CHAPTER 4

THIN LAYER DRYING MODEL

4.1 INTRODUCTION

Drying is one of the preservation methods of agricultural products (Dincer 1998) and also a complex thermo-physical and biochemical process comprising simultaneous heat and mass transfer between the surface of the material and the surrounding media (Hossain and Bala 2002). Mathematical models have proved to be very useful for analyzing these transfer processes during drying. The aim of this study is to develop a mathematical model for predicting the thin layer drying kinetics at different drying conditions of air. Thin layer drying equations are used to estimate the drying curves for the product. Several investigators have proposed numerous mathematical models for the thin layer drying of many agricultural products. This process is advantageous, because a full scale experimentation of different products and configurations of the drying system is time consuming and also costly (Hossain and Bala 2002).

4.2 THIN LAYER MODELS

Thin-layer equations are often used for a description of the drying kinetics for various types of porous materials. Thin-layer drying models that describe the drying phenomenon of biological materials fall mainly into three categories, namely, theoretical, semi-theoretical and empirical. The first takes into account only internal resistance to moisture transfer, while the other two
consider only the external resistance to moisture transfer between the product and the air (Fortes and Okos 1981; Henderson 1974; Whitaker et al., 1969). Assuming that the resistance to the moisture flow is uniformly distributed throughout the interior of the homogeneous isotropic material, the diffusion coefficient $D$ is independent of the local moisture content, and if the volume shrinkage is negligible, Fick’s second law can be derived as follows:

$$\frac{\partial M}{\partial t} = D \nabla^2 M$$  \hspace{1cm} (4.1)

The semi-theoretical models are generally derived by simplifying the general series solutions of Fick’s second law, or a modification of simplified models and valid within the temperature, relative humidity, air flow velocity and moisture content range for which they were developed (Fortes and Okos 1981). Among the semi-theoretical thin-layer drying models, the Two-term model (Equation 4.2), the Henderson and Pabis model (Equation 4.3), the Lewis model (Equation 4.5), the Page model (Equation 4.6) and the Modified Page model (Equation 4.7) are used widely. Sharaf-Eldeen, Blaisdell and Hamdy (1980) presented a two-term model to predict the drying rate of shelled corn fully exposed to air. This model is the first two terms of the general series solution to the analytical solution of Equation (4.2). However, it requires constant product temperature and assumes constant diffusivity. The Two-term exponential model has the form

$$MR = \frac{M - M_e}{M_0 - M_e} = a \exp(-k_0 t) + b \exp(-k_1 t)$$  \hspace{1cm} (4.2)

where $M$, $M_0$ and $M_e$ are the material, initial, and equilibrium moisture contents in a dry basis, respectively, and $a$, $k_0$, $b$, $k_1$ are the empirical
coefficients. The Henderson and Pabis model is the first term of a general series solution of Fick’s second law (Henderson and Pabis 1969)

$$MR = \frac{M - M_e}{M_0 - M_e} = a \exp(-kt) \quad (4.3)$$

This model was used successfully for model drying corn (Henderson and Pabis 1969), wheat (Watson and Bhargava 1974) and peanuts (Moss and Otten 1989). The slope of this model, coefficient k, is related to effective diffusivity when the drying process takes place only in the falling rate period and liquid diffusion controls the process (Madamba et al., 1996).

The Lewis model (Lewis 1921) is a special case of the Henderson and Pabis model where intercept is unity. Lewis described the moisture transfer from agricultural materials as analogous to the flow of heat from a body immersed in cold fluid. Comparing this phenomenon with Newton’s law of cooling, the drying rate is found to be proportional to the difference in moisture content between the material being dried and the equilibrium moisture content in the drying air condition. This can be depicted as,

$$\frac{dM}{dt} = -k \ (M - M_e) \quad (4.4)$$

or after integrating yields

$$MR = \frac{M - M_e}{M_0 - M_e} = \exp(-kt) \quad (4.5)$$

The Page model is a modification of the Lewis model to overcome its shortcomings. This model has produced good fits in predicting the drying
of grain and rough rice (Wang and Singh 1978), shelled corn (Agrawal and Singh 1977) and barley (Bruce 1985)

\[ MR = \frac{M - M_e}{M_0 - M_e} = \exp\left(-kt^n\right) \]  \hspace{1cm} (4.6)

Overhults et al., (1973) also modified the Page model to describe the drying of soybean

\[ MR = \frac{M - M_e}{M_0 - M_e} = \exp\left(-kt^n\right) \]  \hspace{1cm} (4.7)

The empirical models derive a direct relationship between the average moisture content and drying time. They neglect the fundamentals of the drying process and their parameters have no physical meaning. Therefore, they cannot give a clear and accurate view of the important processes occurring during drying, although they may describe the drying curve for the conditions of the experiment (Keey 1972). The empirical models considered in this work are; Wang Singh, Diffusion Approach, Modified Henderson and Pabis, Verma et al., and Midilli and Kucuk as listed in Table 4.1.
Table 4.1  Thin layer drying curve models for variation of moisture ratio (MR) with time (t)

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Model Name</th>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lewis</td>
<td>MR=exp(-kt)</td>
<td>Mujumdar, 1987</td>
</tr>
<tr>
<td>2</td>
<td>Page</td>
<td>MR=exp(-kt^n)</td>
<td>Diamante and Munro, 1993</td>
</tr>
<tr>
<td>3</td>
<td>Modified Page</td>
<td>MR=exp(-kt)^n</td>
<td>Overhults et al., 1973</td>
</tr>
<tr>
<td>4</td>
<td>Henderson and Pabis</td>
<td>MR=a exp(-kt)</td>
<td>Zhang and Litchfield, 1991</td>
</tr>
<tr>
<td>5</td>
<td>Logarithmic</td>
<td>MR= a exp(-kt)+c</td>
<td>Yaldiz and Ertekin, 2001</td>
</tr>
<tr>
<td>6</td>
<td>Two term</td>
<td>MR= a exp(-k_0t) + b exp(-k_1t)</td>
<td>Henderson, 1974</td>
</tr>
<tr>
<td>7</td>
<td>Two-term exponential</td>
<td>MR= a exp(-kt) + (1-a) exp(-kat)</td>
<td>Sharaf-Elddeen et al., 1980</td>
</tr>
<tr>
<td>8</td>
<td>Wang Singh</td>
<td>MR=1 + at + bt^2</td>
<td>Wang and Singh, 1978</td>
</tr>
<tr>
<td>9</td>
<td>Diffusion Approach</td>
<td>MR= a exp(-kt) + (1-a) exp(-kbt)</td>
<td>Yaldiz and Ertekin, 2001</td>
</tr>
<tr>
<td>10</td>
<td>Modified Henderson and Pabis</td>
<td>MR= a exp(-kt) + b exp (-gt) + c exp(-ht)</td>
<td>Karathanos, 1999</td>
</tr>
<tr>
<td>11</td>
<td>Verma et al</td>
<td>MR= a exp(-kt) + (1-a) exp(-gt)</td>
<td>Verma et al., 1985</td>
</tr>
<tr>
<td>12</td>
<td>Midilli and Kucuk</td>
<td>MR= a exp(-kt^n) + bt</td>
<td>Thomson et al., 1968</td>
</tr>
</tbody>
</table>
4.3 STATISTICAL ANALYSIS FOR THE MODEL

Modeling the drying behavior of different products often requires the statistical methods of regression and correlation analysis. Linear and non-linear regression models are important tools used to find the relationship between different variables, especially those for which no established empirical relationship exists. In this study, the constants and coefficients of the best fitting model are determined, involving drying variables such as air temperature, humidity, velocity and product thickness. The effects of these variables on the constants and coefficients of the drying expression are also investigated by the multiple linear regression analysis.

In this study, a regression analysis is performed using Data Fit 8.1.69 (Oakadale Engineering) computer program. In the literature there are several criteria to evaluate the suitability of a model to experimental data. Among these, the correlation coefficient (r), the mean bias error (MBE), the reduced chi-squared ($\chi^2$) and root mean square error (RMSE) are the most widely used ones (Noomhorm et al., 1986). In this study, the constants and coefficients of the best fitting model were determined by fitting the total model employed to the experimental drying curves involving drying variables. The effect of these variables on the constants and coefficients of the drying expression are also investigated by the multiple linear regression analysis. The goodness of the fit of the tested models to the experimental data is the coefficient of determination (r). The reduced ($\chi^2$), the RMSE and the MBE between the experimental and calculated values for the tested models were calculated as follows:

$$r = \frac{\sum_{i=1}^{N} (MR_{pre,i} - MR_{r})^2}{\sum_{i=1}^{N} (MR_{exp,i} - MR_{r})^2}$$  \hspace{1cm} (4.8)
\[ \chi^2 = \sum_{i=1}^{n} \frac{(MR_{\text{exp},i} - MR_{\text{pre},i})^2}{N - n} \]  

(4.9)

\[ \text{RMSE} = \left[ \frac{1}{N} \sum_{i=1}^{N} (MR_{\text{pre},i} - MR_{\text{exp},i})^2 \right]^{\frac{1}{2}} \]  

(4.10)

\[ \text{MBE} = \frac{1}{N} \sum_{i=1}^{N} (MR_{\text{pre},i} - MR_{\text{exp},i}) \]  

(4.11)

where \( MR_{\text{exp},i} \) is the \( i \)th experimentally observed moisture ratio, and \( MR_{\text{pre},i} \) is the \( i \)th predicted moisture ratio. \( N \) is the number of observations and \( n \) is the number of constants (Akpinar et al., 2003; Dincer et al., 2002; Gunhan et al., 2005).

The drying data are fitted to the different semi theoretical/empirical models mentioned in Table 4.1 and the results of the drying models are compared with the experimental values. The models are evaluated, based on the coefficients of correlation (\( r \)), reduced chi-squared (\( \chi^2 \)), RMSE and MBE values.

4.4 THIN LAYER MODEL EVALUATION

The drying data are fitted to the different semi theoretical/empirical models mentioned in Table 4.1 and the results of the drying models are compared with the experimental values as shown in Figure 4.1.
Though almost all the models fit with the experimental values, the best model is identified based on the statistical analysis. The models are evaluated based on the coefficients of correlation ($r$), reduced chi-squared ($\chi^2$), RMSE and MBE values. The details of the statistical analysis are presented in Table 4.2. The observed statistical analysis shows that the Page model obtained the highest value for the coefficient of determination ($r$) and the least value for the reduced chi-square ($\chi^2$) and RMSE when compared with the other models.

Thus, the drying kinetic data for each experimental run are interpreted using the Page model. To account for the effect of the drying variables on the Page model’s constant $k$ (sec$^{-1}$) and coefficient $n$ (dimensionless), the values of $k$ and $n$ are regressed against drying air
humidity, temperature, velocity and product thickness using the multiple regression analysis.

Table 4.2 Results of statistical analyses on the modeling of bagasse

<table>
<thead>
<tr>
<th>Model Name</th>
<th>r</th>
<th>MBE</th>
<th>RMSE</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lewis</td>
<td>0.988672</td>
<td>-0.0096</td>
<td>0.015678</td>
<td>0.000246</td>
</tr>
<tr>
<td>Page</td>
<td>0.996271</td>
<td>-0.0012</td>
<td>0.007166</td>
<td>5.14E-05</td>
</tr>
<tr>
<td>Modified Page</td>
<td>0.994831</td>
<td>-0.0056</td>
<td>0.012639</td>
<td>0.00016</td>
</tr>
<tr>
<td>Henderson and Pabis</td>
<td>0.996123</td>
<td>-0.0002</td>
<td>0.007206</td>
<td>5.52E-05</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>0.996116</td>
<td>-0.0001</td>
<td>0.007219</td>
<td>5.91E-05</td>
</tr>
<tr>
<td>Two term</td>
<td>0.996123</td>
<td>-0.0002</td>
<td>0.007206</td>
<td>5.89E-05</td>
</tr>
<tr>
<td>Two-term exponential</td>
<td>0.988713</td>
<td>-0.00961</td>
<td>0.913173</td>
<td>0.886003</td>
</tr>
<tr>
<td>Wang &amp; Singh</td>
<td>0.985915</td>
<td>0.04406</td>
<td>0.02195</td>
<td>0.000546</td>
</tr>
<tr>
<td>Diffusion Approach</td>
<td>0.988713</td>
<td>-0.00961</td>
<td>0.015626</td>
<td>0.000277</td>
</tr>
<tr>
<td>Modified Henderson and Pabis</td>
<td>0.991413</td>
<td>0.00589</td>
<td>0.012165</td>
<td>0.00021</td>
</tr>
<tr>
<td>Verma et al.,</td>
<td>0.98453</td>
<td>-0.0116</td>
<td>0.018509</td>
<td>0.000388</td>
</tr>
<tr>
<td>Midilli and Kucuk</td>
<td>0.995188</td>
<td>-0.00021</td>
<td>0.008</td>
<td>7.25E-05</td>
</tr>
</tbody>
</table>

All possible combinations of the variables within the range are tested and included in the multiple regression analysis. From the moisture ratios obtained, it is found that, compared to the other models, the Page model predicts the experimental values with minimum error as represented by the regression methods. The Page model constants and coefficients are expressed in terms of the drying parameters (air humidity $H$ (g(Kg of d.a)$^{-1}$), temperature $T$ (°C), velocity $V$ (ms$^{-1}$) and product thickness $H_t$ (mm)) based
on the multiple regression analysis, and its corresponding equations are chosen based on the correlation coefficient value., and they are expressed as:

\[ k = a(H) + b(T) + c(V) + d(Ht) + e \]  \hspace{1cm} (4.12)
\[ n = x + y \log(t) + z(k) \]  \hspace{1cm} (4.13)

The constants of the Equation (4.12) and (4.13) are obtained by solving the equation using the regression analysis for all possible combinations of the drying parameters. The Equation (4.12) and (4.13) are expressed below for the obtained constant values from the analysis,

\[ k = 3.1094667 \times 10^{-3}(H) – 3.1183596869 \times 10^{-3}(T) – 3.947507753 \times 10^{-2}(V) + 0.113762212(Ht) + 0.49123557038 \]  \hspace{1cm} (4.14)
\[ n = -0.86990405 + 0.238750462 \log(t) – 1.1754564904(k) \]  \hspace{1cm} (4.15)

Incorporating these expressions in the Page Model equations, the moisture ratio of bagasse can be estimated with a high accuracy in the measurement ranges of \( H = 9-24 \text{ g/(kg of d.a)} \), \( T = 80-120^{\circ}C \), \( V = 0.5-2.0 \text{ m/s} \) and \( Ht = 20-60 \text{ mm} \). The model and its incorporated relationships between the coefficients and the drying parameters are consistent with the experimental data as evidenced by the good correlation values of

\[ r = 0.99627, \quad \chi^2 = 5.1352 \times 10^{-5}, \quad \text{RMSE} = 0.007166034 \]

4.5 COMPARISON OF THIN LAYER PAGE MODEL WITH EXPERIMENTAL RESULTS

The accuracy of the established model was evaluated by comparing the computed moisture ratios with the observed values as shown in Figures 4.2, 4.3 and 4.4 for varied thickness.
Figure 4.2  Comparison of experimental moisture ratio with those predicted for constant $T = 100^\circ$C, $H_t = 20$ mm, $V = 1$ m/s, $H = 16$ g/kg d.a

Figure 4.3  Comparison of experimental moisture ratio with those predicted for constant $T = 100^\circ$C, $H_t = 40$ mm, $V = 1$ m/s, $H = 16$ g/kg d.a
Figure 4.4 Comparison of experimental moisture ratio with those predicted for constant $T = 100^\circ$C, $Ht = 60$ mm, $V = 1$ m/s, $H = 16$ g/kg d.a

The closeness of the plotted data to the straight line representing equality between the experimental and predicted values illustrates the suitability of the Page model for describing the drying behavior of bagasse.

**4.6 CALCULATION OF EFFECTIVE DIFFUSIVITY**

The effective moisture diffusivity, $D_e$ is calculated, using the method of slopes as described by Doymaz (2004) and Panchariya et al., (2002). As Fick’s second law is used to describe the moisture diffusion process during drying, its solution which expresses the diffusion of liquid in a slab shaped solid in terms of the dry basis moisture content, can be written as follows (Aguerre et al., 1982):

$$MR = \frac{8}{\pi^2} \exp \left[ -D_e \left( \frac{\pi}{Ht} \right)^2 t \right]$$  \hspace{1cm} (4.16)
where \( D_e \) is the effective diffusivity \( (m^2/s) \), \( H_t \) is the thickness of the slab \( (m) \) and \( t \) is the drying time \( (s) \). The solution term of Equation (4.16) is also known as the Henderson and Pabis model. The slope, coefficient, \( k \), of this model is related to the effective diffusivity,

\[
k = D_e \left( \frac{\pi}{H_t} \right)^2
\]

\[(4.17)\]

The effective diffusivities of bagasse are determined by substituting the slopes derived from the linear regression of \( \ln (MR) \) vs. time in the Equation (4.16). Generally, an effective diffusivity is used due to the limited information on the mechanism of the moisture movement during drying and the complexity of the process.

Rizvi (1986) stated that effective diffusivities depend on the drying air temperature and the composition of the material. The effect of the temperature on effective diffusivity is generally described using the Arrhenius-type relationship (Panchariya et al., 2002). The logarithm of \( D_e \) as a function of the reciprocal of absolute temperature is plotted to show a linear relationship between \( \log D_e \) and \( (1/T) \), leading to an Arrhenius-type relationship between effective diffusivity and temperature:

\[
D_e = D_o \exp \left( \frac{-E_a}{RT} \right)
\]

\[(4.18)\]

where \( D_o \) is the diffusivity constant, \( E_a \) is the activation energy \( (J/mol) \) and \( R \) is the gas constant \( (8.3145 \ J/mol \ K) \). The slope of the straight line is \(-E_a/R\) from which the activation energy \( E_a \) is calculated for the drying process. The diffusivity constant \( D_o \) is also calculated from the analysis procedure. Figure 4.5 shows the plot for the linear regression of \( \ln (MR) \) with time for varied temperature to estimate the effective diffusivities.
The effective diffusivity of bagasse during drying varied from $1.63 \times 10^{-10}$ to $3.2 \times 10^{-10} \text{ (m}^2\text{/s)}$ in the temperature range of 80 to 120°C. The logarithm of $D_e$ as a function of the reciprocal of absolute temperature is plotted in Figure 4.6.

### Figure 4.6 Arrhenius type relationship between effective diffusivity and temperature
The results show a linear relationship between \((\log D_e)\) and \((1/T)\), leading to an Arrhenius-type relationship between the diffusion coefficient and the temperature. The slope of the straight line is \((-E_a/R)\) from which \(E_a\) is calculated. The diffusivity constant \(D_o\) and the activation energy \(E_a\) calculated from the linear regression are \(2.43 \times 10^{-7} \, \text{m}^2/\text{s}\) and 19.47 kJ/mol respectively.

4.7 CONCLUSION

Thin layer drying experiments of bagasse were carried out to determine the drying kinetics and also to identify a suitable drying model. Of the twelve thin-layer drying models, which were comparatively tested according to their coefficients of correlation, reduced chi-squared, RMSE and MBE values, the Page model best described the drying behavior of bagasse. The drying rate constants were correlated well with the experimental drying variables like air velocity, temperature, and humidity and layer thickness, using the non-linear polynomial regression model. The drying rate constants were greatly influenced by the layer thickness and the air temperature. When the effects of the drying parameters on the constants and coefficients of the Page model were taken into account, the resulting model gave a good fit \((r = 0.99627)\) to the observed drying behavior of bagasse.

The temperature dependence of the diffusivity coefficients was described by an Arrhenius-type relationship for bagasse. The activation energy for moisture diffusion was found to be 19.47 kJ/mol while using air as drying medium.