1.1 Introduction

According to United Nations there are more than 7 billion living humans on the Earth while, 200 years ago this figure was less than 1 billion. Present upsurge in world population has presented mankind with a new challenge to build a sustainable world which ensures the security of the life for present and upcoming generations. Rapid population growth is jeopardizing all the development efforts and population problem exists as a challenge to the policy makers. The unprecedented expansion in world population and consequential socio economic problems faced by the human beings has stirred the policy planners and social researchers across the globe due to two major considerations; an increasing concern about relation between population growth and depletion of available natural resources and a growing awareness that unrestricted population growth tends to impose a strong constraint on the availability of a reasonable standard of living, health facilities and other basic requirements for survival of the mankind. Uncontrolled population growth is related to various problems
such as excessive consumption of non-renewable natural resources, over strained infrastructure, scarcity of employment opportunities and inequitable distribution of income, etc. There is, therefore, a major move throughout the world to bring down the growth rate of population dynamics.

Demography is a scientific study of the size, composition, spatial distribution of human population, and of changes overtime in these facets due to the changing levels of fertility, mortality, marriage, migration and social mobility. It is not only concerned with the advanced, empirical analysis of the mutual relationship between a population and its cultural and socio-economic conditions but also with the factors governing these relationships. A comprehensive study of human population must include insight from various disciplines. The well-known and well-established approaches to the study of human population dynamics include demographic, socio-cultural and biological investigations. The social and economic development of a country largely depend upon size and structure of the population and therefore the studies regarding the nature and behaviour of these components have always been a point of great interest for every society. Fertility, mortality and migration are three demographic factors which determine the size of a population. Since fertility is responsible for the natural biological maintenance of any population being diminished by death, it plays a key role in studying population dynamics. Fertility levels of any population reflect its reproductive behaviour. Also, among all the demographic factors, fertility is considered to be primarily responsible for the current rapid acceleration in the growth of human population especially in underdeveloped and developing countries. As a result of this, assessment of fertility levels and trends has become a priority area of research and governments in the developing countries are making great deal of efforts towards fertility control.

In order to have improved understanding about population dynamics, it becomes indispensable to analyse the mechanism through which socio-cultural, economic and biological
variables influence the fertility performances of a population. Demographic analysis of population dynamics examines the effect of socio-demographic variables like age, marital status, religious affiliation, educational background, economic status etc.; behavioural factors such as coital frequency and voluntary abstinence etc. and physiological variables like maternal nutritional status, frequency of ovulation, higher rates of spontaneous pregnancy wastage and sub-fecundity owing to endocrine disorders; which influence natality level ‘within’ and ‘between’ the populations. It is very much clear that the socio-economic and behavioural variables can affect the fertility rate only through biological manifestations. Therefore any discipline studying human fertility must be based on understanding of the biological factors and their relationships with other socio-cultural variables. A few biological factors of known importance are:

1. The duration of process - that is, the time, measured from marriage(or the onset of a sexual union), during which the reproductive process can continue.

2. Time taken by a susceptible woman to conceive. This is thought of as being a function of her fecundability, or probability of conception per unit time. Clearly, a logical time unit would be a cycle. However, data comes in units of months or years therefore unit of fecundability is considered in such units. Also, oft times the distribution of cycle lengths is considered.

3. The various outcome of pregnancy and associated probability of each outcome. The pregnancies may turn out to be complete and result in live birth or they may remain incomplete resulting in pregnancy wastage.

4. The duration of the non-susceptibility associated with conceptions.

But, having direct observations on these factors through detailed clinical observations on large populations is quite difficult and also, this would not provide distribution and estimates of biological factors like fecundability etc. This led to developments of various
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statistical techniques and models for studying human reproduction and population dynamics.

1.2 Brief description of modelling

A demographic model represents a demographic phenomenon in the form of mathematical or statistical models defined for two or more demographic variables. A mathematical as well as statistical model of a phenomenon is a theoretical structure, a logico-deductive conceptualized scheme, expressed through a single equation or a set of continuing equations meant to unfold various aspects of phenomenon under consideration. In fact, it is an abstraction of the real phenomenon in which the relevant relations among the real elements are replaced by similar relations among the mathematical entities. A model systematizes the comparison across time as well as over space. Demography is discipline which makes extensive use of modelling since appropriate models can serve both as aids in data analysis and as a means of exploring the possible answers to a whole series of questions. Model building encourages precise thinking about the determinants of process in question. The need for quantification of different aspects of human fertility has been recognized by the demographers since long back for having a better and thorough understanding of this phenomenon. As a result, various scientific techniques and methods have been developed for analysing different aspects of human fertility. Development of mathematical and statistical models is such an attempt in the direction of explaining the complex socio-biological phenomenon of human reproduction which essentially requires a multidisciplinary approach.

Use of mathematical and statistical models in physical and biological sciences has a long history but their use in social sciences has been of recent origin. One of the reasons being the lack of persons qualified in both mathematics and social sciences. Another reason being the general belief of the people that social phenomena involve human beings and hence are too complicated to be put in certain mathematical equations and ignorance of this
The adequacy of any model is judged by the success with which it can predict the effects of changes in the system which it describes and by whether or not it can account for changes which have occurred in the past. Since any model entails simplification of reality, it requires a judgement about which aspect of reality may be neglected or simplified without unduly affecting the conclusions. Such judgements are necessarily influenced by the availability of adequate and reliable data as well as the mathematical simplicity. Because of this simplification, the inferences drawn may some times be invalid or the estimates of the parameters involved in the model may be quite different from the true values. However, there may be some situations where the estimation procedure may be robust to the simplification of the reality and the estimates obtained from simple models may be quite close to reality. Thus, it all depends on the skill of the researcher that how he can make simple models without compromising much with the reality.

The study of biological factors that affect fertility is very important for having a better understanding of mechanism of fertility. The major biological factors involved in human reproduction are: (i) fecundability, (ii) incidence of physiological sterility (iii) chance of conception terminating into a foetal death or in a live birth, (iv) the period of temporary sterility comprising durations of gestation and postpartum amenorrhea associated with a conception. All social, cultural, economical, psychological and other variables influence human fertility through these biological factors. These factors may vary from female to female and for a female may be dependent upon age, parity, duration of marriage, etc. Consideration of all the above mentioned socio-biological factors and underlying variations, modelling human fertility and reproductive behaviour becomes a cumbersome task. How-
ever, once defined, such models abstract the above mentioned socio-biological factors of a reproductive process, formalize the hypothesis about effect of these factors on the process and then help to study the implication of this formalization.

During last four-five decades, large number of mathematical models have been proposed to describe the complex process of human fertility. Mathematical models used in the human population studies are either deterministic or stochastic depending on the nature of the phenomenon under investigation. A deterministic model takes care of situations where the outcome is certain. On the other hand, a probabilistic or stochastic model is capable of modelling those phenomena also where outcome is uncertain. While the mean or expected values given by stochastic models may agree with those of deterministic models based on sufficiently similar assumptions, stochastic models offer an insight into the unforeseen consequences otherwise difficult to understand in complicated systems. Thus, stochastic models are more powerful and informative tools for the study of demographic phenomena. In this way stochastic models help the researchers and planners in estimating parameters and taking decisions under uncertainty. Due to unpredictable character of human behaviour which involves an element of uncertainty, the stochastic models may resemble the social phenomena more closely than the deterministic models. Thus, stochastic models are more powerful and informative tools for the study of demographic phenomena.

Few important uses of demographic models may be enumerated as below:

1. The models give insight into and better understanding of the phenomenon under consideration. In fact a model for a system is formulated keeping in mind the fact that it embodies the essential features of the system. For this the model builder generally makes certain assumptions about the system based on his experience and intuition and tries to describe the behaviour of the system in terms of mathematical equation(s). All these help to understand the phenomenon in a better fashion.

2. As a result of enhanced understanding of the phenomenon under consideration, mod-
els may provide enhanced basis of prediction. In fact the phenomenon may have
different components which may be interrelated and the model incorporates all these
relationships in terms of a set of mathematical equations. Thus the model provides
a method for investigating the possible consequences in the system due to various
alterations in the determinants of the system. This type of use has more relevance
in the context of social sciences than in natural sciences. In fact, in natural sciences
it is usually possible to conduct experiments under controlled conditions and conse-
quences of alterations in various components can be easily assessed. However, social
phenomena involve human beings and hence it is generally not possible or desirable
to conduct such experiments. Thus, the use of models becomes the only alternative
for this situation.

3. Models may be used for estimating various parameters of the system by applying
them to observed data relating to the system. Of course, suitable statistical techniques
of estimation are required for the purpose. Many times the parameters are such that
they are not directly observable and hence one has no option than to estimate them
with suitable techniques.

4. Many times models pave ways in explaining certain apparent inconsistencies in the
observed data relating to the phenomenon under consideration. Models may also
stimulate new ideas and unravel hitherto unknown facts.

The social phenomena, where several social, cultural, psychological and economic factors
act and interact together, are bound to be exceedingly complex. However, many times
simple models based on reasonably good assumptions provide satisfactory and interesting
conclusions that are really important from policy perspective. In recent times, the availabil-
ity and popularity of computers has facilitated the study of complicated models that involve
relatively a large number of variables. Although limitations in the model set up which are
due solely to computational complexities can be removed to a greater extent by using com-
puters and software but, the utility of simple models can not be ignored. The compelling
factor in opting for simple models instead of complex models is not only the mathematical complexity, but the non-availability of suitable and reliable data. Thus, a model howsoever realistic, in the sense that it is based on very realistic assumptions, may not be of much use unless reliable data on each variable could be obtained. These situations of non-availability of reliable data, on various aspects of the phenomena, can efficiently be handled through modelling. Hence, in the process of model formulation, one must create a balance between the needs of underlying theory and of data. Consequently, one strives for a parsimonious model i.e., the simplest model giving the most adequate approximation to the real scenario.

1.3 Literature review

In the present section we present a brief description of the earlier research work done in the related areas. The first attempt to use probability distributions in studying the reproductive process seems to have been made by the Italian statistician and demographer Gini [1924], who introduced the concept of fecundability and tried to estimate it from data on the first live birth based on certain assumptions. Certain modified forms of binomial and Poisson distributions were suggested by Dandekar [1955] for the number of children born to a specified age cohort of females in fixed time period. Brass [1958] has obtained a negative binomial distribution to describe distribution of number of births to a female during a specified time. James [1963] obtained a distribution for number of births to a female which was derived from Dandekar’s discrete time model with an additional assumption that all the infecundable period with live birth conception are included in (0,T). Singh [1961, 1963, 1964, 1968] extended the discrete and continuous time models of Dandekar assuming that a fraction of females are susceptible to the risk of conception through out the period of observation and remaining are incapable to being pregnant during this period. Singh [1961, 1963, 1964, 1968] also extended his models of numbers of births for a heterogeneous group of females where it has been assumed that the probability of conception follows a beta distribution in discrete time model and conception rate follows a type III distribution, in

L. Henry, in a series of papers [Henry, 1953, 1957, 1961a,b] provided various analytical models for births and conceptions. Henry [1953] under a set of assumptions, obtained the expected number of births of a specified order in a time interval \((t, t + \delta t)\). The breakthrough in this line came when Sheps [1964] derived a powerful analytical model in which reproduction process has been viewed as a renewal process with finite states. This general model allows for a variety of pregnancy outcomes and different distributions for the gestation and PPA periods. Sheps and Perrin [1966] derived the distribution of number of births to a female in a time \(T\) after marriage incorporating foetal loss which in an extension of the model given in Singh [1964]. Sheps et al. [1969] showed that Henry’s expression was a particular result from a modified renewal process in which the first interval has a distribution different from the rest of the intervals.

In all the models described above, it is assumed that the female is susceptible to the risk of conception at the start of the observational period. If the beginning of the observational period is not marriage, this assumption may not be true. Pathak and Prasad [1977] modified Singh’s discrete time model by assuming the probability that a fecund female is exposed to the risk of conception in the first unit of the observation is \(\beta\) and probability that she is not exposed to risk of conception is \((1 - \beta)\). Singh and Bhattacharya [1970] have derived expressions for the number of conceptions in a fixed period which was generalised by formulating the distribution of incomplete and complete conceptions in a bivariate form Singh and Bhaduri [1971]. Singh et al. [1974] extended Singh [1968] model by considering both conception rate and chance of onset of sterility to be parity dependent. This model demonstrates the utility of the model in the evaluation of family planning programmes. Mode [1972] estimated the Malthusian parameter taking into account the fact that the reproductive period ends eventually, and that there is a risk of death throughout the life span. Mode
[1975] has given a model for number of births involving some very general expressions incorporating the dependence of the fertility parameters over age and parity.

The present discussions relates to some analytical models which take account of few factors of reproduction. These mentioned works were some of the pioneering works in the field of demographic modelling. Thereafter, many developments have taken place each of which has paved a new path to simplify the complexities of population dynamics. There is vast amount of work on stochastic models for human reproduction and obviously it is difficult to cover such a vast topic in short review. Next we see how the present work contributes to this field.

1.4 Outline of present research work

The research work in present thesis has been directed towards demonstrating the significance of statistical techniques and stochastic modelling in assessing the population dynamics via human reproduction. In addition, possible policy recommendations have been suggested. This research work has been focused on varying dimensions of the study of human reproduction viz. contraception before first pregnancy, estimation of sex ratios at birth under consideration of sex selective induced abortion, estimation of heterogeneity in probability of having a male birth, distribution of women’s age at last conception, visualisation of fertility behaviour under different stopping rules and expected family size associated with fertility desires of couples and it’s realisations.

Chapter 1 (the present one), is introductory in nature and it consists of a brief description of the significance of the studies related to human fertility. In addition, the contribution of statistics and modelling techniques in human reproduction studies has been discussed and a brief outline of the chapters included in the thesis is also included.
There exist ample of research literature investigating the various facet of contraceptive use behaviours in India but the use of contraception by married Indian women, prior to having their first pregnancy has been neglected so far. **Chapter 2** attempts to identify the socio demographic determinants and differentials of contraceptive use or non use by a woman in India, before she proceeds to have her first child. For a country like India, early marriages and early motherhood have been a big concern for policy planners. An unplanned first pregnancy in case of early marriages poses health issues to young mothers and children born to them. Further, for women getting married at appropriate child bearing ages, an unplanned first pregnancy may force the mothers to discontinue their education or occupation precipitously and may limit their intellectual development and economic utility. The study possesses an important place in assessing the success of family planning programmes in developing a planned vision towards beginning of family formation. The analysis has been done using data from the third National Family Health Survey (2005-2006), India.

In several countries like India, girls are more likely than boys to be aborted, to die in infancy, or to have younger siblings, all of which signal that parents want sons. However, standard techniques for measuring sex preferences fail to detect more subtle forms of sex preferences. Excessive desire for sons, especially for at least one son is a major socio-culture problem. Most Indian couples have a desire for two sons or at least one son for old age security as well as continuation of the family name. This desire very much influences the fertility performance of the couples and also sex ratio at birth. There are many biological and social determinants of sex ratio (ratio of number of males and total number of children) at birth. Among these, sex selective induced abortion is considered as most significant. **Chapter 3-I** is a modest attempt to answer the question; Does the son preference together with sex preferred induced abortions alter the sex ratio at birth? Though an individual couple having constant probability of producing male child (p) throughout there fertility span, can not affect the proportion of boys among its offspring by any stopping rule, the sex selective abortion can reduce the fraction of girls in the population. The impact of
practise of sex selective abortion on sex ratio at birth under different hypothetical situations has been examined in this chapter.

The sex ratio at birth has been the focus of much research and debate so has been underlying variation in it. As the variation in sex ratio at birth has gained much attention in past years, heterogeneity in probability of having a male birth as a possible determinant of this variation has gained place on the table. Is the probability of bearing a male (or female) child the same across couples? If not, what is the underlying pattern of heterogeneity? Of course, there are many other factors affecting the sex ratio at birth. Note that heterogeneity might well come from genetic differences among parents or from the influence of some other factors. The aim of Chapter 3-II is to provide a measure of heterogeneity among couples in relatively homogeneous subgroups among heterogeneous population of India. To investigate and establish the presence of such heterogeneity in a defined population, the authors subjected this piece of work to a mathematical model based approach for investigation. Followed by defining an appropriate theoretical model for the number of male children a woman can have, we have applied EM algorithm to get the maximum likelihood estimates of the model and hence the probability of having a male birth. NFHS-3 data for the state of Uttar Pradesh and three southern states in India has been considered for estimation purpose. This study concludes with existence of substantial heterogeneity in probability of having a male birth within and between the populations of the two regional groups under consideration.

Studies of human reproduction have focused more on the life cycle events related to beginning of the child bearing, i.e., age at marriage and age at first birth. The concluding behaviour of women’s age at their last conception and time of conclusion of child bearing has gained less attention. Chapter 4 estimates the probability distribution of the women’s age at their last conception with the help of a stochastic model. Using stochastic model enables us to visualize the probable age at last conception of young women, who yet not have completed their fertility. By considering some stopping rules defining the possible
fertility regulating behaviour of women, we have estimated the probability distribution of women’s age at last conception. The findings suggest that the stopping behaviour of couples influences the women’s age at last conception to a great extent. This may be helpful to planners for having at least a rough idea of estimated proportion of women of different age groups who will be completing their childbearing and willing to go for sterilization after marriage under different stopping rules regarding desired family size and sex composition of children. Accordingly, these estimates will help planners to optimize the cost and service provision for sterilization programs for women.

Fertility enjoys a key role in the domain of demography as the primary factor responsible for rapid population upturn across the globe. There exist a range of measures to assess the levels and trends within and across the populations. Fertility is affected by socio-cultural and consequential behavioural factors which are effective only through biological manifestations. This induces randomness in the human fertility which makes stochastic modelling relevant to unravel the complicated procedure of human fertility. Chapter 5 proposes a new approach to estimate the fertility measures ASFR, TFR, GRR, NRR, mean length of generation and growth rate with help of a stochastic model under some pertinent plans defining the fertility regulating and stopping behaviour of couples. In this chapter an investigation has been made to see how couple’s stopping behaviours may affect various fertility parameters. This work will be helpful to researchers and planners to visualize the behaviours of fertility parameters for various possible stopping rules regarding fertility regulating behaviour of couples in the society. Since it is not possible to perform controlled experiments on humans, modelling techniques can be really helpful in these situations.

The failure of the Indian women of child bearing ages in realising their desired fertility has been one of the main concerns of the Indian policy planners. Chapter 6 attempts to explore the variability in desired family, expected family size associated with these desired families and realisations of these fertility desires with respect to key socio demographic
variables in the state of Uttar Pradesh. Using statistical methods in conjunction with the representative data from Measurement, Learning, and Evaluation (MLE) Project for the Urban Health Initiative in Uttar Pradesh, India, this study establishes the existence of variation in fertility desires, associated expected family sizes and implementation of these desires, with respect to place of residence, caste/religion, educational qualification, age of the respondents at survey time, wealth index and city in which the respondent resides.