Chapter 1

Introduction

1.1 Standard Model and beyond

The Standard Model (SM) of particle physics encompasses our current knowledge of elementary particles. The SM is a successful quantum field theory, based on the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge group, which describes the interactions among elementary particles and three of the four fundamental interactions. In Nature, all the known phenomena observed so far, can be described in terms of four fundamental forces:

- Strong Interaction
- Weak Interaction
- Electromagnetic interaction
- Gravitational Interaction

The \( SU(3)_C \) is the gauge group of strong forces, where 'C' denotes the color quantum number carried by quarks as well as gluons, which are mediators of strong interaction. The symmetry group corresponding to electroweak interaction is \( SU(2)_L \times U(1)_Y \), where 'L' refers to the left-chirality of fermions and 'Y' stands for the weak hypercharge, which is defined by the relation

\[
Q = T_3 + \frac{Y}{2}
\]

('Q' and \( T_3 \) being respectively the electric charge and third component of the weak isospin of the fields involved.

So, SM describes only the first three of four fundamental forces previously mentioned. Gravity effects are negligible at the highest energy scale of particle accelerator experi-
ments performed till date and therefore its exclusion does not affect the explanation of whatever has been observed so far in the world of fundamental particles.

1.2 Particle content of SM

The fermionic sector of the SM consists of six types each of quarks and leptons, which come in three generations or flavors. The transformation properties of these fields under SM gauge group are determined by their respective charges under the gauge groups. The quark fields transform as triplets (fundamental representation) of $SU(3)_C$, whereas the leptons are singlet under this gauge group. The leptons do not possess any color charge and hence they do not take part in strong interactions. SM being a chiral theory, treats left-handed and right-handed fermion fields differently with respect to $SU(2) \times U(1)$ gauge interactions. The left-handed fields transform as doublets (fundamental representation) under $SU(2)$, while the right-handed fields are singlets under this group. Thus we have the particle content for the quark sector as follows

Doublets : \[ \begin{pmatrix} u \\ d \\ c \\ s \\ t \\ b \end{pmatrix}_L 
\]

Singlets : \[ u_R, d_R, c_R, s_R, t_R, b_R. \]

For the leptonic sector we have

Doublets : \[ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_{\mu} \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau} \\ \tau \end{pmatrix}_L \]

Singlets : \[ e_R, \mu_R, \tau_R. \]

The left-handed chiral component of the field $\Psi$ is defined as $\psi_L = P_L \psi = [(1 - \gamma_5)/2] \psi$ and for the right-handed one, $\psi_R = P_R \psi = [(1 + \gamma_5)/2] \psi$. With the convention followed so far, the lepton doublets will have hypercharge $(-1)$ and for lepton singlets it is $(2)$. The quark doublets are of hypercharge $+1/3$, for up-type quark singlets it is $+4/3$ and down-type quark singlets will have hypercharge $(-2/3)$.

The gauge sector of SM contains eight massless vector fields $G^a_{\mu}(a = 1, 2, \ldots 8)$, known as the gluons, the gauge bosons of $SU(3)_C$. The gauge bosons corresponding to the broken $SU(2)_L \times U(1)_Y$ group are, $\gamma$ (photon), $W^\pm$ and $Z$. The gluons are electrically neutral and carry color quantum numbers. So, they have self-interactions (both trilinear and quartic). The $W^\pm$ are massive, charged particles and $Z$ boson is massive but electrically neutral. The $W^\pm$ and $Z$ bosons are self-interacting also, whereas the photon is non self-interacting, massless and neutral.
1.3 Origin of mass: Higgs mechanism

The particle spectrum of SM described so far is incomplete. Till now, we have considered only those interactions among the particles, which arise as a consequence of local gauge invariance. However, invariance under $SU(2)_L \times U(1)_Y$ gauge group implies that all the fermions as well as gauge bosons have to be massless. Introduction of explicit mass terms for them threatens to destroy the good features of SM, such as, renormalizability and unitarity. The solution to this, is to generate the mass term in the Lagrangian by breaking the gauge symmetry spontaneously, that is to say, not in the original Lagrangian but through a selective (non-symmetric) choice of the vacuum. This phenomena is known as 'Spontaneous Symmetry Breaking' (SSB) [1–9].

It has been observed that in the SM the electric charge is conserved and therefore the concerned gauge group of electromagnetism i.e $U(1)_{em}$ is an exact symmetry of the theory. So under SSB, we should have the following symmetry breaking pattern

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$$

(1.2)

This breaking can be achieved by introducing a spin 0 scalar field $\Phi$ that transforms as a doublet under the $SU(2)_L$ with $U(1)_Y$ hypercharge +1. This complex scalar doublet is defined as

$$\Phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right).$$

(1.3)

The scalar potential involving the scalar doublet, which also contains self-interaction term for the doublet, is written as

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$  

(1.4)

The self-interaction of $\Phi$ is such that gauge invariance is broken spontaneously. The vacuum expectation value (vev) $v$ of the neutral component of doublet is expressed in terms of the mass parameter $\mu$ and the self-interaction strength $\lambda$ and is given by

$$v = \sqrt{-\frac{\mu^2}{\lambda}}.$$  

(1.5)

for $\mu^2 < 0$ and $\lambda > 0$.

If the scalar field is shifted with respect to the above vev, then, written in terms of the shifted field, the gauge invariance of the theory appears to be broken. The $U(1)_{em}$ invariance, however, remains unbroken, since the non-zero vev pertains only to the neutral part of the scalar field, which has no electromagnetic interactions.
The resulting mass spectrum consists of massive fermionic and gauge fields as well as a neutral scalar particle known as Higgs boson, which has recently been discovered at the Large Hadron Collider (LHC), CERN, Geneva and has mass $\sim 125$ GeV.

Masses of the gauge bosons are obtained from the Lagrangian
\[ \mathcal{L}_\Phi = (D^\mu \Phi)^\dagger D^\mu \Phi - V(\Phi), \]  
where $D_\mu$ is defined by
\[ D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\sigma^a}{2} W^a_\mu - ig_3 \frac{\lambda^b}{2} G^b_\mu, \]
where $g_1$, $g_2$ and $g_3$ are the gauge couplings for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ groups respectively. The masses of gauge bosons are obtained as

- $m_\gamma = 0$
- $m_W = \frac{1}{2} g_2 v$
- $m_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$

The weak mixing angle $\theta_W$, also called Weinberg angle, which gives the relationship between the masses of $W$ and $Z$ bosons, is defined as
\[ \theta_W \equiv \tan^{-1} \left( \frac{g_1}{g_2} \right). \]  
(1.8)

The massless vector fields that corresponds to photon couples with matter fields with electromagnetic coupling constant $'e'$, the electric charge. This is related to the $SU(2)_L$ and $U(1)_Y$ couplings in the following way
\[ g_2 \sin \theta_W = g_1 \cos \theta_W = e \]  
(1.9)

and lastly, the $\rho$-parameter, which measures the relative strengths of neutral and charged current interactions in the theory, is given by
\[ \rho = -\frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \approx 1. \]  
(1.10)

Masses of fermions are generated via gauge invariant Yukawa interactions between scalar and fermionic fields:
\[ \mathcal{L}_{Yuk} = \sum_{i,j=\text{generation}} \left( -Y^u_{ij} \bar{Q}_i \Phi u_j - Y^d_{ij} \bar{Q}_i \Phi d_j - Y^l_{ij} \bar{L}_i \Phi e_j + \text{h.c.} \right). \]  
(1.11)
where $\tilde{\Phi} = i\sigma_2 \Phi^*$ and $Q$ and $L$ represents the quark and lepton doublets respectively and $Y^u$, $Y^d$, $Y^l$ are the Yukawa coupling matrices for the up-quark, down-quark and charged leptons respectively. After the field $\Phi$ gets the vev $v$, the Yukawa Lagrangian takes the form of $m_{\psi} \overline{\psi}_L \psi_R$ with the mass matrices,

$$m_{ij}^u \propto v Y^u_{ij}, \quad m_{ij}^d \propto v Y^d_{ij}, \quad m_{ij}^l \propto v Y^l_{ij}. \quad (1.12)$$

These mass matrices are in flavor basis and are to be diagonalized to get the mass basis. These Yukawa couplings are free parameters in SM and are fixed by the masses of the corresponding fermions. It should be noted that neutrinos do not have any mass term due to the absence of their right-handed partners.

The predictions of the SM have been tested to a high degree of precision experiments carried out at high-energy colliders like the LEP, Tevatron and currently at the LHC, as well as in low-energy experiments of flavor physics. In almost all cases, experimental observations are in accordance with the predictions of SM. The long elusive Higgs boson has also been discovered at the LHC [10, 11]. Apart from a few discrepancies (e.g., the anomalous magnetic moment of muon) the SM is the most consistent model of particle physics till date. This has established SM as a starting point, as far as building fundamental theoretical models of Nature is concerned. In spite of the huge success of SM, however, there are some observations that encourage us to go beyond the SM. In the next section we point out some of those observations.

1.4 Need for new physics beyond SM

In the following we outline some observations which indicate that the SM suffers from some drawbacks and is not a complete theory to describe the particle domain.

- There are many free parameters ($\sim 20$) in SM, which can only be fixed through experiments. All the masses, couplings and mixing parameters in the quark sector are described by these free parameters of the theory and there is no explanation as to why the parameters have such values.

- The structure of fermionic sector in SM remains unexplained. The masses of the SM fermions range from sub-eV (for neutrinos) to over hundred GeV (for top quark). There is no satisfactory fundamental explanation for this huge hierarchy in masses of fermions. The fermions also come in three ‘generations’, with higher generation
having higher mass. Such a replication is not predicted or explained by anything within the SM.

The mixing in the quark sector also has a generational structure, i.e., the largest mixing occurs between the generations one and two, followed by mixing of two and three and finally, mixing between one and three, which are the feeblest ones. SM does not explain this pattern.

- It is important to ensure that tree-level values of the various SM parameters are stable. The inclusion of higher order terms in general leads to radiative corrections that modify the couplings and masses via the renormalization procedure. Any quadratic correction to the mass for gauge bosons are tamed by the gauge symmetry. For fermions chiral symmetry does this task, leaving only a weak, logarithmic dependence. However, the Higgs boson being a scalar does not have any such way out to cancel quadratic corrections and its mass would be driven to the scale of new physics. For example, correction due to top quark contribution, shifts the mass of Higgs from its tree level value by,

$$\Delta m_H^2 = -\frac{|Y_t|^2}{4\pi^2} \Lambda_{\text{cutoff}}^2$$  \hspace{1cm} (1.13)

where $\Lambda_{\text{cutoff}}$ is some cut off scale up to which SM is well-behaved and beyond which some new physics comes in and $Y_t$ is the top-quark Yukawa coupling, due to which the correction is largest. Now, if $\Lambda_{\text{cutoff}} \sim M_{\text{pl}}$, where $M_{\text{pl}}$ is the Planck scale ($\sim 10^{19}$ GeV), then corrections to Higgs mass squared reach up to $10^{38}$ GeV$^2$. Therefore, in order to maintain a Higgs mass squared consistent with experimental measurements, we need to add counter-terms to the Higgs mass squared, so that the divergences cancel out. This obviously requires a large fine-tuning of the parameters involved. This is called the Fine-tuning/Naturalness/Hierarchy problem.

This so called Hierarchy problem has motivated several new physics scenarios, the most popular one being the existence of Supersymmetry (SUSY), which relates bosonic and fermionic degree of freedom.

- Unification of the electroweak and strong couplings, is the so called Grand Unified Theories (GUT), cannot be achieved within the SM framework. The running of these couplings are such that they do not exactly unify at any given energy scale, if their evolution all the way up is controlled by SM interactions alone.
1.4. NEED FOR NEW PHYSICS BEYOND SM

- Gravity, one of the four fundamental forces of Nature, which becomes important near Planck scale, is not included in SM. SM may at best be treated as an effective theory up to Planck scale and new physics should appear at that scale. One of the alternatives is *String theory*, which hope to predict a quantized description of gravity.

- The first evidence for Dark Matter (DM) came from the measurement of rotation curves of galaxies. The rotation curves were found to fit with the hypothesis that visible part of the galaxy was immersed in a halo of invisible matter i.e Dark Matter. In fact, the Universe consists of only 4.9% of visible matter and the rest consists of DM and Dark Energy. There is no such particle in SM which can fit in properly to explain this DM riddle.

- SM can not explain the observed baryon asymmetry of the Universe, namely, why we have more matter than anti-matter. This will perhaps require a level of CP-violation, which is not present in the SM.

- Lastly, one of the most important findings suggesting the existence of physics beyond the SM is the evidence for non-zero neutrino masses and mixings, observed in the form of neutrino oscillation. In SM, a neutrino is massless because of the absence of the corresponding right-handed partner. In principle, neutrino masses can be easily accommodated in the SM framework by postulating the existence of right-handed heavy sterile neutrinos. But, the extreme smallness of neutrino mass, which are many orders of magnitude smaller than all the fermion masses, calls for a deeper understanding. Also, the bi-large mixing pattern of neutrinos, as evident from oscillation data, is very different from what is noticed in the quark sector. This suggest some new underlying mechanism. The question as to, why there is such a large difference between mass of neutrino and that of the charged lepton belonging to the same $SU(2)_L$ multiplet is still a mystery. If neutrinos are found to be Majorana particles, then lepton number would be violated by their mass-term, which might give us a hint of physics beyond the SM. Several models for explaining the neutrino mass and mixing have been put forward, starting from seesaw mechanisms, largely motivated by high-scale grand unified theories, to low-scale models like SUSY with lepton number violation. But the questions remain yet to be answered.

We have mentioned some of the shortcomings of SM. Clearly, we need some theory beyond SM (BSM) to address these loop-holes. With this motivation we proceed to study
some BSM models based on seesaw mechanism, which might become helpful in understanding and overcoming some of the incompletenesses of the SM.