INTRODUCTION

Humans are natural-born scientists. When we’re born, we want to know why the stars shine. We want to know why the sun rises.

Michio Kaku

1.1 CLASSICAL-QUANTUM CORRESPONDENCE AND NON-LOCALITY

Prior to the discovery of quantum theory, physics appeared to be solely governed on all scales by classical theories, in which physical systems are completely deterministic; each observable being measured with arbitrary precision. In classical physics, all observables are simultaneously measurable, and a specification of initial conditions in systems dictates with certainty the outcomes of measurements. This is not the case in quantum physics [79]. The most distinctive differences between classical and quantum systems have to do with the uncertainty principle and probability amplitudes [63]. Quantum states are described as probability amplitudes, which are rays in a Hilbert space.

The study of non-classical systems is of abiding interest in the physics community, and is where the domain of the work presented in this thesis lies.

1.1.1 The Classical Limit

Historically, the failure to explain black body radiation and atomic spectra through classical formulations led to the development of quantum physics as a means to replace classical physics. Due to the discomfort of scientists at the time with a theory that is governed by probability amplitudes, Einstein and his collaborators deemed the establishment of a quantum theory of the macroworld to be incomplete [33]. While the inequalities established by Bell, Clauser et. al. [12, 28], and their verification [8, 27] indicated otherwise, there is still no definite limitation or boundary established for the applicability of the principles of quantum physics. Until now, as a theory, it cannot fully replace classical physics. The very formulation of quantum physics requires a classical system governed by Hamiltonian dynamics.

Bridging the conceptual gap between the classical and quantum worlds, or “the infamous boundary” in Bell’s words, is considered an important goal, attractive to both theoretical and experimental physicists [52, 55]. Strides have been made in this direction. For instance, Dolde et. al. recently showed quantum entanglement at the macroscopic level and at room temperature by entangling engineered solid state spin qubits in diamond [31].

1.1.2 EPR Correlations

Einstein, Podolsky, and Rosen, hypothesised in 1935 that quantum physics is incomplete, through the help of a gedanken experiment [33, 60], due to the apparent appearance of superluminal com-
munication between entangled parties. This appearance is due to the instantaneous change in information about the probabilities.

To remark on the paradox that was illustrated through this thought experiment, we first understand an element of reality, as per their Criterion of Reality:

“Every element of the physical reality must have a counter-part in the physical theory... If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.” [33]

Equipped with this definition, they raised an apparent contradiction. They first assumed the quantum mechanical wave function provides a complete description of physical reality, from which they drew the conclusion that two physical quantities corresponding to non-commuting operators may share a simultaneous reality. Yet, quantum mechanics asserts that precise knowledge of one of the two non-commuting operators precludes precise knowledge of the second, thus they do not qualify for a simultaneous reality. This is the EPR paradox, which led them to believe that the wave function does not, in fact, provide a complete description of reality.

1.1.3 Local Hidden Variable Theorems

While Einstein acknowledged the existence of entanglement, he could not accept that a change at one place could influence, immediately, affairs at another [14]. The sentiment was that a complete theory doesn’t leave room for indeterminism of the kind quantum physics has. Local hidden variables were explored as a means to deem quantum theory complete, yet fell flat and led to the notion of non-locality [12].

1.1.3.1 The Bell Inequality

In his seminal 1964 paper on non-locality, Bell established an inequality that demonstrated that no physical theory of local hidden variables can ever reproduce all of the predictions of quantum physics [12, 15] (additionally relevant is the precursor work of Bohm [20]). Local realistic theories and quantum physics give rise to conflicting predictions for certain scenarios that involve space-like separated yet correlated events. Specifically, local realistic theories predict that Bell’s inequality will be obeyed, whereas quantum physics predicts that it will be violated [105]. States that violate the Bell inequality are non-local, and so, non-classical. Clauser, Horne, Shimony and Holt redefined the inequalities in a manner more experimentally accessible to verify the implications [28], giving rise to what we henceforth refer to as the Bell-CHSH inequality.
The Bell inequality has, since its formulation, been studied exhaustively, both experimentally [8, 27, 48, 65, 66, 75, 83, 87, 93, 99], and theoretically [10, 13, 27, 28, 43, 49, 98].

Among the first series of tests was [99], using photon pairs produced in atomic radiative cascades. The second generation of tests were performed by Aspect, Clauser, and Shimony [8, 27]. A third generation involved nonlinear splitting of ultraviolet photons to produce pairs of correlated EPR photons [65, 82]. Bell test experiments were said to be subject to two ‘loopholes’ [7]: the possibility of correlations of seemingly separate events being triggered by unknown superluminal signals, and detection efficiencies low enough to allow the possibility of the sub-ensemble of detected events to agree with quantum physics, despite the entire ensemble satisfying the inequalities. The issue of the possibility of not counting for all possible degrees of freedom a physical device might be sensitive to, thereby giving rise to possible ‘fake’ states, is covered in [44]. A local hidden variable model of quantum correlation exploiting the detection loophole is outlined in [46].

An experiment targeting the entanglement of four-photon states is outlined in [50]. Violation with particles of different ‘species’, in particular a single atom and a single photon, is demonstrated in [61]. In 2008, Weihs et. al. claimed to fully enforce the condition of locality for the first time [94]. Sufficient physical distance between the measurement sites yielded the necessary space-like separation of the observations, along with ultra-fast and random settings of the analysers, and completely independent data registration. Attempts have also been made to exhibit experimental violation of a Bell’s inequality in time [64, 67].

In 2015, the two loopholes – of detection and of locality – appeared to finally have been solved in [48]. It would therefore not be out of place to say that the validity of quantum physics rests on a firm experimental pedestal. A recap of the Bell inequality and a preliminary dive into the non-locality of multi-level systems has been provided in Chapter 2.

1.2 CLASSICALITY IN THE LARGE $N$ LIMIT

Despite extensive successes such as those described above, the study of all aspects of quantum physics, or for that matter, all tests of non-locality, remains to be exhausted. The classical limit of quantum physics is one such example, and the exhaustively studied two qubit systems are only the simplest. Coupled $N$-level systems are natural candidates to move in this direction.

There are proofs that a quantum system becomes classical with large $N$ [45, 100], where $N$ denotes the dimension of the Hilbert space for some degree of freedom in the quantum system. Yet, contrary to earlier expectations [59], it was realised that the two hallmarks of non-classicality, non-locality and entanglement, thrive even as $N \to \infty$, as shown in studies in higher dimensional bi-
partite systems [5, 6, 51, 68]. More recently proposed inequalities for testing non-locality in higher dimensional systems [29, 41, 51], demonstrate the same behaviour of thriving non-classicality.

In Chapter 3, we introduce the notion of ‘weak classical limit’ for coupled $N$–level quantum systems as $N \to \infty$, to describe the precise sense in which one attains classicality. This notion reconciles the seemingly contradictory results by showing that so-called classicality is not so much an inherent property of the system, as it is a consequence of limited experimental resources.

1.3 BELL-TYPE INEQUALITIES AND DETECTORS OF NON-CLASSICALITY

The Bell inequality, and other inequalities detecting non-locality – henceforth referred to as Bell-type inequalities – are constructed in such a way that they are obeyed by the set of all local states. Hence a violation of any of these inequalities is an unmistakable signature of non-locality. The converse statement is, however, not guaranteed to be true. That is, a state that obeys a given Bell-type inequality, need not be local (see [25] and references cited therein). Thus, it is pertinent to look at the respective domains of validity of multiple prescriptions of non-locality.

We look specifically at the Bell-CHSH form of the Bell inequality. A formulation that is inequivalent to Bell-CHSH and is often used for higher dimensional correlations is the CGLMP (Collins-Gisin-Linden-Massar-Popescu) prescription [29, 41, 51]. Bell-CHSH involves a single inequality with two dichotomic observables at each site. The latter involves a dimension-dependent inequality that also has two observables per site, however, each observable has $N$ distinct eigenvalues, $N$ being the dimension of each subsystem. For two qubits, CGLMP reduces to Bell-CHSH.

When applied to the family of fully entangled states $|\Psi_E\rangle = (1/\sqrt{N}) \sum_{j=1}^{N} |jj\rangle$, in $N \times N$ dimensions, it was found that the CGLMP identifies non-local states that evade the Bell-CHSH analysis. The predicted violations have been subsequently verified in a series of experiments for maximally entangled states [30, 50, 56, 85, 91], the last one involving coupled systems upto $N = 16$.

It would therefore appear that CGLMP is more effective in detecting non-local states than Bell-CHSH. If true, the states that violate CGLMP would form a superset over the ones that violate Bell-CHSH. In order to verify this claim, one should not be restricted to a particular class of states. It will therefore be prudent to look at how well CGLMP fares for maximally Bell-CHSH non-local states, that is, those states that violate the Bell-CHSH inequality maximally. The short answer, resulting from the foray described in Chapter 4 is: not very well. This is due to a natural, intuitive manifestation of the geometrical structure of local states in a joint correlation space in the
light of polytope theory [9, 25]. Thus CGLMP is not more discriminating of non-local states than Bell-CHSH.

Furthermore, non-locality is not the sole detector of non-classicality, though it is a good detector for coupled systems. Since quantum logic [19] gives rise to non-classical consequences, this very nature could be a powerful tool for the detection of non-classical states. A significant amount of work has been done in this regard, namely [36, 37]. Of interest here is the formalism proposed in [3], which provides a construction of quantum representatives of classical indicator functions of joint events involving multiple observables in a way that the quantum representatives follow a framework that is operationally Boolean, yet gives rise to non-classical probabilities. The basis of the formulation is the notion of pseudo-projection operators as these representatives. This formalism holds a strong promise of yielding a comprehensive framework for the detection of non-classicality, and was largely established for single and coupled qubit systems. We therefore extend this formalism in Chapter 5 to single qutrit systems, laying the groundwork for this formalism to address qutrit correlations.

1.4 STATE-INDEPENDENCE

In the context of information theory, certain quantum protocols have an edge over classical ones, since they harness inherently non-classical quantum features such as those of space-like separated correlations, that is, correlations within systems with space-like separated subsystems. For instance, entanglement and non-locality are powerful properties that can be used as information theoretic tools. However, there’s a lesser explored potential resource: time. By making use of time-like sequential measurements one can construct a state-independent Bell-CHSH inequality, even for a single qubit [23, 84]. In the light of quantum information theory, applications that are state-independent would eliminate the need for complex processes to prepare and manipulate quantum states. In Chapter 6, a state-independent secure quantum key distribution protocol is proposed, with the hope that a way is paved for more state-independent implementations, as well as an expedition of wide-scale implementation of QKD in the field of cryptography.

1.5 STRUCTURE OF THIS THESIS

The results of the above briefly mentioned studies have been collated and presented in the chapters ahead. The structure of this thesis is as follows: we begin with a recap of the Bell inequality, preliminarily studying the non-locality of multi-level systems (Chapter 2). We then move on to establish weak classicality primarily in the light of non-locality in the $N \to \infty$ limit (Chapter 3). Because there are multiple formulations of non-locality, we take the two most commonly employed
formulations and perform a detailed comparison (Chapter 4). We extend the pseudo-probability formalism, a distinguisher of non-classicality built upon the grounds of quantum logic, to single qutrit systems, as the previous work done in this regard was primarily for qubit systems (Chapter 5). We then move beyond the domain of space-like separated correlations to study time-like sequential measurements as a state-independent information theoretic resource, in the light of qubit systems – potentially extendable to higher level systems (Chapter 6).

We hope that this thesis will provide a good basis for the elements of non-classicality in multi-level systems, while paving the way for state-independent quantum information theoretic protocols.