Chapter 1

Introduction

Networks are generic representations of complex systems in which the underlying topology is a graph. Networks are generally used to model empirical data from real world problems where the relationship between given components is of importance and may evolve with time. However, before formally describing a network, I must introduce a few preliminary graph-theoretic terms and other notations that have been used throughout this work.

A vocabulary $\tau$ is a set consisting of relation symbols $P, Q, R, \leq, ...$ function symbols $f, g, h, \cdot, +, ...$ and constant symbols $c, d, 0, 1, ....$

To every relation symbol and every function symbol there is a natural number $\geq 1$ attached to it, the arity of the symbol. For a fixed vocabulary $\tau$, a structure $\mathcal{A}$ for $\tau$ (a $\tau$-structure) is a nonempty set $A$ together with

(i) relations $R^A \subseteq A^n$ for every n-ary relation symbol $R \in \tau$

(ii) functions $f^A : A^m \rightarrow A^k$, where $k \in \mathbb{N}$, for every m-ary function symbol $f \in \tau$

(iii) constants $c^A \in A$ for every constant symbol $c \in \tau$.

An example of a structure is a group $(G, \cdot)$ such that the group axioms, namely closure,
associativity, existence of identity element and existence of inverse for each element, are satisfied. Another example of a structure is a graph \[30\] [198].

### 1.1 Preliminaries of graph theory

A graph \(G\) is a three tuple \((V, E, \psi_G)\) where \(V\) is the set of vertices, \(E\) is the set of edges and \(\psi_G : E \rightarrow [V]^2\) is the incidence function that associates with each edge in \(E\) an unordered pair of vertices of \(G\). For an edge \(e\), such that \(\psi(e) = \{v, u : \text{and} v \in V(G)\} \in E(G)\), one calls the vertices \(u\) and \(v\) as \textit{adjacent} (to each other) or connected (to each other) or neighbours (of one another). An edge is said to connect or join two vertices and the edge is said to be \textit{incident} with the given two vertices. Two same vertices can be connected by multiple edges and one edge can connect one vertex to itself. The cardinality of the set \(V\) is called the \textit{order} of the graph, and the cardinality of the set \(E\) is called the \textit{size} of the graph. A \textit{trivial graph} is graph consisting of a single vertex and no edges.

An edge of the form \(\{v, v\}\) which said to join a vertex to itself is called a \textit{self-loop} or (more commonly) a \textit{loop}. Multiple edges between a given pair of vertices \(\{v, u\}\) are called \textit{multi-edges}. A graph without self-loops and multi-edges is called a \textit{simple graph}.

A graph \(G_0 = (V_0, E_0, \psi_0)\) is called a \textit{subgraph} of a graph \(G = (V, E, \psi)\), if \(V_0\) is a subset of \(V\) and \(E_0\) is a subset of \(E\) and \(\psi_0|_{E_0} = \psi\).

A \textit{directed graph} or \textit{digraph} is a three tuple \(G_D := (V_D, E_D, \psi_D)\) such that \(V_D\) is the set of vertices, \(E_D\) is the set of directed edges or arcs and \(\psi_D : E \rightarrow V \times V\) is the incidence function that associates with each arc in \(E_D\) an ordered pair of vertices of \(G\). for an arc, \(e = (v, u) \in E\) vertices \(v\) and \(u\) are designated as \textit{tail} and \textit{head} respectively. A directed edge is said to be directed from its tail to its head. The tail and the head of a self-loop are the same vertex.

Two graphs \(G\) and \(H\) are \textit{isomorphic}, written \(G \cong H\), if there are bijections
1.1 Preliminaries of graph theory

\( \theta : V(G) \rightarrow V(H) \) and \( \phi : E(G) \rightarrow E(H) \) such that \( \psi_G(e) = \{u, v\} \) if and only if \( \psi_H(\phi(e)) = \{\theta(u), \theta(v)\} \). Alternatively, one can say that two graphs \( G_1 = (V_1, E_1, \psi_1) \) and \( G_2 = (V_2, E_2, \psi_2) \) are called isomorphic if there exists a bijection \( \gamma : V_1 \rightarrow V_2 \) such that \( \{v_i, v_j\} \in E_1 \iff \{\gamma(v_i), \gamma(v_j)\} \in E_2 \), for all \( v_i, v_j \). The structure of a graph is its adjacency and hence a graph may also be represented mathematically as \( G = (V, E) \) such that the incidence function \( \psi \) is implicitly known.

For a graph \( G \), the degree \( d(v) \) of a vertex \( v \) is the number of edges which are incident to \( v \). The monotonic sequence of degrees of vertices belonging to the set of vertices \( V = V(G) \) is called degree sequence of the graph \( G \). A vertex with degree zero is called an isolated vertex. A vertex with degree 1 is called a pendant vertex. A graph is called \( r \)-regular or regular of degree \( r \) if each of its vertices have degree \( r \).

Let \( k_n \) be the number of vertices with degree \( n \) in a graph \( G \). Then the distribution of \( k_n \) as a function of \( n \) is called the degree distribution of the graph \( G \).

A walk in a graph \( G \) is an alternating sequence of vertices and edges,

\[ W = v_0, e_1, v_1, e_2, ..., e_n, v_n \]

such that for \( j = 1, ..., n \), the vertices \( v_{j-1} \) and \( v_j \) are connected by the edge \( e_j \). Moreover, if the edge \( e_j \) is directed from \( v_{j-1} \) to \( v_j \), then \( W \) is a directed walk. The vertices \( v_0 \) and \( v_n \) are called starting vertex and terminal vertex respectively (collectively they are called as end vertices), while all the other vertices in the sequence are termed as internal vertices. A closed walk is a walk in which the starting vertex \( v_0 \) and the terminal vertex \( v_n \) of the walk are identical. In a simple graph a walk may be represented by listing a sequence of vertices: such that the vertices are adjacent. The length of the walk is number of edges (counting repetitions).

A trail in a graph is a walk such that no edge occurs more than once. An Eulerian trail in a graph \( G \) is a walk that contains each edge of \( G \) exactly once.

A path in a graph is a trail such that no internal vertex in the sequence is repeated.
The length of the path is again the number of edges in the sequence. A path of length $N$ is denoted as $P_N$. A cycle is a closed path (i.e., same starting and terminating vertices).

The shortest path length or shortest distance (sometimes simply distance) between two vertices $u$ and $v$ is the length of the shortest path (which is a subgraph of the original graph) where the vertices $u$ and $v$ are the end vertices. The maximum over all shortest distances in a graph is called the diameter of the graph.

A graph is called a connected graph, if for every pair of vertices $u$ and $v$, there exists a path where $u$ and $v$ are end vertices. Otherwise, the graph is called disconnected. A disconnected graph consists of more than one connected subgraphs, and any of these connected subgraphs is called a connected component or component of the disconnected graph.

A tree is a connected graph without any cycle.

A totally or fully or completely connected graph (or more commonly, complete graph) is a graph where every pair of vertices is connected by an edge. The complete graph of order $N$ is denoted by $K_N$. A completely connected subgraph of a graph is called a clique.

A graph $G$ is called a bipartite graph if the vertex set $V(G)$ can be partitioned into two disjoint subsets $V_1$ and $V_2$, such that each edge of $G$ connects a vertex in $V_1$ with a vertex in $V_2$. Hence there is no edge which joins two vertices in the same subset. If every vertex of one subset is connected by edges with all vertices of other subset, then the bipartite graph is called a complete bipartite graph, and is usually denoted by $K_{m,n}$, where $m$ and $n$ are the cardinalities of the two subsets.

A graph $G$ is called a weighted graph if there is a function $w : E \rightarrow \mathbb{R}$ defined on the set of edges such that $w$ assigns a real number to each edge in $G$ called the weight of the edge.
1.2 Random graphs

A random graph in its simplest form can be thought of as a graph in which the edges are random variables. In such a case, the object of study i.e. random graph does not remain one particular graph but becomes an ensemble or family of many possible graphs and the statistical or algebraic properties of this ensemble are of interest [26].

Erdős and Rényi proposed and studied a model [60] for random graphs which is now considered an important tool to study graphs formally. The Erdős and Rényi random graph model often denoted as $G(n, m)$, returns a graph of order $n$ and size (number of edges) $m$, which is chosen uniformly at random from an assembly of all possible graphs of order $n$ and size $m$.

Another random graph model, called the Gilbert’s model represented as $G(n, p)$ [77], is a bit different where the existence of an edge in a graph constructed from $n$ vertices depends on a fixed probability $p$. The whole process can be thought of as choosing a pair of vertices $v_i$ and $v_j$ form the set of vertices and an edge is put decisively between them with probability $p$. This process is continued till all pair of vertices are exhausted and simulated only once. The expected number of edges in this process are $pn(\frac{n-1}{2})$. For this model, in the large limit case the mean degree remains constant at a value $k = p(n-1)$ and the probability that a vertex has a degree $d$ is

$$p_n = \binom{n-1}{d} p^d (1-p)^{n-1-d} \approx \frac{k^d e^{-k}}{d!} \quad (1.2.1)$$

For $n \to \infty$, this equality holds given $d$ is fixed. For this reason, the graph generated by this model is sometimes called a Poisson random graph.

Erdős and Rényi further studied some monotonic-increasing properties on random graphs. In their subsequent works [61][62], they proved that graphs with a few edges less than a particular threshold number of edges are very unlikely to have some such property, whereas a graph with size slightly larger than the threshold size is almost certain to showcase the property under investigation. This phenomenon is also called phase
transition in random graphs [93]. In the study of complex networks, random graphs often serve as a source of reference with respect to some property under investigation such that the deviation of the property in real world complex network as compared to a random network leads to inferences about the peculiar nature of the real world complex network with respect to the property.

1.3 Matrices associated with a graph

There are certain matrices called connectivity matrices that are associated with a graph [14]. The most common connectivity matrix associated with a graph is called an adjacency matrix. Others include Laplacian matrix and normalized Laplacian matrix which are defined in this section. Suppose that $G = (V, E)$ is a simple graph where $|V| = n$. Suppose the vertices of $G$ are listed arbitrarily as $v_1, v_2, ..., v_n$. The **adjacency matrix** $A$ of $G$, with respect to this listing of the vertices is the $n \times n$ binary matrix or $(0, 1)$ matrix with 1 at its $(i, j)^{th}$ entry when $v_i$ and $v_j$ are adjacent, and 0 when they are not adjacent. In other words, if adjacency matrix is $A = [a_{ij}]$, then

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge of } G \\ 0 & \text{otherwise.} \end{cases}$$

Let $d_i$ denotes the degree of a vertex $v_i$ in the simple graph $G$. The **degree matrix** of the graph $G$ is a diagonal matrix given by $D = [a_{i,j}]$, where

$$a_{ij} = \begin{cases} d_i & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

The **Laplacian matrix** associated with the graph $G$ is a matrix $L = [a_{ij}]$ such that

$$a_{ij} = \begin{cases} d_i & \text{if } i = j \\ -1 & \text{if } \{v_i, v_j\} \text{ is an edge of } G \\ 0 & \text{otherwise.} \end{cases}$$

The **normalized Laplacian matrix** associated with the graph $G$ is a matrix $\mathcal{L} = [a_{ij}]$
such that

\[ a_{ij} = \begin{cases} 
1 & \text{if } i = j \text{ and } d_i \neq 0 \\
-\frac{1}{\sqrt{d_i d_j}} & \text{if } \{v_i, v_j\} \text{ is an edge of } G \\
0 & \text{otherwise.} 
\end{cases} \]

The Laplacian matrix \( L \) can be derived from the adjacency matrix \( A \) and degree matrix \( D \) as

\[ L = D - A. \quad (1.3.2) \]

The normalized Laplacian matrix \( \mathcal{L} \) is similarly related to the Laplacian matrix \( L \) as

\[ \mathcal{L} = D^{-\frac{1}{2}} LD^{-\frac{1}{2}} = I - D^{-\frac{1}{2}} AD^{-\frac{1}{2}}, \quad (1.3.3) \]

where \( I \) is the identity matrix, and the matrix \( D^{-\frac{1}{2}} \) is a diagonal matrix with entries \( \frac{-1}{\sqrt{d_i}} \) at its diagonal positions \( (D^{-\frac{1}{2}})_{ii} \).

Characterisation of graphs and studying the different relation between several graph properties based on the spectrum of these connectivity matrices has given rise to the field of spectral graph theory. Some theories developed in this field have found widespread applications in diverse areas, for instance Perron-Frobenius eigenvector of world wide web network [33] is used exclusively in Google search algorithms. In the study of complex systems using graph spectrum, useful information about the system is often inferred based on the values of largest, second largest and smallest eigenvalues of these connectivity matrices [39][191].

### 1.4 Networks and structural indices on networks

Networks are generic formulations of the interactions of the components of a complex systems in which graphs are used as a means of representation such that the various components can be thought of as vertices and the interaction of interest between these components is designated as edges [130]. Orientation of the edges in a network informs the researcher about the structural peculiarities of organisation in a network and this in turn often reflects upon some important information about the complex system under
investigation. The importance of using networks as an approach to model complex systems is the ample possibilities of investigations that are opened up about the complex system using the different structural indices, the results of which can be interpreted in a way which is specific to the complex system under investigation itself. Though often there are more than one method by which a complex system can be modelled using a network, however, given that a considerable network model of the interactive components in the complex systems is achieved i.e. a structural pattern is obtained, the structural information of the organization as characterized by some structural indices may lead to additive information about the complex system which may be open for interpretation. Many of the times networks are designed to study the transfer of entities (information, energy, etc) along its edges which are often also called as links [32][127].

Vertices in a network are sometimes referred to as nodes.

Formally, a network $N$ is a four tuple $N = (V_\lambda, E_\lambda, \psi_\lambda, \Lambda)$ along with an algorithm $A$ such that for $\Lambda \neq \emptyset, i \in \Lambda$, $V_\lambda$ is a set of vertices $V_i$, $E_\lambda$ is a set of edges $E_i$, $\psi_\lambda$ is incidence function $\psi_i : E \rightarrow [V]^2$ where $[V]^2$ is the set of not necessarily distinct unordered pairs of vertices such that $(V_i, E_i, \psi_i)$ is a graph given by the algorithm $A(i)$. The incidence function $\psi$ provides structure to a graph by associating to each edge an unordered pair of vertices in the graph as $\psi(e) = \{v_i, v_j\} : v_i, v_j \in V, \forall e \in E \subseteq [V]^2$. Here $i$ is the temporal component by virtue of which a network can evolve as per the given algorithm $A$ [185].

A network is thus an empirical object while the underlying graph that represents the network is an algebraic object. I call a network as static network if the temporal component $\Lambda$ consist of a single element $i$, otherwise the network is a dynamic network.

In recent years network theory has been used extensively in diverse fields to study a wide range of complex systems [131]. The application of network theory to different fields of research has in itself created nascent and independent avenues of research in humanities, sciences and technology. Networks that are studied in humanities to gain an insight into the organisational and evolutionary aspects of a society include but are
not restricted to social networks, archaeological networks and networks encountered in economics and finance [195][36][197][167][2].

Research conducted in recent years on technological networks such as information networks, infrastructure networks and computer networks has played an important role in improving the quality of life of a common individual [105][121][151][47]. In the development and exploration of modern sciences, networks have found a pivotal role with a myriad of studies being conducted on networks emerging specifically in chemical and biological sciences. Metabolic networks, protein interaction networks, neural networks, gene regulatory networks and ecological networks are just a few examples of networks that have been vastly studied by researcher from diverse backgrounds [131][81][97].

In order to quantify structural aspects or properties of a network, a structural index is often calculated. Let $G$ be a weighted or unweighted, directed or undirected graph and let $X$ represent the set of the set of vertices or edges of $G$ respectively. A real valued function $s$ is called a structural index if and only if the following condition is satisfied

$$\forall x \in X, \ G \cong H \Rightarrow s_G (x) = s_H (\phi (x)),$$

where $s_G (x)$ denotes the value of $s(x)$ in $G$ [32]. Thus, a real valued function defined on either the edge set or the vertex set of a graph (the underlying graph in case of a network), whose value is preserved under isomorphism is a structural index. A simple example of a structural index is the vertex degree, since the degree of a vertex remains invariant under isomorphism, an index that calculates vertex degree is a structural index. As shall be discussed later in the thesis, this index is commonly referred to as degree centrality. Since isomorphic graphs are always cospectral [33], a function that maps a vertex or edge in a graph to any algebraic expression involving the eigenvalues or eigenvector (of any of the connectivity matrix associated with a graph) which produces a real number as its value upon evaluation will be a structural index. Structural indices, both algebraic and spectral, are thus used to quantify the structural aspects of a complex network.
1.5 Ecological networks

Ecosystems with a variety of complex interactions possible between the varieties of components embedded within them are a primary example of complex systems [106][22][141]. Modelling interactive processes embedded in an ecosystem as complex network has been extensively studied in literature leading to an emerging field of ecological networks [20][150][18][134]. Recent advancements in ecological network flow modelling has led to the foundation of Ecological/Ecosystem Network Analysis (ENA), a method to holistically analyse environmental interactions [141][142][143][144][67][186][187][188][68]. Ecological networks can be considered as generic representations of mass-energy flow (usually in the form of carbon exchange) in an ecosystem[91][139]. A network approach to study ecosystems is important because in contrast to studying individual components embedded in an ecosystem, a network representation of the ecosystem may provide an insight into indirect interactions in an ecosystem and thus some emergent properties of the ecosystem can be examined [189][148][67].

Ecological networks have been widely studied in ecology for decades. In their earliest possible form, ecological networks stemmed from the elementary work of Hannon [80] and Finn [71] who applied the ideas of flow analysis to ecological systems, inspired by the work of Leontief [104] on input-output analysis of production in economics. In his book entitled "Ecological Networks", Margalef describes and studies ecological systems as networks and further go on to compare different kind of networks thus obtained for differences and similarities [111]. He is among the first researchers to promote an idea of heterogeneity across ecological networks, which is now a active area of interest for ecologists. The search for commonalities across network structure may indicate that the networks under study that show similarity in their structural properties as characterised by some parameters may have a similar mechanism of network formation. It may also indicate that some structural constraints must be applicable for a network embedded in an ecosystem to be stable under perturbation.
Network theory or rather graph theory found use in representing the mass energy flow in ecosystems from an early age, starting from the pioneer works of Lindenman and Odum [108][136]. They initially described mass-energy flow in an ecosystem as food web networks. The mass-energy flow could be the study of carbon exchange between some species or group of species present in a landscape or ecosystem. There are several processes that result in inter species carbon exchange and the network representation of such processes are collectively called food web networks [139]. The most common and widely studied process that is responsible for a majority of carbon exchange among species present in an ecosystem is predation [139]. The network representation of predation in an ecosystem, where vertices represent species or group of species and the edges represent the relation of carbon exchange between these species through predation and consumption, are called predator-prey networks.

Several predator-prey food web models have been described in literature. One of the first models, proposed by Cohen [43] is very similar to a random graph model in which each edge (indicating a predator-prey relationship) can be present with equal probability in the network, where the probability is given by an estimate for global connectence (ratio of edges present in a network to edges possible) in the network. This model was improvised further as a cascade model [44], in which the species are arranged on the basis of their body size on an axis and an edge is possible between two species of heterogeneous body size i.e. larger species only are allowed to prey on smaller species. This model has been since improved upon by introduction of a niche model by Williams and Martinez [201], in which a given species can prey on other species within a certain size range. Further since, biological knowledge such as adaptation and phylogenetic information has been integrated in such models to present a more realistic modelling approaches [38]. Thus the network structure of these refined models are discrete representations of ecological dynamics in an ecosystem [180].

A recurrent motif of research interest in ecological networks have been the study of relationship between structure of the network and robustness of the ecological system
it represents [73]. The usual approach of investigation has been to study the effects of perturbations (loss of species) on the network organisation. It is possible that the loss of some species may produce only local changes in the network structure while for some other species, their removal could produce a cascade effect on the global network structure and may entirely dismantle or produce components in the network. Thus an idea of keystone species has been developed [116][29][11][12].

Inter-species carbon flow can result from processes such as parasitism, pollination of flowering plants and seed dispersion in a landscape or ecosystem of interest. The networks originating as a result of such processes are host-parasite networks, plant-pollination networks and seed dispersal networks respectively. In host parasite network, vertices represent either a host species or a parasite species and there is an edge present between two vertices if one of the species is a parasite to the other species which is a host species [192][148].

In a plant-pollination network, the vertices are either plant species of pollinator (insects, birds etc.) species and an edge is present only between a plant and pollinator species which represent that the particular pollinator species pollinates the given plant [21][51][133]. In a similar manner, seed dispersal networks comprises of plant species and species that disperse the seeds of such plants as vertices and edges represent the dispersion relation [168]. It must be clear that the underlying graphs in these three kind of networks, i.e. host-parasite networks, plant-pollination networks and seed dispersal networks are bipartite graphs such that the vertices can be partitioned as host vertices and parasite vertices, plant vertices and pollinator vertices and plant vertices and dispersion species vertices respectively such that any edge only connects a vertex from one partition to other in all the cases.

It is generally assumed that the complexity of food-web networks is captured in some simple algebraic measures such as connectance [58] and in literature the structure of these networks is often presumed to be similar to each other [202][139]. Several attempts have been made by ecologists to seek sources of difference in the structure
of bipartite ecological networks in the light of differences in processes generating such networks or based on the differences between ecologically significant structural indices across such networks [72][114][59][146][169].

Moreover, for a long period of time, the framework used for characterisation of bipartite ecological networks has remained unchanged [95]. Though indirect effects have been used relatively recently in literature to describe bipartite ecological networks [203], algebraic structural indices such as vertex degree, which at most is a local measure, are often used to describe or quantify the network globally. Thus describing a bipartite ecological network using some algebraic structural indices may lead to loss of ecological information under consideration when comparing the networks [135].

Additionally, the mass-energy flow could be in the form of flow of genetic or ecological information (migration of species, seed dispersion etc.) across a specific landscape, where the topographical and geographical features in the landscape may aid or resist the flow of ecological information [179]. The topographical features in the landscape thus compartmentalise the whole landscape and these compartments act as components present in the landscape where the interaction between these components (i.e. flow of ecological information among the compartments) is modelled as a network [179][91][35]. Such networks are known as network of ecological connectivity with respect to some focal species of interest [119][120][185][155][164]. An example of a network of ecological connectivity of some species (for instance, tiger) in a landscape is a network where vertices represent large conserved forest areas and edges represent corridors of thin and contiguous forest cover preserved between these conserved forest areas which aid in the movement of focal species along the landscape and thus ecological connectivity between the conserved forest areas is maintained [164].

Maintaining connectivity in a landscape is important in promoting the survival and vitality of species present in the landscape through flow of ecological information in the form of organism movement, seed dispersal and other ecological processes [40][66]. A massive challenge faced by conservation ecologists and land managers in the present age
is forest or habitat fragmentation, where larger forest or habitat patches for some species are increasingly turning into smaller fragmented patches, providing an increased risk and resistance for gene flow along the fragmented landscape [98][25] [173][162]. Thus, maintaining connectivity and mitigating the fragmentation of habitat may be critical for landscape process such as gene flow and dispersal [178][194].

Ecologists thus turned their attention to study of spatial or landscape networks to be able to tackle the problem of habitat loss for land connectivity and its consequences on the metapopulations [190][76][48][64][25][19]. Metapopulation theory has long described the network of forest patches [84][83]. However, by borrowing ideas such as perturbations on network in the form of vertices removal, a characterisation of robustness of a landscape based on the network structure has been achieved by ecologists [73]. Thus studying how network topology affects metapopulation processes has lead to estimates on how long shall a certain species shall survive in the wild [65], thus giving conversation ecologists a time-bound in which to process, frame and act out their policies.

1.6 Aim and organisation of the thesis

A comprehensive study of literature on progress in structural aspects of ecological networks and its implications on the ecosystem has inspired me to ask some questions on the process-pattern aspects and organisation of an ecosystem (a complex system in general). First, I have found that a very few studies have addressed the landscape connectivity of some key species which are susceptible to climate change in the Indian subcontinent using network theory and a lot remain unknown about the nature of challenges faced by these native Indian species in face of increasing landscape fragmentation. Thus I study the ecological connectivity of some moist temperate floral species using some structural indices on forest network in Western Himalayas. The structural indices informs oneself about the nature of information flow (flow of ecological information in the present scenario) across the network and its consequences on the maintenance of
ecological connectivity at a local and global scale.

Second, I find that while species invasion in a landscape is a common phenomenon studied extensively by ecologists, no body of work to the best of my knowledge exist that make use of a network structure to study and investigate invasion of a species in some study area. Thus, I use network theory as a novel approach to study invasion of \textit{Lantana camara} in a part of Rajaji Tiger Reserve and draw conclusions on the nature of invasion of lantana as a process, using the structural indices on network of potential lantana patches derived using knowledge on topographical and geographical features of the landscape integrated with ecological and biological properties of \textit{Lantana camara}.

Third, I have found that the difference in the structure of food web networks is often quantified using some basic algebraic measures which may not be sufficiently apt at describing the local and global structure of these networks and characterise the difference in structure of these networks. I thus use some spectral structural indices to differentiate between the structure of food-web networks. Hence, as I get informed more and more about the advantages of using structural indices, both algebraic and spectral, to study structural organisation of ecosystems, I open up on broad queries formed around pattern-process interactions in an ecosystem and a complex system in general.

The main objective of study in this thesis is thus centred around the following questions that can be asked for a complex network in general. However, I have employed ecological networks to investigate these problems as processes that generate such ecological networks are thoroughly studied and well documented, while at the same time, ample data for ecological networks is available in public domain.

(i) Given the knowledge on the ecological or topographical properties of different interactive components of an ecosystem (a complex system in general), how can one form a network representation of the ecosystem? I wish to know if a legitimate network representation can be achieved for the ecosystem if one is aware of the
ecological properties of the components present in the ecosystem such that the interaction between these components can be modelled as a network.

(ii) Given the network representation of a complex system (an ecosystem in my case), what inference can be made about the regulatory mechanisms of the process that may conduce on the network based on the topological structure of the network? Complex networks are examples of structural patterns that are produced in a real world complex system by the means of modelling a process of interest. Thus if some information can be derived on the way process regulates in the given complex system is informative about the complex system itself. Thus I wish to know how a network representation of an ecosystem informs us about the ecosystem.

(iii) How the topological structure of a network is itself governed by the process that it represent? The structural patterns themselves may emerge in a complex system as governed by the feedbacks obtained as a result of interactions between the several components of a complex system i.e. the process may in turn shape the patterns in complex systems. Thus I wish to find if the different ecological processes that are represented by a network show considerable difference in the structure of the networks based on the differences in the environments or ecosystems.

To be able to answer the above stated queries, I have employed different structural indices as the key informant about the structural properties of the complex network and thus try to gain an understanding of the patterns-processes organisation of a given ecosystem. The structural indices used in this thesis are both algebraic structural indices and spectral structural indices and are used in a way that structural information represented by these structural indices leads me to answer the central problems described in this section. These structural indices are subsequently defined in the following chapters and the structural information that these indices generate for a network in general and for the specific networks under study are discussed in details. The thesis is organised as follows.

In Chapter 2, I study the growth and connectivity on the landscape level of anemo-
1.6 Aim and organisation of the thesis

Chory (wind dispersal) of some Himalayan moist temperate forest species in the Indian
parts of Western Himalayas. The floral species identified in this work are representational
species in these regions and are highly susceptible to climate change and thus
it is important to maintain ecological connectivity of these species in the region. The
primary objective of this study is to identify the regions within the study area which
are crucial for maintaining ecological connectivity and aiding the flow of ecological in-
formation in the entire region.

In Chapter 3, I wish to present a study of potential invasion and spread of *Lantana
camara* in Chilla range present within Rajaji Tiger Reserve (RTR). *Lantana* sp. as an
invasive species is often considered a threat to native ecosystem. I investigate lantana
invasion as a complex network perspective, where relatively dense and large potential
lantana distribution patches are considered as vertices, connected by relatively sparse
and thin potential lantana continuities which are identified as edges. Here, I wish to
identify regions in the study area which aid the spread of lantana in the entire study
area.

In Chapter 4, I use random projections of weighted spectral distribution to com-
pare thirty one predator-prey networks described in previously published studies to
analyse if there is any considerable difference in the structure of these predator-prey
networks as maybe due the difference between the ecological environments from which
these networks originate. To investigate this idea, I classify the predator-prey networks
either as aqueous predator-prey networks if they originate from aqueous environments
or as terrestrial predator-prey networks if the ecosystem they are found embedded in
is a terrestrial ecosystem and try to assimilate if an structural difference is observed in
the random projections. I further try to establish that any difference observed may not
be the property of a random graph.

In Chapter 5, I compare ecological networks originating as a result of three dif-
ferent ecological processes using random projections of weighted spectral distribution
to assess if the underlying ecological processes in these systems produce considerable
difference in the structure of the networks. Absence of any significant difference in the structure of the networks may indicate towards the possibility of a universal structural pattern in these ecological networks. The underlying graphs of the networks derived by the ecological processes, namely host-parasite interaction, plant pollination and seed dispersion are all bipartite graphs and thus several algebraic structural measures fail to distinguish between the structure of these networks. Thus I use spectral structural indices to investigate the possible difference in the structure of these networks.

In Chapter 6, conclusions based on work done in this thesis and a possible future direction for my research is discussed.