Chapter 3

Seesaw mechanism

3.1 Introduction to seesaw

We have already mentioned seesaw mechanism briefly in the previous chapter. Here we present a more elaborate discussion of the seesaw frameworks, which generate Majorana masses for neutrinos. Such a mass term is generally of the form

\[ \mathcal{L}_m = \frac{1}{2} m_L \bar{\nu}_L (\nu_L)^c \]

where \( \nu_L \) is a left-handed neutrino field and \( (\nu_L)^c = C \bar{\nu}_L^T, C = i\gamma^2\gamma^0 \) being the charge conjugation operator. Such Majorana masses are possible since both the neutrino and the anti-neutrino have zero electric charge. Majorana masses for neutrinos are then not forbidden by electric charge conservation. However, such masses for neutrinos violates lepton number conservation by two units. Also, \( \nu_L \) possesses non-zero isospin and hypercharge. So in the SM framework, assuming only Higgs doublets, such Majorana mass terms for left-handed neutrinos are forbidden by gauge invariance. The idea of the seesaw mechanism is to generate such terms effectively, after heavy external fields are integrated out. Ultra-small neutrino masses then can be justified in terms of 'big' mass parameters of these heavy fields.

There are three different realizations of seesaw mechanism. They are all based upon the fact that, according to group theory, two doublets can be decomposed into a triplet and a singlet \( (2 \otimes 2 \equiv 3 \oplus 1) \) combination. Thus the left-handed lepton doublet and the Higgs doublet of SM can couple either to a triplet or a singlet field. The three seesaw mechanisms outlined here bring such 'big' mass parameters from three different sectors, each of them going beyond the SM in its own way.
3.2 Type I seesaw: Fermion singlets

To understand the principle of type I seesaw mechanism [118], let’s start with the neutrino mass matrix once more. We assume that right-handed neutrinos ($\nu_R$) are present in the scenario in addition to the usual left-handed neutrinos ($\nu_L$) of the SM. Therefore, it is now possible to construct a Dirac mass term for the neutrinos as follows

$$\mathcal{L}_{\text{Dirac Mass}} = m_D \bar{\nu}_R \nu_L + h.c \quad (3.2a)$$

$$= \frac{1}{2} (m_D \bar{\nu}_R \nu_L + m_D (\bar{\nu}_L)^c (\nu_R)^c) + h.c. \quad (3.2b)$$

In general Majorana type mass terms are also possible for neutrinos, since they have zero electric charge. The Majorana mass term for left and right-handed neutrinos are written as

$$\mathcal{L}_{\text{Mass}}^L = \frac{1}{2} m_L (\bar{\nu}_L)^c \nu_L + h.c \quad (3.3a)$$

$$\mathcal{L}_{\text{Mass}}^R = \frac{1}{2} m_R (\bar{\nu}_R)^c \nu_R + h.c \quad (3.4a)$$

where ‘L’ and ‘R’ stands for left and right-handedness respectively. Let us define a vector field $n_L$ such that

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}, \quad (\bar{n}_L)^c = ((\bar{\nu}_L)^c \bar{\nu}_R) \quad (3.5)$$

where $(n_L)^c$ is the charge conjugate field of $n_L$. Then the total mass lagrangian for neutrinos can be written as

$$\mathcal{L}_{\text{Mass}}^{\text{total}} = \mathcal{L}_{\text{Dirac Mass}} + \mathcal{L}_{\text{Mass}}^L + \mathcal{L}_{\text{Mass}}^R \quad (3.6)$$

$$= \frac{1}{2} (\bar{n}_L)^c M n_L \quad (3.7)$$

The mass matrix $M$, whose diagonalisation gives the physical neutrino masses is given by

$$M = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \quad (3.5)$$

The positive mass eigenstates of this matrix are

$$m_{1,2} = \frac{1}{2} (m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}) \quad (3.6)$$
In the seesaw scenario the added right-handed neutrinos are assumed to have higher mass well above the electroweak symmetry breaking (EWSB) scale, while the Dirac mass $m_D$ (like other charged fermions) is considered to be of the order of EWSB scale. Therefore, in this scenario, $m_R \gg m_D$. On diagonalisation of $M$, the eigenvalues corresponding to the neutrino mass eigenstates are obtained as

$$m_1 \simeq \frac{m_D^2}{m_R}$$

$$m_2 \simeq m_R$$

It is clear from these equations that, now we have a right-handed neutrino with mass scale $\Lambda_N = m_R$, which also may be the scale of some new physics and at the same time a very light neutrino, with mass suppressed by $\frac{m_D}{\Lambda_N}$. This new mass scale is often taken to be close to the grand unification scale to explain the proposed sub-eV masses of the three left-handed neutrinos. In fact, heavier the new right-handed neutrino, the lighter is the left-handed neutrino. This is the underlying basic principle of seesaw mechanism.

Formally, in type I seesaw, right-handed heavy $SU(2)$ singlet fermion fields ($N_R$) with zero hypercharge are added to the SM fields to produce non-zero neutrino masses. The left-handed lepton doublets then couple to the Higgs field and the newly introduced right-handed fermion fields to produce the Majorana mass terms for the neutrinos. The extra piece of lagrangian for this new heavy right-handed fields is given by [121]

$$\mathcal{L}^{New} = i\bar{N}_R \partial \bar{N}_R - \frac{1}{2} \bar{N}_R M_N N_R^c + \bar{l}_L \tilde{\Phi} Y^c_{l} N_R + h.c$$

where the first two terms represent the kinetic term and Majorana mass term for the right-handed fermion fields and the third one is the Yukawa interaction term. $l_L$ and $\Phi$ are the left-handed lepton doublet and Higgs field respectively. The vev of Higgs field is denoted by ‘v’.

The left-handed neutrino mass term can be estimated from the above lagrangian. The lagrangian can be solved to get the equation of motion. Expansion of the propagator gives a factor $\frac{-i}{p-M_N}$, which for low momenta $p$ can be approximated by $\frac{-i}{M_N}$, with $M_N$ as the mass of heavy right-handed fermion fields. Then integrating out the heavy fields using the equation of motion, namely,

$$\frac{\partial \mathcal{L}^{New}}{\partial N_R} = 0$$
one obtains the effective dimension 5 operator [121]

$$\delta L^{d=5} = \frac{1}{2} \epsilon^{d=5}_{\alpha \beta} (\nabla_{\alpha} \Phi^* \Phi \nabla_{\beta}) + h.c$$ (3.11)

The coefficient $\epsilon^{d=5}$ is given by [121]

$$\epsilon^{d=5} = Y^T_N \frac{1}{M_N} Y_N$$ (3.12)

After electroweak symmetry breaking, the neutrino mass is obtained by inserting the Higgs vev ‘v’ and is expressed as [121]

$$m_\nu = \frac{v^2}{2} \epsilon^{d=5} = Y^T_N \frac{v^2}{2M_N} Y_N$$ (3.13)

From this relation, light Majorana neutrino mass can be generated by properly adjusting the value of the parameters $Y_N$ and $M_N$.

### 3.3 Type II seesaw : Scalar triplets

In this case [119] the scalar sector of the SM is extended by adding at least one $SU(2)$ scalar triplet $\Delta(\Delta_1, \Delta_2, \Delta_3)$ with hypercharge 2. The electromagnetic charged states for the triplet can be obtained from the couplings to the leptons

$$\bar{\tilde{l}}_L (\tau \cdot \Delta) l_L = (-\bar{e}_L)^c (\bar{\nu}_L)^c \begin{pmatrix} \Delta_3 \\ \Delta_1 + i \Delta_2 \\ -\Delta_3 \end{pmatrix} \begin{pmatrix} \nu_L \\ \bar{\nu}_L \end{pmatrix}$$ (3.14)

Since $\nu_L$ and $(\tilde{\nu}_L)^c$ have charge -1 and $\nu_L$, $(\bar{\nu}_L)^c$ have no charge, the charged states of the triplet are given by [121]

$$\Delta^{\pm} \equiv \frac{1}{\sqrt{2}} (\Delta_1 - i \Delta_2), \Delta^0 \equiv \Delta_3, \Delta^0 \equiv \frac{1}{\sqrt{2}} (\Delta_1 + i \Delta_2)$$ (3.15)

The Yukawa interaction term of the new triplet, which violates lepton number by two units, is represented as

$$\mathcal{L}_{yuk} = Y_\Delta \bar{\tilde{l}}_L (\tau \cdot \Delta) l_L$$ (3.16)

where $\bar{\tilde{l}}_L = (-\bar{e}_L)^c (\bar{\nu}_L)^c$, $\tau$'s are the Pauli matrices and $Y_\Delta$ is the Yukawa coupling matrix. The rest of the part of the lagrangian of the triplet field containing the scalar potential
terms is given by [121],

\[
\mathcal{L}_\Delta = M_\Delta^2 \Delta \Delta + \frac{1}{2} \lambda_2 (\Delta^\dagger \Delta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\Delta^\dagger \Delta) + \frac{1}{2} \lambda_4 (\Delta^\dagger T^i \Delta)^2 + \lambda_5 (\Delta^\dagger T^i \Delta)(\Phi^\dagger \tau^i \Phi) + (\mu_\Delta \Phi^\dagger (\tau \cdot \Delta)^\dagger \Phi + h.c)
\]

(3.17)

Here, \(T^i\)'s are the generators of the triplet representation of \(SU(2)\)

\[
T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

(3.18)

In principle, one can start with gauge invariant as well as lepton number conserving triplet Yukawa interactions by assigning lepton number \(L = -2\) to the scalar triplet. \(L\) could then be broken spontaneously once \(\Delta^0\) acquires a vev [144]. However, that would have led to a massless \(SU(2)\) triplet Goldstone boson with unsuppressed coupling with the \(Z\) boson. The consequence is a much larger contribution to the invisible decay width of the \(Z\) than is permitted by the LEP data. Therefore, explicit \(L\)-violation is phenomenologically preferable in this scheme. A way to generate the triplet vev is through the trilinear interaction term in the scalar potential. In this case, minimization of the scalar potential produces a vev \((v_\Delta)\) for the neutral component of the triplet \(\Delta^0\), when the Higgs doublet \(\Phi\) acquires a vev and is given by [116]

\[
v_\Delta \equiv \frac{\mu_\Delta v^2}{M_\Delta^2}
\]

(3.19)

It can be seen that, triplet vev can be made small by choosing a large value for the mass of the triplet field or by choosing the coefficient of the trilinear term to be very small. Keeping a small value for the triplet vev \((v_\Delta)\) is crucial in this model since it is directly related to neutrino masses through the Yukawa coupling term. Another important reason to keep the triplet vev small is to respect the \(\rho\)-parameter constraint, which restricts [145]

\[
\rho = 1.0008^{+0.0017}_{-0.0007}
\]

(3.20)

Since the neutral component of triplet \(\Delta^0\) couples to the \(W\) and \(Z\)-bosons, its vev contributes to their masses. Instead of the SM relations, we now have [34]

\[
M_W^2 = \frac{1}{4} g_w^2 (v^2 + 2 v_\Delta^2)
\]

(3.21)
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\[ M_Z^2 = \frac{1}{4} (g_1^2 + g_2^2)(v^2 + 4v_\Delta^2) \]  
(3.22)

so that, expression for \( \rho \)-parameter now reads as [34],

\[ \rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{1 + 2v_\Delta^2/v^2}{1 + 4v_\Delta^2/v^2} \]  
(3.23)

Then using the constraint on \( \rho \)-parameter, we obtain an upper bound for the allowed value of \( v_\Delta \) as [34],

\[ \frac{v_\Delta}{v} < 0.07 \]  
(3.24)

Majorana masses of neutrinos are generated when the neutral component of the triplet \( \Delta^0 \) acquires vev and the relation between the neutrino mass, Yukawa coupling and triplet vev is expressed as :

\[ M^{ij}_\nu = v_\Delta Y^{ij}_\Delta \]  
(3.25)

Here the coupling \( Y^{ij}_\Delta \) decide the pattern of neutrino mixing, in analogy to the right-handed neutrino mass matrix in type I seesaw. The distinctive feature of type II seesaw lies in a different vev being responsible for neutrino masses, as compared to the other fermion masses. Thus the essence of this scenario is \( v_\Delta \ll v \), thanks to the character of the scalar potential.

### 3.4 Type III seesaw : Fermion triplets

Type III seesaw [120] is very much similar to the type I seesaw mechanism. In the type III seesaw framework at least two heavy \( SU(2) \) right-handed fermion triplets are added instead of fermion singlet fields. This is because at least two non-vanishing neutrino masses have to be generated in order to explain the oscillation data. The fermion triplet fields \( \Sigma (\Sigma_1, \Sigma_2, \Sigma_3) \) have zero hypercharge. The Majorana mass term as well as the dynamics of the fields is governed by the lagrangian [121]

\[ \mathcal{L}_\Sigma = i \Sigma_R^\dagger \delta \Sigma_R - \left[ \frac{1}{2} \Sigma_R^\dagger M_\Sigma \Sigma_R + \Sigma_R^\dagger \Sigma_\Sigma (\tilde{\Phi}^1 \tau_1 L_L) + h.c \right] \]  
(3.26)

The covariant derivative of the above equation is given by

\[ D_\mu = \partial_\mu - ig_1 Y \frac{1}{2} B_\mu - ig_2 T^a \frac{1}{2} W^a_\mu \]  
(3.27)
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Where, $T^a$'s are the $3 \times 3$ generators of $SU(2)$ introduced in the previous section. $Y_{\Sigma}$ is the Yukawa coupling and $l_L, \Phi$ are the SM lepton and Higgs doublets. The relation between the $SU(2)$ components of $\Sigma$ and the electric charge eigenstates are as follows [121]

$$\Sigma^\pm \equiv \frac{\Sigma_1 \mp i \Sigma_2}{\sqrt{2}}, \quad \Sigma^0 \equiv \Sigma_3$$

(3.28)

In the case of type III seesaw also, the Lagrangian can be solved to get equation of motion for the fields. Following the same procedure as in the case of type I seesaw mechanism, after expanding the propagator and integrating out the heavy fields, the effective five dimensional operator is obtained as [121]

$$\delta L^{d=5} = \frac{1}{2} c_{\alpha \beta}^{d=5} (\bar{l}_\alpha \tau \Phi)(\Phi^\dagger \tau l_\beta) + h.c$$

(3.29)

where the coefficient is [121]

$$c_{\alpha \beta}^{d=5} = Y_{\Sigma}^T \frac{1}{M_{\Sigma}} Y_{\Sigma}$$

(3.30)

Majorana neutrino masses are obtained when the Higgs field acquires a vev and is given by [121]

$$m_\nu = \frac{v^2}{2} Y_{\Sigma}^T \frac{1}{M_{\Sigma}} Y_{\Sigma}$$

(3.31)

It has been already pointed out that type III seesaw is very much similar to type I seesaw mechanism. We comment on some general features of these two models for neutrino mass generation. In both the cases, the effect of heavy new degrees of freedom (singlet and triplet fields in the case of type I and type III respectively) in low energy phenomena can be manifested by adding higher-dimensional operators to the SM. It is well known that, given the SM gauge symmetries and particle content, only one type of dimension-five operator is allowed and its generic form is given by [117]

$$\frac{1}{\Lambda} (L \Phi)(L \Phi) + h.c \equiv \frac{v^2}{\Lambda} \nu \nu + h.c$$

(3.32)

All other operators that can be constructed are of dimension six or higher. In the above relation $L$ and $\Phi$ are SM lepton and Higgs doublets respectively, 'v' is vev of the neutral component of Higgs and reflects the fact that neutrino mass is generated through this operator after electroweak symmetry is broken. The most interesting general feature of the seesaw models is the existence of a new physics scale $\Lambda$, which can be identified with the mass scale of heavy fermion triplet(singlet) $M_{\Sigma}$ ($M_N$) of type III (type I) seesaw mechanism. For $\Lambda \sim 10^{15}$ GeV or GUT scale, it is possible to obtain sub-eV range masses.
for light neutrinos in agreement with the current experimental data. Such a high value of
this new mass scale certainly motivates a possibility that the higher dimensional operator
is indeed generated by some new physics beyond SM.

Of all the three seesaw scenarios described above, the works presented in this thesis
are based on type II and type III seesaw framework.

3.5 Two-zero texture and the inadequacy of a single triplet

In the previous chapter we have seen that strong evidence has been accumulated in
favour of neutrino oscillation from the solar, atmospheric, reactor and accelerator neutrino
experiments over the last few years. A lot, however, is yet to be known, including
the mass generation mechanism and the absolute values of the masses, in addition to
mass-squared differences which affect oscillation rates. Also, a lot of effort is on to ascertain
the nature of neutrino mass hierarchy, including the signs of the mass-squared differences.
We have also pointed out that a gateway to information of the above kinds is the
light neutrino mass matrix, in a basis where the charged lepton mass matrix is diagonal.
In this context we have discussed a possibility that frequently enters into such investigations
is one where the mass matrix has some zero entries, perhaps as the consequence of
some built-in symmetry of lepton flavours. At the same time such ‘zero textures’ lead to a
higher degree of predictiveness and inter-relation between mass eigenvalues and mixing
angles by virtue of having fewer free parameters. We have also pointed out that two-zero
textures have a rather wide acceptability. It has been hinted in [146] that none of the seven
possible two-zero-texture cases can be achieved by assuming only one scalar triplet. We
write the allowed seven two-zero textures for the $3 \times 3$ symmetric Majorana mass matrix
of the light neutrinos, denoted by $M_\nu$ once more for the sake of completeness:

\[
\begin{align*}
\text{Case } A_1 : M_\nu & \sim \begin{pmatrix} 0 & 0 & x \\ 0 & x & x \\ x & x & x \end{pmatrix}, & \text{Case } A_2 : M_\nu & \sim \begin{pmatrix} 0 & 0 & 0 \\ x & x & x \\ 0 & x & x \end{pmatrix} \\
\text{Case } B_1 : M_\nu & \sim \begin{pmatrix} x & x & 0 \\ x & 0 & x \\ 0 & x & x \end{pmatrix}, & \text{Case } B_2 : M_\nu & \sim \begin{pmatrix} x & 0 & x \\ 0 & x & x \\ x & x & 0 \end{pmatrix} \\
\text{Case } B_3 : M_\nu & \sim \begin{pmatrix} x & 0 & x \\ 0 & 0 & x \\ x & x & x \end{pmatrix}, & \text{Case } B_4 : M_\nu & \sim \begin{pmatrix} x & x & 0 \\ x & x & x \\ 0 & 0 & x \end{pmatrix}
\end{align*}
\]
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Case C : \( M_\nu \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix} \) \hspace{1cm} (3.36)

These textures are defined in a basis where the charged lepton mass matrix is diagonal. In the context of Type-II seesaw the Yukawa couplings of scalar triplets \( \Delta_k \) are given by \([147]\) :

\[
\mathcal{L}_Y = \frac{1}{2} \sum_{k=1}^{2} y^{(k)}_{ij} L_i^T C^{-1} i \tau_2 \Delta_k L_j + \text{h.c.,} \hspace{1cm} (3.37)
\]

where \( i, j = e, \mu, \tau \), \( C \) is the charge conjugation matrix, the \( y^{(k)}_{ij} \) are the symmetric Yukawa coupling matrices of the triplets \( \Delta_k \). From the above Lagrangian, the neutrino mass matrix is obtained as \([147]\)

\[
\mathcal{M}_{\nu ij} = w_k y^k_{ij} \hspace{1cm} (3.38)
\]

with \( w_k \) s being the vev of the triplets. Among the neutrino mass terms, some are allowed, while others are not, as the consequence of a particular texture. This fact can be associated with a conserved global \( U(1) \) symmetry, under which all fields have some charge. Under this symmetry, the lepton doublets \( L_i \) and the scalar triplet \( \Delta_k \) transform as \([147]\)

\[
L_i \rightarrow p_i L_i \hspace{1cm} \text{and} \hspace{1cm} \Delta_k \rightarrow p_0 \Delta_k \hspace{1cm} (3.39)
\]

with phase factors \( |p_i|, |p_0| = 1 \). An examination of each of the allowed textures reveals that the three phase factors for different lepton flavors i.e, \( p_e, p_\mu, p_\tau \) have to be different from each other. The Higgs doublet transforms trivially under the horizontal symmetry, thus enabling the charged-lepton mass matrix to be diagonal. Now, let us look at the consequence of such a symmetry when just one triplet is present. We shall see that this assumption leads us to a contradiction.

In all the seven possible two-texture-zero cases, the \( \mu\tau \) element of \( \mathcal{M}_\nu \) is non-zero. Thus, the corresponding Yukawa coupling element \( y_{\mu\tau} \) must be non-zero, and the resulting interaction term must conserve the \( U(1) \) charge. This implies \([147]\)

\[
p_0 p_\mu p_\tau = 1 \hspace{1cm} (3.40)
\]

We first examine the Cases \( B_1, B_2, B_3, B_4 \) and \( C \). For these five cases, \( y_{ee} \neq 0 \). Therefore, upon applying the symmetry operation we have,

\[
p_0^2 p_e^2 = 1 \hspace{1cm} (3.41)
\]
The inequality of the $U(1)$ charges for the different neutrino flavour eigenstates then results in $y_{e\mu}$ picking up a phase factor [147]:

$$p_0 p_e p_\mu = \frac{p_\mu}{p_e} \neq 1$$  \hspace{1cm} (3.42)

This leads to the conclusion

$$y_{e\mu} = 0.$$  \hspace{1cm} (3.43)

Proceeding in the same way with the $e\tau$ element of Yukawa coupling, we obtain

$$p_0 p_e p_\tau = \frac{p_\tau}{p_e} \neq 1$$  \hspace{1cm} (3.44)

which again implies

$$y_{e\tau} = 0.$$  \hspace{1cm} (3.45)

On the other hand, it is seen that in none of the cases $B_1, B_2, B_3, B_4$ and $C$, the Yukawa couplings $y_{e\mu}$ and $y_{e\tau}$ are both zero. Thus none of these five textures is viable when only one triplet is present in the scenario.

We next address the two remaining cases, namely $A_1$ and $A_2$. For both of these, one has $y_{\mu\mu} \neq 0$. Thus we have again after the symmetry operation,

$$p_0 p_\mu^2 = 1$$  \hspace{1cm} (3.46)

However, that would again mean [147]

$$p_0 p_\mu p_\tau = \frac{p_\tau}{p_\mu} \neq 1$$  \hspace{1cm} (3.47)

This in turn destroys the viability of these two texture as well. Thus one is forced to conclude that none of the seven possible two-zero-texture cases can be achieved by an abelian horizontal symmetry assuming only one scalar triplet. But, when two or more triplets are present, then there will be more freedom in terms of the charges possessed by them, and the phase factor relations will be less constraining. Thus the contradictions that appear with a single triplet can be avoided, so that at least some of the seven possible two-zero textures are allowed. Therefore it is important to examine the phenomenological consequences of an augmented triplet sector if Type-II seesaw has to be consistent with two-zero textures. We proceed in that direction in the following chapters.