormalization program.

Four physicists C. Becchi, A. Rout, R. Stora and I. V. Tyutin (independently) [3, 4, 5, 6] found a very interesting symmetry transformation of FP effective action known as BRST Transformation. The analytical form of the BRST transformation and its relevance to renormalization and anomaly cancellation were described by Becchi, Rouet and Stora in a series of papers culminating in the 1976, “Renormalization of gauge theories” [3, 4, 5]. The equivalent transformation and many of its properties were independently discovered by Tyutin [6]. Its significance for rigorous canonical quantization of a Yang-Mills (YM) theory and its correct application to the Fock space of instantaneous field configurations were elucidated by T. Kugo and I. Ojima [7]. These symmetry transformations have following characteristics. They are (i) infinitesimal (ii) global (i.e. independent of space and time) (iii) anti-commuting (iv) nilpotent. Sometimes the nilpotency is proved using equation of motion of one or more fields then it is referred as on-shell nilpotent. However, BRST transfor-
formation can be made off-shell nilpotent by introducing Nakanishi-Lautrup (NL) type auxiliary fields to the theory. These transformations are extremely useful in characterizing various field theoretic models and renormalization of gauge theories are known to be greatly facilitated by the use of BRST transformations. These transformations enables one to formulate Slanov-Taylor (ST) [8] identities in a compact and mathematically convenient form. There is another symmetry of gauge fixed action known as anti-BRST symmetry. In this symmetry the role of ghost field changes with anti-ghost field [9, 10]. The anti-BRST symmetry does not add anything substantial to BRST quantization procedure but is important in the geometrical description of the superspace formulation of gauge theories [11, 12]. The foundation of this thesis is based on solid platform of BRST formulation.

It has been found that the usual FP procedure which yields quadratic ghost action is not applicable to some supergravity models where quadratic ghosts are needed to preserve unitarity [13] and nilpotency of the BRST operator is ensured only by using the equation of motion for certain fields in the gauge fixed action. Such theories are said to have open algebra. In some theories the ghost action itself has additional gauge symmetry which needs further gauge fixing. These theories are called reducible gauge theories. For such theories, the field spectrum is enlarged by introducing further ghost of ghosts. FP procedure doesn’t work for general reducible theories or when the gauge algebra is not closed. In order to cover a wider class of gauge theories, a powerful technique of BRST quantization was proposed by I. A. Batalin and G. A. Vilkovisky known as field/anti-field (or BV) formalism [14, 15]. In this technique the effective action is extended by introducing anti-fields which satisfy more general and rich mathematical relation known as quantum master equation (QME). The interconnection between BRST formulation and field/anti-field formalism is a very exciting topic of recent research [16]. Field/anti-field formalism is based on the BRST symmetry with an infinitesimal, global and anti-commuting parameter. Field/anti-field
formalism is studied in path integral quantization method which uses Lagrangian formalism. This formalism has been reviewed in [17].

Another powerful technique of BRST quantization in the Hamiltonian approach is BFV formalism developed by I. A. Batalin, E. S. Fradkin and G. A. Vilkovisky [18, 19, 20]. This method is used to construct BRST transformation of constrained systems [21]. It is not only applicable to the systems with first class constraints but also applicable to the systems with second class constraints [22]. This technique relies on BRST transformations which are independent of the specific gauge condition. In this technique, the BRST charge is constructed from the set of first class constraints of the theory by introducing a pair of ghost field and corresponding momenta for each set of constraints. For the system of second class constraints, the BRST charge is constructed after converting the second class constraints to first class constraints via various techniques. This method uses the enlarged phase space where Lagrange multipliers and their corresponding momenta are treated as a dynamical variables [23]. The main features of BFV approach are as follows: (i) it does not require closure (off-shell) of the gauge algebra and therefore does not need an auxiliary field, (ii) this formalism relies on BRST transformation which is independent of gauge-fixing condition and (iii) it is also applicable to the first order Lagrangian. Hence it is more general than the strict Lagrangian approach [24]. There are various ways to study BRST formulation such as BV-BRST and BFV-BRST formalism as described above. We mainly consider different generalizations of BRST symmetry in the context of BV-BRST and BFV-BRST formalisms.

To convert second class constraints to first class constraints we use a general method known as BFFT (Batalin-Fradkin-Fradkina-Tyutin) method developed by four physicists I. A. Batalin, E. S. Fradkin, T. E. Fradkina and I. V. Tyutin [25, 26, 27]. This is an iterative technique to change the second class constraint to first class constraint. This method has been used to study many of the mathematical models in recent years [28, 29, 30, 31, 32, 33, 34, 35].
BRST symmetry has been generalized in many ways. In 1993, Lavelle and Macmullan [36] found a generalized BRST symmetry adjoint to usual BRST symmetry in case of QED. This generalized BRST is non-local and non-covariant. The motivation behind the emergence of this symmetry was to refine the characterization of physical states given by the BRST charge. Since locality has been considered to be the main cause of infinities in the usual quantum field theory, people have been turning to non-local quantum field theory [37, 38]. Non-local gauge symmetry plays an important role in non-local quantum field theories. Later, Tang and Finkelstein [39] found another generalized BRST symmetry which is non-local but covariant. Such a BRST is not necessarily nilpotent but can be made nilpotent under certain condition in auxiliary field formulation. This symmetry imposes a constraint on the physical states, which determines the physicality more strongly than previous BRST symmetries. Later two physicists, Yang and Lee [40] also presented a local and non-covariant BRST symmetry in the case of Abelian gauge theories.

S. D. Joglekar and B. P. Mandal [41] further generalized the BRST transformation by allowing the parameter to be finite and field dependent. Such generalized BRST transformations are also symmetry of the effective theory and they are nilpotent. However, the path integral measure is not invariant and give rise to a non-trivial Jacobian. The Jacobian is shown to produce exponential term of local fields which changes effective action to give rise to another new effective action. Such generalized BRST transformations have found many applications [42, 43, 44, 45, 46, 46, 48] namely, to find correct prescriptions for the poles in the axial gauge field propagator [42, 43], to regularize the energy in the Coulomb gauge [44] etc. Recently a new technique of finite BRST transformation has been developed by some Russian physicists in a series of papers [49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. Also some important results about BRST for various physical systems have been developed recently [60, 61, 62].

In usual BRST transformation, variation of the kinetic part of the effective action independently vanishes whereas the variation of gauge
fixing part cancels with the variation of ghost part of the effective action. One of the important generalizations of the BRST symmetry as local and covariant BRST symmetry is known as dual-BRST symmetry [63]. Under dual-BRST symmetry, the variation of gauge fixing part independently vanishes whereas the variation of the kinetic part cancels with the variation of ghost part of the action. So far in the literature, dual-BRST symmetry has been treated as an independent symmetry because of its analogy to the co-exterior derivative in the language of differential geometry. Therefore sometimes it is referred as co-BRST symmetry. The usual BRST symmetry is analogous to the exterior derivative. The anti-commutators of exterior derivative and co-exterior derivative gives a Laplacian operator analogous to the bosonic symmetry [40, 63, 64, 65, 66].

Another generalization of the BRST transformations can also be made for YM theory in which the anti-commuting parameter is spacetime dependent [67]. These are not exact symmetries of the theory, however they do lead to a non-trivial Ward-Takahashi (WT) identity. This non-trivial WT identity could lead to new consequences which are not contained in the usual WT identity. Such generalized BRST transformations are realized as the broken orthosymplectic symmetry found in the superspace formulation of YM theory [68].

BRST and anti-BRST symmetries are treated as an independent symmetry only if they absolutely anti-commute amongst themselves. Similarly, dual BRST and anti-dual BRST symmetries are independent symmetries only if they absolute anti-commute. In order to make them absolutely anti-commutative, a restriction is invoked. Such restrictions are known as Curci-Ferrari (CF) restrictions [9]. Although, it is necessary to invoke these restrictions but reason behind imposing such restrictions are not clear in the Lagrangian framework. It is also not known, what kind of constraints they are in the language of Dirac’s prescription of constraint analysis.

The consequences of BRST symmetry, formulated as Slanov-Taylor (ST) identities, are central to the discussion of renormalizability, unitar-
ity, gauge independence of the theory. Any attempt that sheds light on, offer a reformulation of, understanding of BRST symmetry and YM theory is, therefore of significance to particle physics. This motivates us to construct various generalizations of BRST symmetry and their applications in quantum field theory.

This thesis is mainly based on the construction of three important aspects of BRST symmetry. BRST symmetry, dual-BRST symmetry and generalized BRST symmetry with a finite field dependent parameter. The BFV Hamiltonian formalism has been explored in the context of BRST symmetries. This formalism has been applied to mathematical models like particle on a torus, particle on torus knot etc. The FFBRST formalism has also been explored in context of various field and string theory models like Maximal Abelian (MA) gauge in YM theory, Nambu-Goto string in light-cone gauge and bosonic string in harmonic gauge etc. In first chapter we will introduce the important results related to BRST transformation. In II chapter we will discuss about various mathematical techniques required to solve problems related to BRST transformations. In the III chapter we will discuss about maximal Abelian gauge and its use in addressing the confinement problem. Then using the FFBRST transformation we will try to study the confinement problem in more general Lorenz gauges. In IV chapter we will discuss about various nilpotent symmetries related to particle on torus. In the same chapter we will develop BRST and anti-BRST symmetries for particle on torus knot for the first time. In V chapter we will discuss about Weyl degrees of freedom in Nambu-Goto string in light-cone gauge using finite field BRST transformation. In VI chapter we will address the problem of ghost number anomaly in conformal gauge in bosonic string by connecting it to action in harmonic gauge using FFBRST transformation. In the VII chapter we will summarize the total work done. This thesis is divided into the following seven chapters. The detailed content of these chapters are given below.

Chapter I is dedicated to the general introduction of BRST sym-
metries and related topics like generalized BRST symmetries, basic techniques like field/anti-field formalism in Lagrangian approach and BFV formalism in Hamiltonian approach. Brief discussion about various chapters is also presented.

In Chapter II, we will discuss mathematical techniques related to BRST formalism in detail, both in Lagrangian as well as Hamiltonian approach. At first we will discuss field/anti-field formalism or BV formalism in detail. There we will discuss about classical/quantum master equations and generation of BRST transformations. Then we will talk about Hamiltonian BRST formalism in which we will discuss about Dirac’s constraints analysis where we will discuss about first and second class constraints. Then we will discuss about (BFFT) formalism of conversion of second class constraints to first class constraints. Then we will discuss about BFV formalism. At last we will discuss about FFBRST transformation.

In Chapter III, we will apply a generalized (BRST) formulation to establish a connection between the gauge-fixed SU(2) YM theory formulated in the Lorenz gauge and in MA gauge. It is shown that the generating functional corresponding to the FaddeevPopov (FP) effective action in the MA gauge can be obtained from that in the Lorenz gauge by carrying out an appropriate FFBRST transformation. The present FFBRST formulation might be useful to see how quark confinement is realized in the Lorenz gauge.

In Chapter IV, we will investigate all possible nilpotent symmetries for a particle on torus. We explicitly construct four independent nilpotent BRST symmetries for such systems and derive the algebra between the generators of such symmetries. We show that such a system have rich mathematical properties and behaves as double Hodge theory. We further construct the FFBRST transformation for such systems by integrating the infinitesimal BRST transformation systematically. Further we develop BRST symmetry for a particle on the surface of a torus knot by analyzing the constraints of the system. The theory contains second
class constraint and has been extended by introducing the Wess-Zumino term to convert it into a theory with first class constraints. BFV analysis of the extended theory is performed to construct BRST/anti-BRST symmetries for the particle on a torus knot. We will show how various effective theories on the surface of the torus knot are related through the generalized version of the BRST transformation with finite field dependent parameter. In last section BRST/anti-BRST charge for particle on torus knot will be constructed using the technique used in ref. [148].

In Chapter V we will show how Weyl degrees of freedom can be introduced in the Nambu-Goto (NG) string in the path-integral formulation using the re-parametrization invariance of path integral measure. We first identify Weyl degrees of freedom in conformal gauge using BFV formulation. Further we change the NG string action to the Polyakov action. The generating functional in light-cone gauge is then obtained from the generating functional corresponding to the Polyakov action in conformal gauge by using suitably constructed FFBRST transformation.

In Chapter VI we consider Polyakov theory of Bosonic strings in conformal gauge which is used to study conformal anomaly. However it exhibits ghost number anomaly. We show how this anomaly can be avoided by connecting this theory to that of in background covariant harmonic gauge which is known to be free from conformal and ghost number current anomaly, by using suitably constructed FFBRST transformation.

Chapter VII has an overall conclusion of the thesis.