SYNOPSIS

Introduction

The remediation of a contaminated aquifer is a complex process which involves huge cost and time and requires proper planning to optimize the cost and effectiveness of the remediation. A very crucial aspect in the remediation process is to identify the locations and estimate the release magnitude of the unknown contaminant sources. Due to the complexity of groundwater transport process, models are required to identify the location and extent of contaminant plume. Groundwater contaminant transport modeling involves the solution of advection-dispersion-reaction (ADR) equation. Identification of the sources of contamination from the spatial and temporal measurements of contaminant concentrations in the aquifer is an inverse problem which requires solving the ADR equation backwards in time and an optimizer to minimize the error involved.

Grid or mesh based methods such as FDM or FEM are commonly used to model the groundwater flow and transport process in an aquifer. However, due to the difficulty and high computational costs of creating a mesh, meshfree methods have been recently developed. Meshfree methods require only a set of scattered nodes within and on the boundary to represent the modeling domain and no information are required on the connectivity of the nodes. Several meshfree methods have been developed in the last few decades (Liu and Gu, 2005). Meshfree Radial Point Collocation Method (RPCM) is a simple meshfree method (Kansa, 1990) based on the point collocation using Radial Basis Functions (RBF) as the interpolation function. In this study, meshfree RPCM is proposed for simulation of the groundwater flow and transport process.

A Simulation-Optimization (SO) approach is one of the effective techniques used for groundwater source identification. In this technique, the groundwater contaminant source identification problem is formulated as forward-time simulation in conjunction with an optimization model. Gorelick et al. (1983) are the first to apply simulation-optimization approach to groundwater contamination source identification using linear programming and regression as the optimizer. In recent times, many optimization tools based on artificial intelligence such as particle swarm optimization (PSO) has been evolved. In the present study, an attempt is made to develop groundwater contaminant source identification model with the SO approach using Meshfree RPCM as the simulation model and PSO as the optimizer. PSO offers advantages in terms of simplicity in implementation and easy integration with simulation models.

Within this context, the objectives of the present study are:

- Development of coupled groundwater flow and transport simulation models using meshfree radial point collocation methods (RPCM) for both confined and unconfined aquifer problems.
SYNOPSIS

- Development of optimization model using Particle Swarm Optimization (PSO).
- Coupling of meshfree RPCM simulation model and PSO based optimization model to develop Simulation-Optimization (SO) models for groundwater contaminant source identification.
- Validation of the developed models with benchmark problems from published literature.
- Application of the developed SO model to hypothetical and field case studies and to cases with uncertainties in concentration data.

Groundwater Flow and Solute Transport Simulation

The governing equations describing the groundwater flow in a two dimensional inhomogeneous confined and unconfined aquifers can be written as (Bear, 1979):

\[
\frac{\partial}{\partial x} \left[ T_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ T_y \frac{\partial h}{\partial y} \right] = S \frac{\partial h}{\partial t} + Q_w \delta(x - x_i)(y - y_i) - q \tag{1}
\]

\[
\frac{\partial}{\partial x} \left[ K_x h \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y h \frac{\partial h}{\partial y} \right] = S_y \frac{\partial h}{\partial t} + Q_w \delta(x - x_i)(y - y_i) - q \tag{2}
\]

Subject to the initial conditions: \( h(x, y, 0) = h_0(x, y) \) \( \forall x, y \in \Omega \) and the boundary conditions: \( h(x, y, t) = h_1(x, y, t) \) \( \forall x, y \in \partial \Omega_1 \); \( \frac{\partial h}{\partial n} = q_1(x, y, t) \) \( \forall x, y \in \partial \Omega_2 \) or \( Kh \frac{\partial h}{\partial n} = q_2(x, y, t) \) \( \forall x, y \in \partial \Omega_2 \). Here \( h(x, y, t) \) is the piezometric head (m) which is the state variable; \( K_x, K_y \) are the hydraulic conductivities (m/d) in \( x \) & \( y \) directions; \( T_x, T_y \) are the transimissivities (m2/d) in \( x \) & \( y \) directions; \( S \) is the storage coefficient; \( S_y \) is the specific yield and \( Q_w \) is the source or sink term (m3/d/m2). The flow region is represented by \( \Omega \) while the boundary of the domain is denoted by \( \partial \Omega \) (\( \partial \Omega_1 \cup \partial \Omega_2 = \partial \Omega \)). \( \frac{\partial}{\partial n} \) denotes the normal derivative to the boundary. \( h_0(x, y) \) is the initial head in the flow domain (m), \( h_1(x, y, t) \) is the known head value of the boundary head (m) and \( q(x, y, t) \) is the known inflow rate (m3/d/m). Once the hydraulic head is computed, the seepage velocity \( V_x, V_y \) can be evaluated as: \( V_x = -\frac{K_x}{\theta} \frac{\partial h}{\partial x} \); \( V_y = -\frac{K_y}{\theta} \frac{\partial h}{\partial y} \) where \( \theta \) is the porosity of the medium.

The governing equation for two dimensional solute transport of a single chemical constituent in groundwater is given by (Freeze and Cherry, 1979),

\[
R \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( D_{xx} \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_{yy} \frac{\partial C}{\partial y} \right) - \frac{\partial}{\partial x} (V_x C) - \frac{\partial}{\partial y} (V_y C) - c' \frac{C}{\theta b} - R \lambda C + \frac{q_w C}{\theta} \tag{3}
\]

where, \( D_{xx}, D_{yy} \) are the components of dispersion coefficient tensor in \( x \) and \( y \) direction (m2/day); \( D_{xx} = \frac{a_L v_x^2 + a_T v_x^2}{v_x^2} \); \( D_{yy} = \frac{a_L v_y^2 + a_T v_y^2}{v_y^2} \), \( a_L \) and \( a_T \) are the dispersivity in longitudinal (\( x \) and
transverse \((y)\) directions and \(V^2 = V_x^2 + V_y^2\); \(C\) is the concentration of the dissolved species \((\text{kg/m}^3)\); \(\lambda\) is the reaction rate constant \((\text{day}^{-1})\); \(w\) is the elemental recharge rate with solute concentration \(c^*\); \(b\) is the aquifer thickness; \(R = 1 + \rho_b K_d/\theta\) is the retardation factor, in which \(\rho_b\) is the media bulk density, \(K_d\) is the sorption coefficient and \(q_w\) is the volumetric pumping rate from a source. The initial condition is, \(C(x, y, 0) = f(x, y, 0) \forall (x, y) \in \Omega\). The boundary conditions are of the form, \(C(x, y, t) = g_1(x, y, t) \forall (x, y) \in \partial \Omega_1\) (Dirichlet boundary condition); \((D_{xx} \partial C/\partial x)n_x + (D_{yy} \partial C/\partial y)n_y = g_2(x, y, t) \forall (x, y) \in \partial \Omega_2\) (Neumann boundary condition). Here, \(\Omega\) is the flow domain, \(\partial \Omega = \partial \Omega_1 \cup \partial \Omega_2\) denotes the boundary of the modeling domain; \(f\) is a given function in \(\Omega\); \(g_1, g_2\) are functions along the boundaries; and \(n_x, n_y\) are the components of the unit outer normal vector to the boundary. In this study, meshfree RPCM (Liu and Gu, 2005) has been used for solving the groundwater flow and transport equations. In this method, the interpolation of the unknown field variable is done by using radial basis functions (RBF) while discretization of the governing equations is achieved by point collocation.

**Meshfree Radial Point Collocation Method (RPCM)**

The unknown field variable is first approximated using trial or shape functions. A local support domain of a point \(x\) determines the nodes, in its neighborhood, to be used to approximate the function value at \(x\). The approximation of a function \(h(x)\) within a local support domain can be constructed as a linear combination of \(n\) radial basis functions as: \(h(x) = \sum_{i=1}^{n} a_i R_i(x) = R^T(x) a\); where \(R_i(x)\) is a radial basis function (RBF) such as a Multi-Quadrics or Gaussian RBF (Liu and Gu, 2005), \(n\) is the number of points in the support domain, \(a_i\) are unknown coefficients to be determined. The interpolation of the function at the \(k^{th}\) point has the following form:

\[
h(x_k, y_k) = h_k = \sum_{i=1}^{n} a_i R_i(x_k, y_k) \quad k = 1, \ldots, n
\]

which yields \(n\) simultaneous linear algebraic equations with \(n\) unknowns. Solving the system of equations for the unknown coefficients \(a_i\) and substituting back these coefficients, the interpolation can be expressed as,

\[
h(x, t) = \Phi^T(x) h_s(t)
\]

where \(\Phi^T(x) = \{\phi_1(x, y) \quad \phi_2(x, y) \quad \ldots \quad \phi_n(x, y)\}\) is known as the shape function and, \(h_s = \{h_1, h_2 \ldots h_n\}^T\) is the vector of nodal head values at the support domain nodes.

The discretization of the governing equation for groundwater flow and solute transport (eqns. 1, 2 and 3) is done by simple collocation at all the internal nodes. Thus by collocation at the point \(x_r(x_r, y_r)\), the governing equation (1), for a homogeneous and isotropic aquifer, is discretized as:
For time discretization, Cranck-Nicholson time stepping method is applied. Eqn. (6) is finally transformed as:

\[
T_r \left[ \frac{\partial^2 \Phi^T}{\partial x^2} + \frac{\partial^2 \Phi^T}{\partial y^2} \right] h_s(t) = S_r \left( \frac{\partial h}{\partial t} \right)_t + Q_w \delta(x_r - x) - q_r
\]  

(6)

A similar discretization is performed for the solute transport equation which yields:

\[
S_r \Phi^T - \theta \Delta t (\frac{\partial^2 \Phi^T}{\partial x^2} + \frac{\partial^2 \Phi^T}{\partial y^2}) h_s^{t+\Delta t} = S_r h_s^t + \Delta t (1 - \theta) (\frac{\partial^2 \Phi^T}{\partial x^2} + \frac{\partial^2 \Phi^T}{\partial y^2}) h_s^t - Q_w \delta(x_r - x) + q_r
\]  

(7)

where \( \theta = 0.5 \). A similar discretization is performed for the solute transport equation which yields:

\[
R \Phi^T - \theta \Delta t \left( \frac{\partial^2 \Phi^T}{\partial x^2} + \frac{\partial^2 \Phi^T}{\partial y^2} - V_x \frac{\partial \Phi^T}{\partial x} - V_y \frac{\partial \Phi^T}{\partial y} \right) C_s^{t+\Delta t} = RC_r^t + (1 - \theta) \Delta t \left[ \frac{\partial^2 \Phi^T}{\partial x^2} + \frac{\partial^2 \Phi^T}{\partial y^2} - V_x \frac{\partial \Phi^T}{\partial x} - V_y \frac{\partial \Phi^T}{\partial y} \right] C_s^t
\]  

(8)

The discretized equations (7) and (8) are established for all the internal nodes. Appropriate boundary conditions are applied at the boundary nodes. The nodal discretized equations of eqn. (7) can be assembled in matrix form as (Guneshwor et al., 2016),

\[
[K]_{N \times N} [h]_{N \times 1}^{t+\Delta t} = [F]_{N \times N} [h]_{N \times 1}^t + [Q]_{N \times 1}
\]  

(9)

where \( N \) is the total number of nodes including internal and boundary nodes. \( \{h\}^{t+\Delta t} \) is the unknown nodal heads (or concentrations) at current time, \( t = t + \Delta t \) while \( \{h\}^t \) is its values at the previous time step. \( [K] \) is the global coefficient matrix (LHS of eqn. 7), \( [F] \) is the coefficient matrix associated with previous time-step and \( \{Q\} \) is the vector representing contributions of the sources or sinks terms.

In a similar way, the nodal discretized equation of eqn. (8) can be obtained. Further a similar discretization is performed for unconfined aquifers using eqn (2). Based on the above formulation, a coupled groundwater flow and transport simulation model (CFTM) in 2D using RPCM called meshfree CFTM-RPCM has been developed. The developed model has been verified for its accuracy and effectiveness with respect to available analytical and numerical solutions for various problems.

**Particle Swarm Optimization (PSO)**

The Particle Swarm Optimization (PSO) is a nature-inspired swarm intelligence algorithm proposed by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995). A swarm is a collection of multiple units known as particles that interact with each other leading to a complex behavior. PSO relies on the
interactions of these particles to find the optimum value of a function. The ability of the PSO algorithm to optimize a given objective function comes from the velocity and position update steps of the algorithm. For each particle in the swarm, the velocity is updated using the following equation (Clerc and Kennedy, 2002):

\[ v_i(t + 1) = w v_i(t) + c_1 r_1 (x_i^p(t) - x_i(t)) + c_2 r_2 (x_g(t) - x_i(t)) \]  

(10)

The position is then updated as,

\[ x_i(t + 1) = x_i(t) + v_i(t + 1) \]  

(11)

Here, \( i \) is the index of the particle. Thus, \( v_i(t) \) and \( x_i(t) \) are the velocity and position respectively of the \( i^{th} \) particle at time \( t \). The parameters \( w, c_1 \) and \( c_2 \) are coefficients to be-specified by the user and their ranges are: \( 0 \leq w \leq 1.2, 0 \leq c_1 \leq 2 \) and \( 0 \leq c_2 \leq 2 \). \( c_1 \) and \( c_2 \) controls the cognitive and social aspects of the particle respectively while \( w \) controls the inertia of the particle. The values \( r_1 \) and \( r_2 (0 \leq r_1 \leq 1 \text{ and } 0 \leq r_2 \leq 1) \) are random values generated for each velocity update. \( x_i^p(t) \) is the individual best candidate solution for the \( i^{th} \) particle at time \( t \), while \( x_g(t) \) refers to the swarm’s global best candidate solution at time \( t \). Based on the above formulation, a PSO model has been developed and verified for its accuracy and effectiveness.

Simulation-Optimization Model for Source Identification

In Simulation-Optimization (SO) models, the groundwater contaminant source identification problem is formulated as forward-time simulation in conjunction with an optimization model. In this approach, several forward-time simulations of the groundwater solute transport equation are run with different sets or combinations of the potential sources and their strengths. The predicted solute concentrations of these forward runs are compared against the measured spatial and temporal concentration data. Since there are an infinite number of plausible sets of the potential sources, an optimization model is required to find that set of potential sources which lead to minimum difference between the simulated and observed concentrations (the objective function). In this study an SO model (RPCM-PSO-SO model) is being developed using meshfree RPCM as the simulation model and the PSO as the optimizer. Since the forward-time simulation of the groundwater solute transport model needs to be called many times to search for the optimal solution, it may take very large computational time. To overcome this, a concentration response matrix is developed (Gorelick et al., 1983) for evaluation of the concentrations of the contaminant at the observation wells from the forward simulations.

Figure 1 shows the flow chart of the simulation-optimization approach for groundwater contaminant source identification. This approach avoids the mathematical complexities associated with direct inversion of the groundwater flow and solute transport equation.
Figure 1: Flow chart of the developed Simulation-Optimization model (RPCM-PSO-SO model)

Objective Function for Optimization

As depicted in the flow chart (Figure 1), the simulation-optimization (SO) model seeks to match the concentrations predicted by the model to the measured (observed) concentrations. This matching is done by minimizing the sum of a function of the differences between the simulated and measured concentrations. In this study, the objective function used for minimization is the sum of squared differences between predicted and observed concentrations i.e.

$$S = \sum_{i=1}^{NO} \sum_{j=1}^{NT} (C_{i,j}^{pred} - C_{i,j}^{obs})^2$$  \hspace{1cm} (12)
where, $NO = \text{no. of observation bore wells}$, $NT = \text{no. of time steps}$, $C_{i,j}^{\text{pred}} = \text{Predicted concentration at } i^{th} \text{ observation point at the } j^{th} \text{ time step}$, $C_{i,j}^{\text{obs}} = \text{Observed/measured concentration at } i^{th} \text{ observation point at } j^{th} \text{ time step}$.

Application of the Meshfree RPCM Simulation Model

The meshfree RPCM simulation model is applied to one case study of confined aquifer near Surat, Gujarat (Figure 3). The aquifer is bounded by a lake on the north, north-east, west and south-west boundaries. There are no water bodies on the rest of the boundary. The area to be modeled is approximately 4.5 km$^2$. A total of 1008 nodes were used corresponding to a separation between the nodes of 49.6 m along x-direction ($\Delta x$) and 42.8 m along the y-direction ($\Delta y$). The field measured hydrogeological parameters of the site are obtained from Singh and Sarma (2009). It is observed that the transmissivity of the area varies from a minimum of 30.0 m$^2$/day to a maximum of 170.0 m$^2$/day.

There is a recharge zone within the model domain which corresponds to a water pond and is known to be leaking water to the aquifer and is observed as a mound in the water table map of the area. The boundaries adjacent to the lake are treated as constant head boundaries while the rest of the boundaries are treated as flux boundaries. The flux boundary values are estimated and adjusted during calibration of the model. The recharge through the pond is estimated from the make-up level of the pond and is adjusted during calibration. Figure 3 shows the head distribution (contours) and velocity vectors from the RPCM flow model. An areal contaminant (assumed to be TDS) source is assumed to be leaking with a concentration of 1000 ppm. The longitudinal dispersivity ($\alpha_l$) for this problem is 20 m and the transverse dispersivity ($\alpha_T$) is taken as 10% of the longitudinal dispersivity. The transport model was simulated for a period of 10 year. Meshfree RPCM model was applied to compute the head and concentration distribution in the aquifer. The results were compared to the field measured heads and FEM predicted concentration distributions (Figure 2) and found to be in good agreement (Guneshwor et al. 2016). This validates the applicability of the meshfree CFTM-RPCM model.
Application of the Meshfree RPCM –PSO Simulation-Optimization Model

The developed SO model has been applied to a number of hypothetical and field case problems. Results of two case studies are presented here – the first is a steady state case involving a hypothetical aquifer and the second is a transient field case study.

Case Study 1 (Steady state case): Figure 4 shows the hypothetical aquifer considered for the steady state case. An underground pipe lies in an unsaturated zone above the water table and carries effluent from one end to the other. This problem was studied by Gorelick et al.(1983) and all the parameters are taken from the above work. The aquifer is 10 m thick and is of size 160 m x 240 m with a constant hydraulic conductivity of 0.864 m/day. It is assumed that the effluent (pollutant) has been flowing continuously within the pipe and contains high concentrations of the non-reactive pollutants – chloride and tritium. Several observation borewells are located within the modeling region which is used for detecting and measuring the concentration of the pollutant found in the groundwater. It is assumed that the relatively small volume of effluent that has leaked into the aquifer does not change the original, steady groundwater flow pattern.
Figure 4: Hypothetical aquifer with an underground pipeline carrying effluent

Figure 5: Chloride Concentration distributions (mg/L)

Here the concentration distributions are at steady state. It is required to locate the leak(s) and determine the magnitude of the solute and water flux from each leak from the concentration distributions of the pollutant observed in the water samples of the observation borewells.

Table 1: Source predictions by the meshfree RPCM-PSO-SO model of the present study as compared to other SO models

<table>
<thead>
<tr>
<th>Potential Source Location</th>
<th>True Leak Magnitudes (L/d)</th>
<th>RPCM-PSO-SO model Predicted Leak Magnitudes (L/d)</th>
<th>Leak magnitudes predicted by Gorelick et al. (1983)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Linear Programming (L/d)</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>III</td>
<td>518.4</td>
<td>518.2</td>
<td>518.2</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>VII</td>
<td>0</td>
<td>3.2</td>
<td>2.0</td>
</tr>
<tr>
<td>VIII</td>
<td>259.2</td>
<td>254.8</td>
<td>256.5</td>
</tr>
<tr>
<td>IX</td>
<td>0</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
The meshfree RPCM groundwater flow and transport simulation was carried out to create the concentration data used in this problem. In the transport model two sources (leaks) were placed at the locations III and VIII. The concentrations of Chloride and Tritium in the effluent are 15,000 mg/L and 15,000 µCi/L respectively. The water flux at first leak (location III) and second leak (location VIII) are respectively 259.2 L/day and 518.4 L/day. Figure 5 shows the chloride concentration distribution from the simulation model. The source predictions by the meshfree RPCM-PSO-SO model of the present study is shown in Table 1 where it has been compared with the SO model used by Gorelick et al. (1983). It is seen that the RPCM-PSO-SO model predictions are comparable to other models.

Case Study 2 (Transient case): The aquifer used for the transient field case study is shown in figure 3. Six hypothetical sources $S_1, S_2, S_3, S_4, S_5$ and $S_6$ are assumed to be releasing TDS (Total Dissolved Solids) contaminant through leaching over a period of four years according to the concentration data given in Table 2. It is assumed that after the 4 years of release, the sources stop releasing any contaminants. Four observation wells viz. $OB-1$, $OB-2$, $OB-3$ and $OB-4$ records the contaminant (solute) concentrations due to the releases from the above sources for a simulation period of 10 years.

Table 2: Strength of the sources during the leaching period

<table>
<thead>
<tr>
<th>Release Year</th>
<th>Source strength (in ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$</td>
</tr>
<tr>
<td>1st year</td>
<td>1000</td>
</tr>
<tr>
<td>2nd year</td>
<td>0</td>
</tr>
<tr>
<td>3rd year</td>
<td>900</td>
</tr>
<tr>
<td>4th year</td>
<td>0</td>
</tr>
</tbody>
</table>

The concentration values at the observation bore wells are recorded for every 30 days (monthly) to construct the breakthrough curves at the wells (Figure 6). The breakthrough curves at the four observation bore wells serve as the measured concentration data and are used as the input to the source identification model. The goal of the source identification model is to reconstruct the release history of the sources (Table 2) from this given concentrations data. The result of the source identification model is presented in Table 3.
The source strength predictions are shown in brackets (bold) alongside the actual release strengths. 50 x 10^3 iterations of PSO was used to generate this result. It is observed that the source predictions are accurate to within 5% of the true values. It may be observed that a few artificial or spurious sources are also predicted. However their magnitudes are very small and can therefore be safely neglected.

Further, the robustness of the proposed RPCM-PSO-SO model for source identification was examined by considering additional scenarios which simulates cases with limited amount of concentration data, missing or incomplete concentration data, intermittent or irregular data collection etc. Effect of the location of the observation borewells with respect to the sources was also examined by placing the wells randomly and in the contaminant plume path. It was observed that the source identification model was able to reconstruct the release histories satisfactorily in all the above cases. The application of the meshfree RPCM-PSO-SO model was further extended to cases with uncertain
concentration data since errors or uncertainties are inevitable in field measurements. The measurement uncertainties were simulated by perturbing the concentration data with random errors sampled from a normal distribution. Various ways of dealing with erroneous concentration data were examined such as, smoothing of the input data to recover the underlying relationship, imposing global and local constraints on the sources from field information, incorporating field information to rule out spurious predictions, giving weights when such information is conceivable or available etc. The developed source identification model was able to cope with the uncertain input data but the prediction accuracies are reduced with increase in uncertainty. From the above case studies, it can be concluded that the meshfree RPCM-PSO-SO model can be effectively used for groundwater source identification.

Conclusions

Following important conclusions are drawn from the present study:

- A Meshfree RPCM model is developed for groundwater flow and solute transport simulation for both confined and unconfined aquifers. The models are verified with available analytical and numerical solutions using FDM and FEM and found to be in good agreement. The developed model was also applied to a field case study. The results are compared to field measurements and with the results from FEM computation. The results were found to be satisfactory validating the applicability of the RPCM model.

- For the purpose of groundwater source identification, a simulation-optimization (SO) model using meshfree RPCM as the simulator and PSO as the optimizer was developed. The model was verified with other methods available in the literature and found to be effective.

- The source identification model was applied to a field case problem. Robustness of the model was examined by simulating various field scenarios such as limited amount of concentration data, missing concentration data, intermittent or irregular data collection etc. The effects of the location of the observation borewells, with respect to the contaminant plume path, on the model predictions were also examined. The model has been found to be effective for all types of the problems.

- The source identification model was also examined for its capability to handle concentration data with measurement errors. Both moderate and high levels of measurement errors were simulated and it was found that the model could handle the uncertainties involved.

- The RPCM-PSO-SO model developed identified the location and magnitude of the sources to acceptable accuracy and will be a useful tool in remediation of contaminated aquifers.