CHAPTER-4

MODELING OF FLUID FLOW IN HYDRAULIC FRACTURES

4.1 Material Balance Equation for flow of gas in Hydraulic Fracture:

4.1.1 Assumptions:

1) No mass transfer between the fluids.
2) Average width of the fracture is \( w_f \).
3) The entire flow process is isothermal. (T= Constant).
4) Hydraulic fracture is perpendicular to the horizontal well bore.
5) The geometry of the hydraulic fracture is rectangle.

The schematic representation of gas flow in the hydraulic fracture is shown in Figure 4.1

![Figure 4.1: Schematic flow of gas in the hydraulic fracture.](image)

Material Balance Equation for the gas flow through the hydraulic fracture:

\[ \text{(Accumulation with in system)} = \text{(Gas flow in through system boundaries)} - \text{(Gas flow out through system boundaries)} + \text{(Generation with in system)} - \text{(Consumption within system)}. \]
As control volume decreases, the volume = $\Delta x.\Delta y.\bar{w_f}$. 

$$\Rightarrow \left( v_{gfx}\rho_{gf} l_{x+\Delta x}\bar{w_f} \right) \left( -v_{gfy}\rho_{gf} l_{y+\Delta v}\bar{w_f} \right) = \left( \frac{-v_{dgm}\rho_{gm}}{z=\frac{w_f}{x}} \right) \left( \frac{-v_{gmz}\rho_{gm}}{z=\frac{w_f}{x}} \right) = \frac{\Delta\left( (\Delta y\Delta x,\bar{w}).\rho_{gf,S_{gf}} \right)}{\Delta t} \Rightarrow 1.$$ 

Dividing equation 1 by $\Delta x.\Delta y.\bar{w_f}$, We got 

$$\Rightarrow \frac{-v_{gfx}\rho_{gf} l_{x+\Delta x} + v_{gfy}\rho_{gf} l_{y+\Delta y}}{\Delta x} + \frac{-v_{dgm}\rho_{gm}}{z=\frac{w_f}{x}} + \frac{-v_{gmz}\rho_{gm}}{z=\frac{w_f}{x}} = \frac{\Delta\left( (\bar{w}).\rho_{gf,S_{gf}} \right)}{\Delta t} \Rightarrow 2.$$ 

Taking limit $\Delta x, \Delta y, \Delta z$ and $\Delta t \rightarrow 0$, 

$$\Rightarrow -\frac{\partial (v_{gfx}\rho_{gf})}{\partial x} - \frac{\partial (v_{gfy}\rho_{gf})}{\partial y} \frac{(v_{gmz}\rho_{gm})}{z=\frac{w_f}{x}} = \frac{\partial (\bar{w}).\rho_{gf,S_{gf}}}{\partial t} \Rightarrow 3.$$ 

Considering Darcy’s law, We have 

$$\Rightarrow v_{gfx} = \frac{-k_{gf}}{\mu_{gf}} \frac{\partial (\rho_{gf})}{\partial x} \rightarrow \text{for x-direction}$$ 

But, as the flow in the hydraulic fracture is two phase flow, the concept of relative permeability comes into act. 

Relative permeability of gas with respect to water $(k_{rgf})$ 

$$\Rightarrow k_{rgf} = \frac{k_{gf}}{k_f} \text{ and } k_{gf} \leq 1.$$ 

Now, $k_{gf} = k_{rgf} \times k_f$ 

Where, $k_f$ = Absolute Permeability in fracture with proppant.[$m^2$].
\( k_{gf} \) = Effective permeability of gas in the fracture.

\( k_{rgf} \) = Relative permeability of gas in the fracture.

Now,

\[
\begin{align*}
-\nu_{fx} &= \frac{-k_{rgf} \frac{k}{\nu_{gf}} \frac{\partial (p_{gf})}{\partial x}}{\mu_{gf}} \quad \text{for x-direction.} \\
-\nu_{fy} &= \frac{-k_{rgf} \frac{k}{\nu_{gf}} \frac{\partial (p_{gf})}{\partial y}}{\mu_{gf}} \quad \text{for y-direction.} \\
-\nu_{gmx} &= \frac{-k_{gm} \frac{\partial (p_{gm})}{\partial z}}{\mu_{gm}} \quad \text{for z-direction.}
\end{align*}
\]

Substituting \( \nu_{fx}, \nu_{fy}, \nu_{gmx} \) in equation 3, we get

\[
\begin{align*}
\rho \frac{\partial}{\partial x} \left\{ \phi \rho_{gf} \right\} - \rho \frac{\partial}{\partial y} \left\{ \phi \rho_{gf} \right\} - \rho \frac{\partial}{\partial z} \left\{ \phi \rho_{gm} \right\} = \frac{\partial}{\partial t} \left\{ \phi \rho_{gf} \right\} \quad \text{4.}
\end{align*}
\]

Where,

\( \beta_c = \) Transmissibility conversion factor = 1.127 \( \text{scf/D-psi} \).

From equation of state,

Formation Volume Factor \( (B_g) = \frac{\rho_{gsc}}{\alpha_c \rho_{gf}} \)

From equation 4,

\[
\begin{align*}
\rho \frac{\partial}{\partial x} \left\{ \phi \rho_{gf} \right\} &+ \rho \frac{\partial}{\partial y} \left\{ \phi \rho_{gf} \right\} + \rho \frac{\partial}{\partial z} \left\{ \phi \rho_{gm} \right\} = \frac{\partial}{\partial t} \left\{ \phi \rho_{gf} \right\} \quad \text{5.}
\end{align*}
\]

From literature, \( \phi_f = \Phi_o \epsilon_f (p_f - p_o) \).

\[
\frac{\partial \phi_f}{\partial p_f} = \phi_f c_f.
\]

Now,

\[
\frac{\partial \phi_f}{\partial t} = \frac{\partial \phi_f}{\partial p_f} \cdot \frac{\partial p_f}{\partial t}
\]

\[
\Rightarrow \frac{\partial \phi_f}{\partial t} = \phi_f c_f \cdot \frac{\partial p_f}{\partial t}.
\]
Equation 5 can be rewritten as

\[
\frac{\partial}{\partial x} \left( \frac{\beta c k_{rgf} k \partial (p_{gf}) \rho_{gsc}}{\mu_{gf} \partial x \alpha_{cB_g}} \right) + \frac{\partial}{\partial y} \left( \frac{\beta c k_{rgf} k \partial (p_{gf}) \rho_{gsc}}{\mu_{gf} \partial y \alpha_{cB_g}} \right) + \left( \frac{\beta c k_{gm} \partial (p_{gm})}{\mu_{gm} \partial z \rho_{gsc} \frac{\rho_{gsc}}{\alpha_{cB_g}}} \right) \frac{-w_f}{2} = \left[ \frac{\rho_{gsc}}{\alpha_{cB_g}} \right] S_{gf} \phi_f c_f \frac{\partial p_f}{\partial t} \rightarrow 6.
\]

Dividing equation 6 with \(\Delta x \Delta y \Delta z\), we get

\[
\frac{\partial}{\partial x} \left( \frac{\beta c k_{rgf} k \partial (p_{gf}) \rho_{gsc}}{\mu_{gf} \partial x \alpha_{cB_g}} \Delta x \right) + \frac{\partial}{\partial y} \left( \frac{\beta c k_{rgf} k \partial (p_{gf}) \rho_{gsc}}{\mu_{gf} \partial y \alpha_{cB_g}} \Delta y \right) + \left( \frac{\beta c k_{gm} \partial (p_{gm})}{\mu_{gm} \partial z \rho_{gsc} \frac{\rho_{gsc}}{\alpha_{cB_g}}} \right) \frac{\Delta X \Delta Y}{2} = \left[ \frac{V_{bf} \rho_{gsc}}{\alpha_{cB_g}} \right] (1 - S_{wf}) \phi_f c_f \frac{\partial p_f}{\partial t} \rightarrow 7.
\]

In this case, the flow is a multiphase flow i.e. gas and water will flow through the fracture. So, the saturation values of gas and water will change with respect to time during production. Initially, water saturation will be nearly 100% and then the water saturation decreases and the gas saturation increases. A well bore has been considered in the 5th layer from the top of the reservoir. As the hydraulic fractures can be any number, modeling has been done from 1 fracture to 5 fractures. During the modeling of gas flow in the fracture, the gas flows from the matrix into the fracture.

**4.1.2 Discretization Method:**

The developed equation representing the gas flow in the hydraulic fracture is a nonlinear partial differential equation (PDE). For discretization of this nonlinear PDE, Finite difference method has been used.
By applying finite difference method to equation 7, we get

\[
\frac{\beta c k_{rgf}}{\mu_g f} A_x \frac{\rho_{gs c}}{A_c B_g} \frac{1}{\Delta x} (p_{i+1,j,k}^{n+1} - p_{i,j,k}^{n+1}) - \left( \frac{\beta c k_{rgf}}{\mu_g f} A_x \frac{\rho_{gs c}}{A_c B_g} \frac{1}{\Delta x} \right) i_{1/2,j,k}^{n+1} = \left( \frac{\beta c k_{rgf}}{\mu_g f} A_y \frac{\rho_{gs c}}{A_c B_g} \frac{1}{\Delta y} \right) i_{j,1/2,k}^{n+1} (p_{i,j,k}^{n+1} - p_{i,j-1,k}^{n+1})
\]

\[
+ \left[ \frac{\beta c l_{kgm}}{\mu_g m} A_y \frac{\rho_{gs c}}{A_c B_g} \frac{1}{\Delta y} \right] (p_{i,j+1,k}^{n+1} - p_{i,j,k}^{n+1}) + \left[ \frac{\beta c l_{kgm}}{\mu_g m} A_x \frac{\rho_{gs c}}{A_c B_g} \frac{1}{\Delta x} \right] (p_{i+1,j,k}^{n+1} - p_{i,j,k}^{n+1}) \]

\[
= \left[ \left( V_B \frac{\rho_{gs c}}{A_c B_g} \right) (1 - S_{wf}) \phi_f C_f \right] \frac{p_{i,j,k}^{n+1} - p_{i,j,k}^n}{\Delta t} \quad \rightarrow \quad 8.
\]

As we have chosen the spatial discretization terms at new time level i.e. (n+1), the applied finite difference method can be considered has implicit finite difference method.

Considering transmissibility \((T_{gx}) = \frac{\beta c k_{rg}}{\mu_g} \frac{\rho_{gs c}}{A_c B_g}\)
The eqn 8 can be written as
\[
\Rightarrow \left[ \left( \frac{K_f A x_T g x}{\Delta x} \right)_{i+j+\frac{1}{2},k} (P_{i+1,j,k}^{n+1} - P_{i,j,k}^{n+1}) - \right. \\
\left. \left( \frac{K_f A y T g y}{\Delta y} \right)_{i+j+\frac{1}{2},k} (P_{i,j,k}^{n+1} - P_{i-1,j,k}^{n+1}) \right] + \left[ \left( \frac{K_f A y T g y}{\Delta y} \right)_{i,j+\frac{1}{2},k} (P_{i,j+1,k}^{n+1} - P_{i,j,k}^{n+1}) - \right. \\
\left. \left( \frac{K_f A y T g y}{\Delta y} \right)_{i,j-\frac{1}{2},k} (P_{i,j-1,k}^{n+1} - P_{i,j,k}^{n+1}) \right] = \left[ \left( \frac{2}{w_f} T g z \Delta x \Delta y \right) + \left( \frac{V_b \rho g s c (1-S w f) \phi f c_f}{\alpha c B g} \right) \right] (P_{i,j,k}^{n+1} - P_{i,j,k}^n) \] \rightarrow 9.

Writing eqn 9 in the following form
\[
\lambda_{g_{i+j+\frac{1}{2},k}} (P_{i+1,j,k}^{n+1} - P_{i,j,k}^{n+1}) + \lambda_{g_{i+j\frac{1}{2},k}} (P_{i,j,k}^{n+1} - P_{i-1,j,k}^{n+1}) + \lambda_{g_{i+j+\frac{1}{2},k}} (P_{i,j+1,k}^{n+1} - P_{i,j,k}^{n+1}) - \lambda_{g_{i+j-\frac{1}{2},k}} (P_{i,j-1,k}^{n+1} - P_{i,j,k}^{n+1}) = X_{i,j,k} (P_{i,j,k}^{n+1} - P_{i,j,k}^n) \] \rightarrow 10.

Where,
\[
\lambda_{g_{i+j+\frac{1}{2},k}} = \left( \frac{K_f A x_T g x}{\Delta x} \right)_{i+j+\frac{1}{2},k} \\
\lambda_{g_{i+j\frac{1}{2},k}} = \left( \frac{K_f A y T g x}{\Delta x} \right)_{i+j\frac{1}{2},k} \\
\lambda_{g_{i+j+\frac{1}{2},k}} = \left( \frac{K_f A y T g y}{\Delta y} \right)_{i+j+\frac{1}{2},k} \\
\lambda_{g_{i+j-\frac{1}{2},k}} = \left( \frac{K_f A y T g y}{\Delta y} \right)_{i,j-\frac{1}{2},k} \\
X_{i,j,k} = \left[ \left( \frac{2}{w_f} T g z \Delta x \Delta y \right) + \left( \frac{V_b \rho g s c (1-S w f) \phi f c_f}{\alpha c B g} \right) \right] \\
\]

Eqn 10 can be written in the form of
\[
B_{i,j,k} P_{i,j,k-1}^{n+1} + S_{i,j,k} P_{i,j-1,k}^{n+1} + W_{i,j,k} P_{i-1,j,k}^{n+1} + C_{i,j,k} P_{i,j,k}^{n+1} + E_{i,j,k} P_{i,j+1,k}^{n+1} + N_{i,j,k} P_{i,j+1,k}^{n+1} + A_{i,j,k} P_{i,j+1,k}^{n+1} = Q_{i,j,k} 
\]
Where,

\[ B_{i,j,k} = \lambda_g g_{i,j,k - \frac{1}{2}}. \]

\[ S_{i,j,k} = \lambda_g g_{i,j - \frac{1}{2},k}. \]

\[ W_{i,j,k} = \lambda_g g_{i - \frac{1}{2},j,k}. \]

\[ C_{i,j,k} = \left[ \lambda_g g_{i+\frac{1}{2},j,k} + \lambda_g g_{i-\frac{1}{2},j,k} + \lambda_g g_{i,j+\frac{1}{2},k} + \lambda_g g_{i,j-\frac{1}{2},k} + \lambda_g g_{i,j,k+\frac{1}{2}} + \lambda_g g_{i,j,k-\frac{1}{2}} \cdot X_{i,j,k} \right] \]

\[ E_{i,j,k} = \lambda_g g_{i+1,j,k}. \]

\[ N_{i,j,k} = \lambda_g g_{i,j+\frac{1}{2},k}. \]

\[ A_{i,j,k} = \lambda_g g_{i,j,k+\frac{1}{2}}. \]

\[ Q_{i,j,k} = (-X_{i,j,k}) p_i^n. \]

4.2 Material Balance Equation for flow of water in the Hydraulic Fracture:

4.2.1 Assumptions:

1) The flow of water occurs only in X and Y directions only.
2) Width of the fracture is \( \bar{w}_f \).
3) The entire flow process is isothermal. \( T=\text{Constant} \).
4) Hydraulic fracture is perpendicular to the horizontal wellbore.
5) The geometry of the hydraulic fracture is rectangle.

Material Balance Equation for the gas flow through the hydraulic fracture:

\( \text{(Accumulation with in system)} = \text{(Gas flow in through system boundaries)} - \text{(Gas flow out through system boundaries)} + \text{(Generation with in system)} - \text{(Consumption within system)}. \)

As control volume decreases, the volume = \( \Delta x.\Delta y.\bar{w}_f \).
\[
\left(\frac{v_{wx} \rho_{wx}}{\Delta x} \Delta x\right) + \left(\frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y\right) + \left(\frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y\right) + \left(\frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y\right) = \Delta \left(\Delta x \Delta y \bar{w}_t \theta_f \rho_{wf} S_{wf}\right) \Delta t
\]

\[
\Delta \left(\Delta x \Delta y \bar{w}_t \theta_f \rho_{wf} S_{wf}\right) \Delta t
\]

\[\Rightarrow \frac{v_{wx} \rho_{wx} \Delta x \Delta y}{\Delta x \Delta y} + \frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y + \frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y + \frac{-v_{wy} \rho_{wy} \Delta y}{\Delta y} \Delta y = \frac{\Delta \left(\Delta x \Delta y \bar{w}_t \theta_f \rho_{wf} S_{wf}\right)}{\Delta t} \]

Dividing equation 11 by \(\Delta x \Delta y \bar{w}_t\), we get

\[\Rightarrow \frac{-v_{wx} \rho_{wx}}{\Delta x} \Delta x + \frac{-v_{wy} \rho_{wy}}{\Delta y} \Delta y + \frac{-v_{wy} \rho_{wy}}{\Delta y} \Delta y + \frac{-v_{wy} \rho_{wy}}{\Delta y} \Delta y = \frac{\Delta \left(\Delta x \Delta y \bar{w}_t \theta_f \rho_{wf} S_{wf}\right)}{\Delta t} \]

Taking limit \(\Delta x, \Delta y\) and \(\Delta t \rightarrow 0\),

\[\Rightarrow \frac{\partial (v_{wx} \rho_{wx})}{\partial x} + \frac{\partial (v_{wy} \rho_{wy})}{\partial y} = \frac{\partial (\theta_f \rho_{wf} S_{wf})}{\partial t} \]

Considering Darcy’s law, we have

\[\Rightarrow v_{wx} = \frac{-k_{wf}}{\mu_{wf}} \frac{\partial (p_{wf})}{\partial x} \rightarrow \text{for x-direction} \]

But, as the flow in the hydraulic fracture is two-phase flow, the concept of relative permeability comes into action.

Relative permeability of gas with respect to water \((k_{rwf})\)

\[\Rightarrow k_{rwf} = \frac{k_{wf}}{k_f} \text{ and } \frac{k_{wf}}{k_f} \leq 1. \]

Now, \(k_{wf} = k_{rwf} k_f\)

Where, \(k_f = \text{Absolute Permeability in fracture with proppant} \left[\text{m}^2\right]\).

\(k_{wf} = \text{Effective permeability of gas in the fracture}\).

\(k_{rwf} = \text{Relative permeability of gas in the fracture}\).
Now,

\[ -u_{wx} = \frac{-k_{rwf} k_f}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial x} \] for x-direction.

\[ -u_{wy} = \frac{-k_{rwf} k_f}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial y} \] for y-direction.

Substituting \( u_{wx}, \ u_{wy} \) in equation 13, we get

\[ \alpha \left( -\frac{\beta c k_{rwf} k \rho_{wf}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial x} \right) \alpha \left( -\frac{\beta c k_{rwf} k \rho_{wf}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial y} \rho_{wf} \right) = \alpha \left( (\rho_f) \frac{\rho_{wf} S_{gf}}{\partial t} \right) \] \[ \Rightarrow 14. \]

Where,

\( \beta_c = \text{Transmissibility conversion factor} = 1.127 \ \frac{scf}{D-psi}. \)

From equation of state,

Formation Volume Factor \( (B_w) = \frac{\rho_{wsc}}{\alpha c \rho_{wf}} \)

From equation 14,

\[ \alpha \left( \frac{\beta c k_{rwf} k \rho_{wsc}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial x} \right) \alpha \left( -\frac{\beta c k_{rwf} k \rho_{wsc}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial y} \alpha c B_w \right) = \alpha \left( (\rho_f) \frac{\rho_{wsc} S_{wf}}{\partial t} \right) \] \[ \Rightarrow 15. \]

From literature, \( \phi_f = \phi_o \alpha c f (P_f - P_o) \).

\[ \frac{\partial \phi_f}{\partial p_f} = \phi_c f. \]

Now, \[ \frac{\partial \phi_f}{\partial t} = \frac{\partial \phi_f}{\partial t} \frac{\partial p_f}{\partial t} \]

\[ \Rightarrow \frac{\partial \phi_f}{\partial t} = \phi_c f \frac{\partial p_f}{\partial t}. \]

Equation 15 can be rewritten as

\[ \alpha \left( \frac{\beta c k_{rwf} k \rho_{wsc}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial x} \right) \alpha \left( -\frac{\beta c k_{rwf} k \rho_{wsc}}{\mu_{wf}} \frac{\partial (P_{wf})}{\partial y} \alpha c B_w \right) = \left[ (\frac{\rho_{wsc}}{\alpha c B_w}) S_{wf} \phi_f c_f \right] \frac{\partial p_f}{\partial t} \] \[ \Rightarrow 16. \]

Dividing equation 6 with \( \Delta x, \Delta y, \Delta z \), we get
\[
\frac{\partial}{\partial x} \left( \frac{\beta C k_{rwf} k}{\mu_{wf}} A_{x} \frac{\partial (P_{wf} \rho_{wsc})}{\partial x} \frac{\partial (P_{wf} \rho_{wsc})}{\partial c_{w}} \right) \Delta x + \frac{\partial}{\partial y} \left( \frac{\beta C k_{rwf} k}{\mu_{wf}} A_{y} \frac{\partial (P_{wf} \rho_{wsc})}{\partial y} \frac{\partial (P_{wf} \rho_{wsc})}{\partial c_{w}} \right) \Delta y = \left[ \frac{V_{b} \rho_{wsc}}{a_{c} B_{w}} (1 - S_{gf}) \phi_{f} c_{f} \right] \frac{\partial \rho_{f}}{\partial t}
\]

In this case, the flow is a multiphase flow i.e. gas and water will flow through the fracture. So, the saturation values of gas and water will change with respect to time during production. Initially, water saturation will be nearly 100% and then the water saturation decreases and the gas saturation increases. A well bore has been considered in the 5th layer from the top of the reservoir. As the hydraulic fractures can be any number, modeling has been done from 1 fracture to 5 fractures. Equation 17 represents the flow of water in the fracture during gas production from shale reservoirs.

### 4.2.2 Discretization Method:

The developed equation representing the gas flow in the hydraulic fracture is a nonlinear partial differential equation (PDE). For discretization of this nonlinear PDE, Finite difference method has been used.

![Discretization and notation indication for a 2D pressure equation for Hydraulic Fracture](image)

*Figure 4.3: Discretization and notation indication for a 2D pressure equation for Hydraulic Fracture.*

As we have chosen the spatial discretization terms at new time level i.e. (n+1), the finite difference method applied here is implicit finite difference method.
By applying finite difference method to equation 17, we get

\[ \frac{\beta_c k_{rwf} k}{\mu_w f} A_x \left( \frac{\rho_{wsc} 1}{\alpha_c B_w \Delta x} \right)_{i+\frac{1}{2}, j, k} \left( P_{i+1, j, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \frac{\beta_c k_{rwf} k}{\mu_w f} A_x \left( \frac{\rho_{wsc} 1}{\alpha_c B_w \Delta x} \right)_{i-\frac{1}{2}, j, k} \left( P_{i, j, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \\
\left( \frac{\beta_c k_{rwf} k}{\mu_w f} A_y \left( \frac{\rho_{wsc} 1}{\alpha_c B_w \Delta y} \right)_{i, j+\frac{1}{2}, k} \left( P_{i, j+1, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \\
\left( \frac{\beta_c k_{rwf} k}{\mu_w f} A_y \left( \frac{\rho_{wsc} 1}{\alpha_c B_w \Delta y} \right)_{i, j-\frac{1}{2}, k} \left( P_{i, j-1, k}^{n+1} - P_{i, j, k}^{n+1} \right) \right) = \left( \frac{V_B \rho_{wsc} (1 - S_{gf}) \varnothing_f c_f}{\alpha_c B_w} \Delta t \right) \frac{P_{i, j}^{n+1} - P_{i, j}^{n}}{\Delta t} \]

\[ \text{Considering transmissibility} \ (T_{wx}) = \frac{\beta_c k_{rw}}{\mu_w} \frac{\rho_{wsc}}{\alpha_c B_w} \]

The eqn 18 can be written as

\[ \frac{K_f A_{xT_{wx}}}{\Delta x} \left( P_{i+1, j, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \\
\left( \frac{K_f A_{yT_{wx}}}{\Delta y} \right)_{i, j+\frac{1}{2}, k} \left( P_{i, j+1, k}^{n+1} - P_{i, j, k}^{n+1} \right) + \\
\left( \frac{K_f A_{yT_{wy}}}{\Delta y} \right)_{i, j-\frac{1}{2}, k} \left( P_{i, j-1, k}^{n+1} - P_{i, j, k}^{n+1} \right) = \left( \frac{V_B \rho_{wsc} (1 - S_{wf}) \varnothing_f c_f}{\alpha_c B_g} \right) \frac{P_{i, j}^{n+1} - P_{i, j}^{n}}{\Delta t} \]

Writing eqn 19 in the following form

\[ \lambda_{w, i+\frac{1}{2}, j, k} \left( P_{i+1, j, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \lambda_{w, i-\frac{1}{2}, j, k} \left( P_{i, j, k}^{n+1} - P_{i-1, j, k}^{n+1} \right) + \lambda_{w, i+\frac{1}{2}, j, k} \left( P_{i, j+1, k}^{n+1} - P_{i, j, k}^{n+1} \right) - \\
\lambda_{w, i-\frac{1}{2}, j, k} \left( P_{i, j-1, k}^{n+1} - P_{i, j, k}^{n+1} \right) = X_{i, j, k} \left( P_{i, j, k}^{n+1} - P_{i, j, k}^{n} \right) \]

Where,

\[ \lambda_{w, i+\frac{1}{2}, j, k} = \left( \frac{K_f A_{xT_{wx}}}{\Delta x} \right)_{i+\frac{1}{2}, j, k} \]

\[ \lambda_{w, i-\frac{1}{2}, j, k} = \left( \frac{K_f A_{xT_{wx}}}{\Delta x} \right)_{i-\frac{1}{2}, j, k} \]
\[
\lambda_{w_{ij+\frac{1}{2}k}} = \left(\frac{K_f \Delta y_{wy}}{\Delta y_{ij+\frac{1}{2}k}}\right)
\]
\[
\lambda_{w_{ij-\frac{1}{2}k}} = \left(\frac{K_f \Delta y_{wy}}{\Delta y_{ij-\frac{1}{2}k}}\right).
\]
\[
X_{i,j,k} = \left(\frac{v_p \rho_{wsc} (1 - S_{gr}) \phi_{cf}}{\alpha_c B_w \Delta t}\right)
\]

Eqn 20 can be written in the form of

\[
S_{i,j,k} p_{i,j-1,k}^{n+1} + W_{i,j,k} p_{i-1,j,k}^{n+1} + C_{i,j,k} p_{i,j,k}^{n+1} + E_{i,j,k} p_{i+1,j,k}^{n+1} + N_{i,j,k} p_{i,j+1,k}^{n+1} = Q_{i,j,k}
\]

Where,

\[
S_{i,j,k} = \lambda g_{ij-\frac{1}{2}k},
\]
\[
W_{i,j,k} = \lambda g_{ij-\frac{1}{2}l,j,k},
\]
\[
C_{i,j,k} = \lambda g_{ij+\frac{1}{2}l,j,k} + \lambda g_{i-\frac{1}{2}l,j,k} + \lambda g_{ij+\frac{1}{2}k} + \lambda g_{ij-\frac{1}{2}k} + X_{i,j,k},
\]
\[
E_{i,j,k} = \lambda g_{i+\frac{1}{2}l,j,k},
\]
\[
N_{i,j,k} = \lambda g_{ij+\frac{1}{2}k},
\]
\[
Q_{i,j,k} = (-X_{i,j,k}) p_{i,j,k}^{n}.
\]

As, in a shale gas reservoir there may be N number of hydraulic fractures in order to increase the interconnectivity of the wellbore with the matrix, which will increase the rate of production. In this model, we have done my simulation starting with single hydraulic fracture and continued till 5 hydraulic fractures. The pictorial representation of the different positions of hydraulic fractures is shown in Figure 4.4(a,b,c and d), generally equispaced.
Figure 4.4(a): Schematic representation of a Single Hydraulically Fractured Reservoir.

Figure 4.4(b): Schematic representation of a Two Hydraulically Fractured Reservoir.

Figure 4.3(c): Schematic Representation of a Three Hydraulically Fractured Reservoir.
As the hydraulic fractures are surrounded by matrix blocks and the flow of gas will be initially from the matrix to the fracture, while simulating the hydraulic fractured reservoir the non-linear PDE’s of both the matrix and the fractures are simultaneously solved for getting the flow rates of gas from the hydraulic fracture to the wellbore.

4.3 **Algorithm:**

The developed nonlinear partial differential equation is compiled using JAVA for determining the pressure variation in the hydraulically fractured blocks during gas production from shale reservoirs. Figure 4.5 represents the algorithm, used for solving the nonlinear PDE for gas flow inside the matrix.
Declaration of Matrix Variables.

Input of Δx, Δy and Δz and total number of days, minimum no of days is 1 interval.

Calculation of total no of intervals.

Input the values of pressure, temperature, Langmuir volume, langmuir pressure, k_r, p_wf and different gas compositions

Δy of matrix is calculated based on Δy of the wellbore.

Φ_0, no of hydraulic fractures, height of fractures and length of wellbore is taken as input.

Each block of the reservoir (9*9*9) is given the pressure value as an input initially.

A wellbore at the center Z parallel to Z axis is created by putting the wellbore pressure to it.
Declaration of array coefficients for Matrix.

If time is less than the total time interval

YES

The array coefficients are calculated based on if the indicated position comes in wellbore, above and below the wellbore or other.

The Pressure Values are printed in output file.

NO

The coefficients are accordingly placed in the matrix using the position indexing variables, then the matrix multiplication is carried out to find the array of pressures of the blocks for time interval (t+1).

The excel file is generated with $\mu$, $B_g$, $Z$ and pressure values for each block.

The Declaration of fracture variables is done and initialized accordingly.

Pressure of matrix after 1 time interval is taken as initial pressure of reservoir for fracture.

The wellbore pressure is reentered at the center and parallel to Z-axis.
Figure 4.5: Algorithm representing the procedure for solving the gas flow in the hydraulic fractures.

The derived nonlinear partial differential equation for flow of gas in induced or hydraulic fractures is compiled using JAVA programming language and the code is attached in ANNEXURE-II.