Chapter 1

Introduction
Theory of graphs has established itself as an important tool in a wide variety of subjects, ranging from operational research to chemistry and from genetics to linguistics and from electrical engineering to geography.

It is believed that the study of famous Königsberg bridge problem in 1736 by Leonhard Euler was supposed to be the birth of graph theory. The theory of trees was developed by G. R. Kirchhoff in 1847 for its applications in electrical network. Arthur Caley in 1857 introduced trees to enumerate the isomers of hydrocarbon series $C_nH_{2n+2}$. The first systematic representation of four color problem was made by A. De Morgan in 1850s provided the reason for the initiation of many new problems in graph theory.

There are many interesting fields of research in graph theory. Some of them are labeling of discrete structures, algorithmic graph theory, enumeration of graphs and theory of domination are among worth to mention. The research work presented in the thesis mainly deals with the concept of dominating sets in graphs and problems related to it. The theory of domination traces its origin in 1850s when some enthusiastic thought about the minimum number of queens to be placed on $8 \times 8$ chessboard so that all the squares are either attacked by a queen or occupied by a queen. In 1862 de Jaenisch determined that the minimum number of such queens is five. The problem is ultimately to find a dominating set of five queens which can be precisely stated as follows. A vertex subset of graph $G$ is called a dominating set if every vertex is an element of vertex subset or is adjacent to at least one element of that subset.

Theory of domination was formalized by Berge [4] in 1958 and Ore [36] in 1962. A detailed survey on theory of domination presented by Cockayne and Hedetniemi [16] has received considerable attention. The first comprehensive title "Fundamentals of Domination in Graphs" by Haynes et al. [28] contains the noteworthy discussion on domination and subset related problems such as independence, covering, matching, etc. Theory of domination has close interaction with many branches of mathematics, engineering and social sciences. The content is spread over five chapters apart from list of references.
The first chapter is targeted to describe a summary of the thesis while the immediate chapter-2 serves as prerequisite for the remaining chapters. We introduce all the basic terminology and notations which are necessary for the advancement of the flow of discussion.

The strong and weak domination among graph are variants of domination which arise naturally in certain practical situations. For example, consider a network of roads connecting a number of locations. In such a network, the degree of a vertex \( v \) is the number of roads meeting at \( v \). Suppose \( d(u) \geq d(v) \). Naturally, the traffic at \( u \) is heavier than that at \( v \). If we consider the traffic between \( u \) and \( v \), preference should be given to the vehicles going from \( u \) to \( v \). Thus, in some sense, \( u \) strongly dominates \( v \) and \( v \) weakly dominates \( u \). The third chapter is aimed to report some results on strong and weak dominating sets. This concept was introduced by Sampathkumar and Pushpa Latha [42].

The distance between two vertices in a graph is a simple but surprisingly useful notion which is defined as length of the shortest path between two vertices \( u \) and \( v \). It has led to the definition of several graph parameters such as diameter, radius, average distance and metric dimension. Generalizations of shortest paths connecting pairs of vertices to shortest trees, called steiner trees, that connect three or more vertices defines steiner distance. The discussion related to steiner distance in the context of domination theory is carried out in chapter-4. We have established some characterizations and proved some results on steiner domination in graphs.

In the definition of distance, if the length of longest path between two vertices \( u \) and \( v \) is considered then it is known as detour distance. The last chapter-5 is aimed to discuss detour domination in graphs. We have studied this concept in various contexts.

Throughout the thesis, many open problems are posed and future scope of research is indicated. Most of the content is published in scholarly, peer-reviewed and indexed journals with good impact factor which is one of the salient features and significance of the research work embodied in the thesis.
A list of symbols is given just after the table of contents while a bibliography in MLA (Modern Language Association) is provided in alphabetical order of the surname of the first author at the end. The work reported in the thesis is also presented in prestigious conferences, seminars and workshops. A list of publications and an appendix containing reprints of research papers arising from the thesis is attached as Annexure.
List of Publications Arising From the Thesis


   ISSN: 2249 - 3328 (Print) ISSN: 2319 - 5215 (Online)

   Impact Factor: 5.120 (Cosmos Impact Factor)


   ISSN: 2231 - 5330


   (http://fs.gallup.unm.edu/IJMC-3-2017.pdf)

   ISSN: 1937 - 1055


   (http://dx.doi.org/10.17654/DM018040445)

   ISSN: 0974 - 1658

   Impact Facor: 0.61


   ISSN: 0976 - 3228, E - ISSN: 2455-9601


ISSN: 2249 - 3328 (Print) ISSN: 2319 - 5215 (Online)

Impact Factor: 5.120 (Cosmos Impact Factor)


(http://www.malayajournal.org/articles/MJM06020013.pdf)

ISSN: 2321 - 5666(Online) ISSN: 2319-3786(Print)

DOI:10.26637/MJM0602/0013

Impact Factor: 5.54(Index Copernicus)

Details of the Work Presented in Conferences

1. The paper entitled as “Steiner Domination of Some Wheel Related Graphs” was presented in The Eleventh Annual Conference of Academy of Discrete Mathematics and Applications (ADMA) and Graph Theory Day-XI held at B. S. Abdur Rahman University, Vandalur (Chennai) during 10-12 June, 2015.

2. The paper entitled as “On Detour Domination in Graphs” was presented in XXX Gujarat Science Congress-2016 on “Challenges for Sciences and Technology Education During Coming Decades: Preparing for Sustainable Gujarat” jointly organized by K.S.K.V Kachchh University, Bhuj and Gujarat Science Academy during 6-7 February, 2016.

3. The paper entitled as “Some Results on Detour Domination in Graphs” was presented and received first rank in competition in 9th National Level Science Symposium 2016 on Recent Trends in Science and Technology organized by Christ College, Rajkot sponsored by GUJCOST on February 14, 2016.

4. The paper entitled as “Strong Domination in Some Cycle Related Graphs” was presented in The 7th National Conference on “Emerging Vistas of Technology in 21st century” organized by Faculty of Engineering and Technology, Parul University Vadodara during April, 8-9, 2016.

5. The paper entitled as “Strong Domination Number of Some Cycle Related Graphs” was presented in the International Conference on Discrete Mathematics -2016 and Graph Theory Day-XII sponsored by TEQIP-II organized at Siddaganga Institute of Technology, Tumkur during June, 9-11, 2016.

6. The paper entitled as “Weak Domination Number of Some Graphs” was presented in workshop on Indian Women and Mathematics: Regional Workshop on Research and Career Opportunities organized by IIT Gandhinagar, Gujarat during December 20-21, 2016.
7. The paper entitled as “Some Results on Weak Domination in Graphs” was presented in the National Conference on Algebra, Analysis and Graph Theory - 2017 organized by the Department of Mathematics, Saurashtra University, Rajkot held during February 9-11, 2017.

8. The paper entitled as “On Steiner Domination in Graphs” was presented in 13th Annual Conference of Academy of Discrete Mathematics and Applications (ADMA) and Graph Theory Day-XIII jointly organized by Department of Mathematics, SSN College of Engineering, Kalavakkam (Banglore) during June 8-10, 2017.

9. The paper entitled as “Detour Domintaion in the Context of Switching of a Vertex” was presented at the National Conference on Applied Mathematical Sciences, jointly organized by Department of Mathematics(DST-FIST supported), Gujarat University, Ahmedabad and Department of Applied Sciences and Humanities, Parul University, Vadodara held during April 14-15, 2018.

10. The paper entitled as “Detour Domination in Graphs in the Contexts of Degree Splitting” was presented in 14th Annual Conference of Academy of Discrete Mathematics and Applications (ADMA) and Graph Theory Day 2018 at Dhirubhai Ambani Institute of Information and Communication Technology Gandhinagar during June 6-10, 2018.