CHAPTER 6

SELECTIVE HARMONIC ELIMINATION OF SINGLE PHASE VOLTAGE SOURCE INVERTER USING ALGEBRAIC HARMONIC ELIMINATION APPROACH

6.1 INTRODUCTION

The problem of eliminating harmonics in switching converters has been the focus of research for many years. Present day available PWM schemes can be broadly classified as carrier modulated sine PWM and precalculated programmed pulse width modulation (PPWM) schemes. If the switching losses in an inverter are not a concern (i.e., switching on the order of a few kHz is acceptable), then the sine-triangle PWM method and its variants are very effective for controlling the inverter (Mohan et al 2004). This is because the generated harmonics are beyond the bandwidth of the system being actuated and therefore these harmonics do not dissipate power. On the other hand, for systems where high switching efficiency is of utmost importance, it is desirable to keep the switching frequency much lower. In this case, another approach is to choose the switching times (angles) such that a desired fundamental output is generated and specifically chosen harmonics of the fundamental are suppressed (Pate1 and Hoft 1973, Enjeti et al 1990, Sun and Grotstollen 1994). This is referred to as selective harmonic elimination or programmed harmonic elimination as the switching angles are chosen (programmed) to eliminate specific harmonics. This is also called as Computed Pulse Width Modulation (CPWM) or Programmed Pulse Width Modulation (PPWM). The Programmed PWM techniques optimize a
particular objective function such as to obtain minimum losses, reduced torque pulsations, selective elimination of harmonics and therefore the most effective means of obtaining high performance results. It is interesting to note that the various objective functions are chosen to generate a particular programmed PWM technique essentially constitutes the minimization of unwanted effects due to the harmonics present in the inverter output spectra. In view of this, little or no difference between each one of the programmed techniques is observed when significant number of low order harmonics is eliminated. However, each one of the programmed PWM techniques is associated with the difficult task of computing specific PWM switching instants to optimize a particular objective function. This difficulty is particularly encountered at lower output frequency range due to the necessity of large number of PWM switching instants. Also in most cases only a local minimum can be obtained after considerable computational effort. Despite these difficulties programmed PWM exhibit several distinct advantages in comparison to the conventional carrier modulated sine PWM schemes which are listed below:

1. For a given inverter switching frequency, the first uneliminated harmonic is almost double that for a natural or regular-sampled PWM scheme, thus resulting in a far superior pole switching waveform harmonic spectrum.

2. A much higher pole switching waveform fundamental amplitude is attainable before the minimum pulse-width limit of the inverter is reached.

3. About 50% reduction in the inverter switching frequency is achieved when compared with the conventional carrier modulated sine PWM scheme.
4. Higher voltage gain due to over modulation is possible. This contributes to higher utilization of the power conversion process.

5. Due to the high quality of the output voltage and current, the ripple in the DC link current is also small. Thus a reduction in the size of the DC link filter components is achieved.

6. The reduction in switching frequency contributes to the reduction in switching losses of the inverter and permits the use of GTO switches for high power converters.

7. Elimination of lower order harmonics causes no harmonic interference such as, resonance with external line filtering networks typically employed in inverter power supplies.

8. The use of precalculated optimized programmed PWM switching patterns avoids on line computations and provides straightforward implementation of a high performance technique.

In this chapter a simple new algebraic approach using MATLAB is proposed to obtain the notching angles and it is applied to a single phase voltage source inverter for selective harmonic elimination. The simulated results are compared with the developed PIC microcontroller (PIC 18F452) based hardware model.

6.2 PRINCIPLES OF HARMONIC ELIMINATION TECHNIQUE

The two-state output waveform of the single-phase inverter is approached from an analytical point of view and a generalized method for theoretically eliminating any number of harmonics is developed. The basic square wave is chopped a number of times and a fixed relationship between
the number of chops and possible number of harmonics that can be eliminated is derived. Figure 6.1 shows a generalized output waveform with N chops per half-cycle. It is assumed that the periodic waveform has half-wave symmetry and unit amplitude.

Therefore,

\[ f(\omega t) = -f(\omega t + \pi) \]  

(6.1)

where, \( f(\omega t) \) is a two state periodic function with N chops per half cycle.

Let \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{2m} \) define the N chops as shown in Figure 6.1. A Fourier series can represent the waveform as follows:

\[ f(\omega t) = \sum_{n=1}^{\infty} \left[ a_n \sin(n\omega t) + b_n \cos(n\omega t) \right] \]  

(6.2)

where

\[ a_n = \frac{1}{\pi} \int_{0}^{\pi} f(\omega t) \sin(n\omega t) d\omega t \]  

(6.3)
and \[ b_n = \frac{1}{\pi} \int_0^{2\pi} f(\omega t) \cos(\omega t) \cos(n\omega t) \, dt \] (6.4)

Substituting for \( f(\omega t) \) in equation (6.3) and using half wave symmetry property,

\[ a_n = \frac{2}{n\pi} \sum_{k=0}^{2N} (-1)^k \alpha \int_{\alpha_k}^{\alpha_{k+1}} \sin(n\omega t) \, dt \] (6.5)

where \( \alpha_0 = 0, \alpha_{2N+1} = \pi \) and \( \alpha_0 < \alpha_1 < \alpha_2 < \ldots < \alpha_{2N+1} \)

From equation (6.5), evaluating the integral

\[ a_n = \frac{2}{n\pi} \sum_{k=0}^{2N} (-1)^k \left[ \cos(n\alpha_k) - \cos(n\alpha_{k+1}) \right] \]

\[ a_n = \frac{2}{n\pi} \left[ \cos(n\alpha_0 - \cos(n\alpha_{2N+1}) + 2 \sum_{k=1}^{2N} (-1)^k \cos(n\alpha_k) \right] \] (6.6)

But \( \alpha_0 = 0 \) and \( \alpha_{2N+1} = \pi \). Hence,

\[ \cos(n\alpha_0) = 1 \] (6.7)

\[ \cos(n\alpha_{2N+1}) = (-1)^n \] (6.8)

Therefore, (6.6) reduces to

\[ a_n = \frac{2}{n\pi} \left[ 1 - (-1)^n + 2 \sum_{k=1}^{2N} (-1)^k \cos(n\alpha_k) \right] \] (6.9)

Similarly

\[ b_n = -\frac{4}{n\pi} \sum_{k=1}^{2N} (-1)^k \sin(n\alpha_k) \] (6.10)
Utilizing the half wave symmetry property of the wave form, $a_n = 0$ and $b_n = 0$ for even $n$. Therefore for odd $n$, from equations (6.9) and (6.10)

$$a_n = \frac{4}{n\pi} \left[ 1 + \sum_{k=1}^{2N} (-1)^k \cos n\alpha_k \right]$$  \hspace{1cm} (6.11)$$

$$b_n = \frac{4}{n\pi} \left[ -\sum_{k=1}^{2N} (-1)^k \sin n\alpha_k \right]$$  \hspace{1cm} (6.12)

Equations (6.11) and (6.12) are functions of $2N$ variables, $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{2N}$. In order to obtain a unique solution for the $2N$ variables, $2N$ equations are required. By equating any $N$ harmonics to zero, $2N$ equations are derived from equations (6.11) and (6.12).

The equations derived by equating $b_n = 0$ for $N$ values of $n$, are solved by assuming quarter-wave symmetry for $f(\omega t)$, i.e.,

$$f(\omega t) = f(\pi - \omega t)$$  \hspace{1cm} (6.13)$$

From the quarter-wave symmetry property, the following relations are obvious, with regard to Figure 6.1:

$$\alpha_k = \pi - \alpha_{2N-k+1} \text{ for } k = 1, 2, \ldots, N$$  \hspace{1cm} (6.14)$$

Therefore using equation (6.14),

$$\sin n\alpha_k = \sin n(\pi - \alpha_{2N-k+1})$$

$$\sin n\alpha_k = \left[ \sin n\pi \cos n\alpha_{2N-k+1} - \cos n\pi \sin n\alpha_{2N-k+1} \right] \text{ for } k = 1, 2, \ldots, N$$  \hspace{1cm} (6.15)
For odd \( n \)
\[
\sin n\pi = 0, \cos n\pi = -1
\]

Substituting in equation (6.15)
\[
\sin n\alpha_k = \sin n\alpha_{2N-k+1}, \text{ for } k = 1, 2, \ldots, N
\]
\[(6.16)\]

Substituting equation (6.16) in equation (6.12),
\[
b_n = \frac{4}{n\pi} \sum_{k=1}^{N} (-1)^{k+1} \left( \sin n\alpha_k - \sin n\alpha_{2N-k+1} \right) = 0
\]
\[(6.17)\]

From equation (6.14)
\[
\cos n\alpha_k = \cos n(\pi - \alpha_{2N-k+1}), \text{ for } k = 1, 2, \ldots, N
\]
\[(6.18)\]

For odd \( n \), equation (6.18) becomes
\[
\cos n\alpha_k = -\cos n\alpha_{2N-k+1}, \text{ for } k = 1, 2, \ldots, N
\]
\[(6.19)\]

Substituting equation (6.19) in equation (6.11)
\[
a_n = \frac{4}{n\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos n\alpha_k \right]
\]
\[(6.20)\]

For a two-state waveform of the type shown in Figure. 6.1, any \( N \) harmonics can be eliminated by solving the \( N+1 \) equations obtained from setting equation (6.20) to zero. The waveform is chopped \( N \) times per half-cycle and is constrained to possess odd quarter-wave symmetry.
6.3 ALGEBRAIC APPROACH FOR SOLVING HARMONIC ELIMINATION EQUATIONS

Mathematical analyses play a vital role in most of the process implementations. In power electronic circuits, one of the main problems is the formulation of the gate pulse for switching of the power electronic devices. When the gate pulse is a pulse of constant duty cycle, the problem becomes simpler. But in the case of a pulse width modulated (PWM) waveform, there is a great deal of calculations and they vary according to the PWM generation method used.

Mathematical analysis is used in this chapter to solve the non-linear simultaneous harmonic equations. The non-linear equations are the transcendental equations of the switching instants of the PWM waveform. These transcendental equations have to be solved either as such or by suitable transforms (Sun and Grotstollen 1994) in order to obtain the final refined angles to be used in the generation of PWM pulses for the elimination of desired harmonics. Some non-linear iterative methods are used to generate the angles by finding the solutions of the given set of equations. The whole procedure is finally automated using MATLAB programs.

6.3.1 Harmonic Elimination Equations

The generalized harmonic equation of power electronic inverters is of the form given by the expression

\[ V_{2k+1} = \frac{4V_{dc}}{(2k + 1)\pi} \sum_{i=0}^{N} h_i \cos(2k + 1)\alpha_i \]  \hspace{1cm} (6.21)
where

\( V \)  - Voltage output of the inverter
\( V_{dc} \)  - Magnitude of the DC input voltage
\( h_i \)  - Change of waveform level
\( \alpha \)  - Switching (notching) angle of the inverter
\( N \)  - The number of harmonic equations
\( k \)  - Switching angle number (varies from 0 to \( N-1 \))

The equation (6.21) is obtained from the Fourier series expansion of the inverter voltage equation (Sun et al 1994). In general,

Number of harmonic equations to be solved (\( N \)) = Number of harmonics to be eliminated (\( N \))+1.

The additional equation is for the fundamental component of the waveform. The change of waveform level implies the difference between the final value of the PWM pulse at an instant and the value of the wave before switching. This point can be made clear by assuming an exemplary waveform with three notch angles as shown in Figure 6.2. It is obvious that only the first two lower order harmonics can be eliminated using the three equations. The even order harmonics automatically get eliminated due to the symmetry of the wave.

![An exemplary PWM waveform for N=3](image)

**Figure 6.2** An exemplary PWM waveform for \( N=3 \)
The constraint to be satisfied by the waveform (Sun and Beineke 1996) in Figure 6.2 is that,

\[ \alpha_1 < \alpha_2 < \alpha_3 < \frac{\pi}{2} \]  

(6.22)

When the inverter is triggered by the waveform of Figure 6.2, a bipolar waveform (Vassilios et al 2004, Vassilios et al 2006) as shown in Figure 6.3 is obtained. This is because, in the case of a single phase bridge inverter, two sets of switches conduct for the first half of the wave, while the other two sets of switches conduct for the remaining half. Due to the complementary nature of these power electronic switches, a bipolar waveform of Figure 6.3 is obtained. Figure 6.3 is the logical representation of the output voltage waveform of a single phase bridge inverter.

![Bipolar output waveform](image)

**Figure 6.3 Bipolar output waveform corresponding to the pulse of Figure 6.1**

The wave of Figure 6.3 is also called as first type two levels waveform, popularly known as TWL1 waveform (Sun and Grotstollen 1992). The change of voltage level during first transition (at \( \alpha_1 \)) is \( h_1 = 1 - (-1) = 2 \). For second transition (at \( \alpha_2 \)), \( h_2 = -1 - 1 = -2 \). Similarly the other values of \( h_i \) can be calculated. When the equation (6.21) is expanded for \( N=3 \), the three simultaneous equations (6.22), (6.23) and (6.24) are obtained.
where $M = V_1$ denotes that the magnitude of the fundamental component is set to a pre-specified level $M$. Also the second and third equations are equated to zero, which denotes that the voltage level of the harmonics to be eliminated (third and fifth harmonics in this case) should be zero. In other words, the harmonic content pertaining to these two harmonic should not be present in the output waveform, once the converter/inverter is triggered with a PWM generated using the solution of the above equations. In order to solve the above equations, numerical methods are used.

### 6.3.2 Solution by Numerical Methods

It is found that the solving of the above transcendental (or trigonometric) functions of trigger angles is not possible as such. There are two popular and successful methods of solving non-linear transcendental equations. They are Seidel’s method and Newton’s method (www.math.fullerton.edu). But Seidel’s method can be applied only if all the simultaneous equations are functions of the same trigger angle. In simple words, Seidel suggests a method to assume the trigonometric function as a simple variable (i.e., $A = \cos (a)$, $B = \cos (b)$ and so on). The procedure then involves solving the equations for the variables ($A, B, \text{etc.}$) by trial and error. Finally, the inverse trigonometric functions are applied to the variables to get the final solutions (since $a = \cos^{-1}(A)$, $b = \cos^{-1}(B)$ and so on). But this method is not applicable for the set of equations obtained from the harmonic equations. This is because the first set of equations are the cosine functions of the trigger
angle itself, whereas the second and third set of equations are cosine functions of 3\textsuperscript{rd} and 5\textsuperscript{th} multiples of the angles in first equation. So even if Seidel method were applied, it would only worsen the problem by creating a set of 3 equations with 9 unknowns. As a result, Newton’s method is adopted for obtaining the solution for the equations.

### 6.3.3 Initial Switching Angle Generation

Any iterative mathematical method cannot be evaluated without the usage of initial values of the unknown parameter. Newton’s method is no exception to this rule. As a result, the convergence of Newton’s method and hence its effectiveness solely depends on the quality of initial guess. If the initial guess is too far from the final solution, then the Newton’s method may not even converge to the final value. So it is of utmost importance to not only provide the Newton’s method with an initial guess, but also provide it with a good enough guess. A very good guess consequently reduces the number of iterations and hence improves the speed of the whole method. This is very useful in real-time implementation of the method.

The real problem is that there is no clue to decide a good initial guess for Newton’s method. So the problem becomes quite challenging and the method becomes a trial and error method if the initial guesses are not known. So the initial angles are generated by converting the equation into a Cauchy problem (Sun and Grotstollen 1992) and then plotting a set of trajectories of triggering angles as a function of the preset level of the fundamental (M). This helps in the generation of good initial angles for a given value of M. These initial values are then passed into Newton’s method and finally the accurate trigger angles are obtained. This section can be further sub-divided into two parts, namely, the conversion of the equations into Cauchy problem and linearising the curves by using the method of least squares.
6.3.3.1 Cauchy problem formulation

For the waveform considered, the values for initiating the Cauchy problem is given by a general formula,

\[ \alpha_k = \frac{180^\circ \times k}{2N + 1} \text{ and } k=1, 2… N \]  \hspace{1cm} (6.25)

Before proceeding to the formation of Cauchy problem, the given equations have to be converted into matrix form. This is because majority of numerical methods handle the equations only in the form of matrix. So the matrix corresponding to the three equations (6.22), (6.23), (6.24) is,

\[
\begin{pmatrix}
1 & 2 & 3 \\
1 & 2 & 3 \\
1 & 2 & 3 \\
\end{pmatrix}
\begin{pmatrix}
\cos \alpha_1 \\
\cos \alpha_2 \\
\cos \alpha_3 \\
\end{pmatrix}
= 
\begin{pmatrix}
1 & 2 & 3 \\
1 & 2 & 3 \\
1 & 2 & 3 \\
\end{pmatrix}
\begin{pmatrix}
\pi M \\
\pi M \\
\pi M \\
\end{pmatrix}/4
\]

\hspace{1cm} (6.26)

Now the popular Jacobian matrix can be formed by differentiating the equation partially with respect to \( \alpha \), and is given by,

\[
J(\alpha) = 
\begin{pmatrix}
-2\sin \alpha_1 & 2\sin \alpha_2 & -2\sin \alpha_3 \\
-6\sin 3\alpha_1 & 6\sin 3\alpha_2 & -6\sin 3\alpha_3 \\
-10\sin 5\alpha_1 & 10\sin 5\alpha_2 & -10\sin 5\alpha_3 \\
\end{pmatrix}
\]  \hspace{1cm} (6.27)

In this case, the unknown vector is the differential equation involving the derivatives of \( \alpha \). As a result, it can be expressed in the form of a Cauchy problem as follows:
\[ J(\alpha) \frac{d\alpha}{dM} = b[1 \ 0 \ 0, \ldots, 0]^T \]  \hspace{2cm} (6.28)

where \( \alpha \bigg|_{M=M_0} = \alpha_0 \) and \( b = \frac{\pi}{8} \) for TWL1 waveform.

or this can be expressed more elaborately as,

\[
\begin{pmatrix}
-\sin \alpha_1 & \sin \alpha_2 & -\sin \alpha_3 \\
-\sin 3\alpha_1 & \sin 3\alpha_2 & -\sin 3\alpha_3 \\
-\sin 5\alpha_1 & \sin 5\alpha_2 & -\sin 5\alpha_3
\end{pmatrix}
\begin{pmatrix}
\frac{d\alpha_1}{dM} \\
\frac{d\alpha_2}{dM} \\
\frac{d\alpha_3}{dM}
\end{pmatrix}
= \begin{pmatrix}
\pi / 8 \\
0 \\
0
\end{pmatrix} \hspace{2cm} (6.29)
\]

In this case, the trigger angles are initiated by using the equation (6.25). So the values are,

\( \alpha_1 = \frac{180^0}{7}, \alpha_2 = \frac{360^0}{7} \) and \( \alpha_3 = \frac{540^0}{7} \)  \hspace{2cm} (6.30)

at \( M = M_0 = 0 \)

If the above values are chosen, then it can be proved that the fundamental, third and fifth harmonics have zero amplitudes for these initial angles. The equation (6.28) can be called as a typical Cauchy problem. Here, if the value of increment for \( M \) is small enough, then the method proves to be pretty effective. So the increment value for \( M \) is chosen as 0.001, though the value could be less than this value. As a result \( \Delta M \) will be nearly equal to \( dM \) (Sun and Grotstollen 1992). Now the following steps are to be followed to get the trajectories of \( \alpha \) versus \( M \):

1. Initially, set \( M = M_0 = 0, \alpha_k = \frac{180^0 \ast k}{2N+1}, k=1 \) to \( N \).
2. Solve the equation (6.29) to get the next points of \( \alpha \).
3. Increment the value of \( M \), by \( \Delta M = 0.001 \), so that \( M_{k+1} = M_k + \Delta M \). The next value is \( \alpha_{k+1} = \alpha_k + d\alpha \), where \( d\alpha = \Delta M \times d\alpha/dM \).

4. Again jump to step (2). Repeat this until \( \alpha_k > 0, \alpha_k > 0 \).

The above procedure is developed in the form of a M-file program. The program accepts the number of harmonic equations (N) as input, and generates a set of trajectories of \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) as a function of \( M \). It was found that the trajectories tend to zero, as \( M \) reaches the value 1.068. This is the limiting value of \( M \) for the case of \( N=3 \). This value varies according to the number of harmonics to be eliminated.

It was observed that the trajectories are linear for most of the portion. So it is a improved idea to linearise the trajectories to enable easy storage and enhanced speed of calculation, in case of real-time implementation. As a result, least square approximation method is used. The trajectories obtained from the program are shown in Figure 6.4. Similarly the trajectories obtained for other values of \( N \) are shown in Figure 6.5(a-f).

![Figure 6.4 Trajectories for least square approximation (for N=3)](image-url)
Figure 6.5 Trajectories for least square approximation (N=4 to 9)
6.3.3.2 Linearising by least square approximation

As seen from Figure 6.4, the trajectories are linear to almost the whole region. Linearization of these trajectories is carried out from least square curve fitting technique. Finally, the linear equation (Sun et al 1994) obtained will contain $\alpha$ as a linear or scalar function of $M$.

The method of least squares gives two normal equations for linearising any non-linear curve given by:

$$A \sum X + Bn = \sum Y$$

and

$$A \sum X^2 + B \sum X = \sum XY$$

(6.30)

where $A$ and $B$ are the unknown variables to be found, and $(X, Y)$ is the coordinates of the curve at selected points of the curve and $n$ represents the number of data points.

The greater the number of $(X, Y)$ coordinates, the greater is the accuracy of the method. In this case,

$$X = M \quad \text{and} \quad Y = \alpha_i, \quad \text{where } i = 1, 2, N.$$

It should be noted that the value of $n$ of in equation (6.30) denotes the no. of $(X, Y)$ coordinates, while the $N$ used throughout this chapter denotes the no. of harmonic equations.

As a result, the new set of harmonic normal equations is,

$$A \sum M + Bn = \sum \alpha_i$$

and

$$A \sum M^2 + B \sum M = \sum M \alpha_i$$

(6.31)
In this case, the actual coordinates of \((M, \alpha)\) are obtained from the Cauchy problem implementation procedure itself, rather than from the actual procedure of inferring the \(Y\) coordinates from \(X\) in the graph. This is to increase the accuracy of the initial points. The linear equations for the case of \(N=3\), have been found to be

\[
\begin{align*}
\alpha_1 &= 0.4660 - 0.1693M \\
\alpha_2 &= 0.9704 - 0.0866M \\
\alpha_3 &= 1.4105 - 0.4428M
\end{align*}
\]  
\tag{6.32}

Now it is clear that the value of \(\alpha\) can be found for a given value of \(M\). Also, in this case, the value of \(M\) should be less than that of \(M_{\text{max}} = 1.068\) for \(N=3\). For \(M=1\), the values of \(\alpha\) can be calculated from equation (6.32). These values are as follows:

\[
\begin{align*}
\alpha_1 &= 0.2967 \text{ radians (16.99°)} \\
\alpha_2 &= 0.8838 \text{ radians (50.63°)} \\
\alpha_3 &= 0.9677 \text{ radians (55.45°)}
\end{align*}
\]  
\tag{6.33}

These values represent the radians or in degrees and they can be converted to the actual time instant of switching of the wave as follows:

\[
\alpha_{\text{sec}} = \alpha_{\text{rad}} \frac{T}{2\pi} \quad \text{or} \quad \alpha_{\text{sec}} = \alpha_{\text{deg}} \frac{T}{180^0}
\]  
\tag{6.34}

where \(T\) represents the time period of the wave.

For example, for a fundamental frequency of 50 Hz, the time period is 0.02 seconds. So the value \(T=0.02\) seconds. The equation (6.33) can be used to find the \(\alpha\) values in seconds, to be used in actual PWM pulse
formulation. These radian or degree values can be converted to any other frequency value. This is an extremely important feature, because it destroys the country boundaries and enables the method to be used in any country and for device functioning at virtually any frequency.

6.3.4 Generation of Final Notching Angles

The initial angles found in the section 6.3.3.2 have to be used to initialize the Newton’s method to generate the final accurate notch angles. If these initial angles are fed as such, then the harmonics are not completely eliminated due to the error in the initial angles. Only if these angles are refined further to the maximum extent, the PWM wave can be generated and applied to the actual circuit. So here, the Newton’s method is used to refine the initial angles with the help of Gauss elimination method. The Gauss elimination method is used only as a sub-set of Newton’s procedure, in order to solve the simultaneous equations. It should be noted that the Gauss elimination method is used during each iteration of the Newton’s method. The maximum no. of iterations is automatically fixed by the program, depending upon the value of error which can be tolerated.

6.3.4.1 Newton’s iterative method

This is a popularly known procedure and is also referred to as Newton-Raphson’s iterative method. This method is well known for its fast convergence, provided that the method is supported by good initial guesses.

The Newton’s method defines the solution procedure for the set of equations given by,

\[ J(\alpha) \Delta \alpha = f(\alpha) \quad (6.35) \]
Then the iteration procedure is as follows:

1. The values of initial angles \( \alpha = \alpha_0 \) are substituted in the matrices \( J \) and \( f \).
2. The value of \( \Delta \alpha \) is found by,
   \[
   \Delta \alpha = J(\alpha)^{-1} f(\alpha)
   \]
   using Gauss elimination procedure.
3. The values of \( \Delta \alpha \) obtained are compared with the tolerance error value.
4. If the value of \( \Delta \alpha \) is lesser than the error tolerance, then exit the loop.
5. If not, the new values of \( \alpha \) are found by \( \alpha_{(\text{new})} = \alpha_{(\text{old})} - \Delta \alpha \).
6. The new values of \( \alpha \) are substituted in the matrices \( J \) and \( f \). Then the loop is repeated from step (2).

The working of Gauss elimination can be studied in brief in the next section. This is because the Gauss elimination method is the central part of Newton’s method. So, one should know the purpose of employing the Gauss elimination, before going into a deep study of its working. Now after knowing the working of Newton’s method, it would be easier to understand the working of Gauss elimination method.

### 6.3.4.2 Gauss elimination method

There are a plenty of methods used for solving simultaneous equations. These include Gauss elimination method, Gauss Seidel method, relaxation method, and so on. Gauss elimination method is chosen because of its simplicity and ease of programming. Also many methods have an important requirement that the equations must be diagonally dominant, which
is not a problem in Gauss elimination. These reasons make Gauss elimination method, the best choice for our application.

This method works on the principle of back-substitution of values of variables. The method is used to solve the simultaneous equations by formulating them into an augmented matrix formed by adjoining the known matrices on the left and right hand sides. Then the augmented matrix is reduced to an upper triangular matrix and then back substituted to find the unknown variables.

Let the set of simultaneous equations to be solved, is in the form of equation (6.35).

The procedure for solving using Gauss elimination is as follows:

1. Substitute the values of $\alpha$ available from Newton’s method in matrices $J$ and $f$.
2. Formulate the augmented matrix by adjoining the matrices $J(\alpha)$ and $f(\alpha)$. Let the augmented matrix be
   
   \[ A(\alpha) = [J(\alpha) : f(\alpha)] \]  
   
   (6.37)
3. The augmented matrix is reduced to an upper triangular matrix by appropriate row transformations. Here, upper triangular matrix refers to a matrix that contains the lower part of the matrix (below the diagonal, to the lower-left of the matrix) with all zeroes. For such a matrix the last row contains only the last variable (the last variable, in this case is $\alpha_3$). So the value of the last variable can be found directly by dividing the last element of the last row, by the element just to the left of it in the last row.
4. Now the back substitution is used to find the rest of the variables. This is done by substituting the values of known variables in each row, moving from the last row to the first.

If there are N harmonic equations, then the order of the augmented matrix will be N by N+1. In other words, the matrix A will contain N rows with (N+1) columns. The first N columns are the elements of the matrix J, while the (N+1)th column is the matrix f. As a result, after step 3, the matrix A will be of the form in which, the ith row will be a function of the last (i-1) variables. So as the row number increases, the no. of variables involved decreases. So the problem is solved from the last row to the first row. It would be obvious that the last row will be a function of the last variable alone, while the first row will be a function of all the variables.

When the combination of Newton’s and Gauss elimination methods are applied, the final refined angles are obtained. In this case, the final values of α were found to be,

\[ \alpha_1 = 0.2916 \text{ rad (16.707º)} \]  
\[ \alpha_2 = 0.8113 \text{ rad (46.48º)} \]  
\[ \alpha_3 = 0.8976 \text{ rad (51.43º)} \]

These values are converted to a wave of any frequency using equation (6.34). The effectiveness of Newton’s method can be known from the trajectories plotted by the program. The trajectories are shown in Figure 6.6.
It is seen from the trajectories that the method converges in just 4 to 5 iterations. This re-establishes the fact that the choice of Newton’s method is indeed useful in the real-time implementation of the method. Though the iterations are less, the difference between the initial and final values of trigger angles is significant. It is only this difference that makes the harmonic content to zero (if not, at least to the lowest possible value).

6.4 IMPLEMENTATION OF COMPUTED PULSE WIDTH MODULATION

The term “computed PWM” (CPWM) is coined because the generation of PWM by this method is just by computing an interpolated repeated sequence and directly using these values to generate the waveform. As a result, large number of trials is involved in this method. Also the time required to form the interpolated sequence is far less than that required for the sine-triangular comparison method. Since there are no direct tools for creating the wave form that is needed for this work, the required wave is created from the zero-crossings of a variable frequency triangular wave, which switches at pre-specified or programmed instants (Enjeti et al 1990). This method has
been found to simple but effective method to generate a wave of required shape.

### 6.4.1 Implementation in MATLAB (SIMULINK)

To find out the notching angles a program is developed as per the flowchart shown in Figure 6.7. The notching angles are obtained for a given value of N and modulation index (M). The notching angles are directly loaded into Simulink model and then it is executed. The main block (core) in the implementation of this method is the “repeated sequence interpolated” block of SIMULINK. The output from this block is compared with a constant of zero value. This implies that the zero-crossings of the sequence are detected. The coordinates of the sequence are formulated from the shape of the desired PWM wave. The basic logic is to form the quasi-triangular sequence such that its zero crossings generate a transition in the PWM level, i.e., a notch. The SIMULINK implementation of this method is shown in Figure 6.8.

The “repeated sequence interpolated” generates a quasi-triangular waveform, which is sent to a multiplexer along with the constant of zero value. These two signals are again compared using a relational operator and then converted to “double” data type, in order to aid in the triggering of the output pulse to the converter/inverter circuit. Finally a 2nd order digital filter is used to verify whether the generated output is a modulated version of a wave of required frequency. It has to be noted that any modulated wave, on low pass filtering, gives the reference (sine wave in this case) signal, which has been modulated. The output of the model of Figure 6.8 is given in Figure 6.9. The output waveform may resemble the waveform obtained from sine PWM in overall shape, but the values of notch angles are accurately the same as that of those obtained from the calculations.
Figure 6.7 Flow chart for the mathematical analysis

START

INPUT N

GENERATE CURVES OF $\alpha$ Vs M USING CAUCHY THEORY

LINEARISE THE CURVES BY LEAST SQUARE APPROX.

REFINE ANGLES USING NEWTON'S METHOD

SOLVE SIMULTANEOUS EQNS. USING GAUSS ELIMINATION

OUTPUT FINAL NOTCH ANGLES

STOP

Figure 6.8 Simulink representation of the CPWM approach
The method used in here uses a reverse process of formation of triangular wave from PWM wave, in order to get the PWM wave itself. The PWM waveform cannot be directly generated using SIMULINK. There is a block called “timer”, which enables the generation of any given pulse form, but it can only generate the pulse once. But here the necessity is to generate the pulse over and over again for the specified time period. On the other hand, the “repeated sequence interpolated” block can generate the waveform of specified period and repeat it, the constraint being that it can only generate triangular waveform. So the only way out is to generate a triangular waveform and use its zero-crossings to generate a PWM of required shape. But the accuracy of the method depends solely upon the sampling time of the “repeated sequence interpolated”. But the accuracy of the method was best, with a sampling period of 10 microseconds. So this value of sampling interval is used.

6.4.2 Simulink Model Parameters

The formulation of triangular wave may be slightly difficult for the first time. But with a little practice, the method proves quite easy to use. The formation of the coordinates for triangular wave involves writing down the values of notch angles for one time period and creating a notch of triangular
wave for the mid-points of the adjacent points of PWM, the output changing from 1 to 0 and then to -1 and then again to 0 and to 1, and so on.

The method can be made clear by taking the case of N=3 and forming a triangular wave sequence for it. Now the Table 6.1 shows the notch angles of the desired PWM waveform and its corresponding destination output levels. This is formed from Figure 6.9. Here T is the time period of the desired waveform.

Now from these values, it is needed that the triangle should attain its peak values (-1 or +1) in between two notches and should cross zero line, at every notch. So the triangular wave has essentially 15 zero crossings in this case. Now the triangular wave sequence can be formulated as in Table 6.2. It is seen from the Table that the zero crossings occur at the points mentioned in Table 6.1, while the values 1 and -1 alternate about it.

Table 6.1 Sequence to obtain desired PWM waveform (N=3)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Time Period (secs)</th>
<th>Logic Output Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$\alpha_1$</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$\alpha_2$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$\alpha_3$</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$0.5T - \alpha_3$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$0.5T - \alpha_2$</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>$0.5T - \alpha_1$</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>$0.5T$</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>$0.5T + \alpha_1$</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>$0.5T + \alpha_2$</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>$0.5T + \alpha_3$</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>$T - \alpha_3$</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>$T - \alpha_2$</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>$T - \alpha_1$</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>$T$</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 6.10 shows the block properties window (SIMULINK) of the “repeated sequence interpolated” block. It can be clearly seen that the values in the Table 6.2 are fed as such in the Figure 6.10. The values of trigger angles can be fed as such in the form of actual numbers, but it would hinder the understanding of the method. So the variables alpha1, alpha2, etc. are used. These variables have to be initialized before executing the model file. The initialization can be done from the “model properties” window of SIMULINK. The computed values are assigned to the variable using the “model initialization” section of the model properties, as shown in Figure 6.11.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Time Period (secs)</th>
<th>Logic Output Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \alpha_1/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>( \alpha_1 )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( (\alpha_1 + \alpha_2)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>( \alpha_2 )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>( (\alpha_2 + \alpha_1)/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>7</td>
<td>( \alpha_3 )</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>( T/4 )</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>( (T-2* \alpha_3)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>( (T- \alpha_2- \alpha_3)/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>11</td>
<td>( (T-2* \alpha_2)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>( (T- \alpha_1- \alpha_2)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>( (T-2* \alpha_1)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>( (T- \alpha_1)/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>15</td>
<td>( 0.5T )</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>( (T+ \alpha_1)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>( (T+2* \alpha_1)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>( (T+ \alpha_1+ \alpha_2)/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>19</td>
<td>( (T+2* \alpha_2)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>( (T+ \alpha_2+ \alpha_3)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>( (T+2* \alpha_3)/2 )</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>( 0.75*T )</td>
<td>-1</td>
</tr>
<tr>
<td>23</td>
<td>( T- \alpha_3 )</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>( (2*T- \alpha_2- \alpha_3)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>( T- \alpha_2 )</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>( (2*T- \alpha_1- \alpha_2)/2 )</td>
<td>-1</td>
</tr>
<tr>
<td>27</td>
<td>( T- \alpha_1 )</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>( (2*T- \alpha_1)/2 )</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>( T )</td>
<td>0</td>
</tr>
</tbody>
</table>
It is clear that the CPWM method is far better than the sine-triangular comparison method in terms of speed of formulation, ease of implementation and the accuracy of waveform. So the CPWM method is indeed the best suited method for this kind of application. Further this method can be easily implemented in hardware, which is an added advantage.

6.5 SIMULATION AND HARDWARE RESULTS

Based on the developed algebraic approach the notching angles are generated to selectively eliminate the harmonics produced by the inverters.
A Computed PWM waveform is generated using these angles. This waveform is then applied to the Simulated PWM inverter as shown in Figure 6.12 for different values of N and modulation index (M). The specifications of the inverter circuit are as follows:

- DC input voltage : 230V
- Load Resistance : 400 Ohms

![Simulink model of a single phase voltage source inverter](image)

**Figure 6.12  Simulink model of a single phase voltage source inverter**

The circuit consists of four IGBT’s: IGBT1, IGBT2, IGBT3, and IGBT4 which are connected as shown in the Figure 6.12. Pulse generator 1 triggers IGBT1 and IGBT2 during positive half cycles of the output wave. During negative cycles of output IGBT3 and IGBT4 are triggered by pulse generator2.

To corroborate the results obtained in the simulation, a hardware prototype model using microcontroller is implemented to selectively eliminate
the harmonics. The block diagram of the proposed hardware model is shown in Figure 6.13. The power circuit is designed for an output voltage of 230V with a output frequency of 50 Hz and the rated output power is 500 watts. Four IGBTs, (IRG4BC20S) are used as the main switches which are connected in full bridge configuration. The power circuit diagram is shown in Figure 6.14. To generate computed PWM signals PIC microcontroller (PIC 18F452) is used. The microcontroller based implementation of computed PWM generation is shown in Figure 6.15. The driver circuit for the inverter is shown in Figure 6.16. The load connected to the output terminals is 400 ohms. The components of the hardware are shown in Figure 6.17. The complete test setup shown in the Figure 6.18. To capture the waveforms the four channel digital storage oscilloscope (GWINSTEK, GDS2104) is used. For measuring gate voltages the oscilloscope probes are directly connected. For output voltage measurements differential module is used along with the oscilloscope and the multiplication of the differential module is 400. For voltage measurements true RMS meters (Tektronix, TX1) are used. For harmonic analysis and measurement the Fluke 434 power quality analyzer is used.

Figure 6.13 Block diagram of the microcontroller based single phase inverter model
Figure 6.14 Power circuit diagram of the inverter model

Figure 6.15 Microcontroller based implementation of computed PWM
Figure 6.16 Driver circuit for the inverter
In this section the harmonic analysis of the single phase voltage source inverter with square wave gate pulse is analyzed and then harmonic analysis is carried out for computed PWM gate pulses for $M=1$ and the variation of fundamental RMS voltage for $0.1<M<1$ for $N=3$. 
6.5.1 Harmonic Analysis of Inverter Circuit with Square Gate Pulse

The proposed simulink and hardware model was tested for a simple square wave gate pulse. The switching waveforms and output voltage waveform and harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.19 and 6.20 respectively. The output voltage waveform and harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.21 and 6.22 respectively. Table 6.3 shows the harmonic content expressed as a percentage of fundamental for each order of the harmonic.

![Figure 6.19 Switching waveforms and output voltage waveforms](image1)

**Figure 6.19** Switching waveforms and output voltage waveforms

![Figure 6.20 Harmonic analysis of pulse triggered single phase inverter](image2)

**Figure 6.20** Harmonic analysis of pulse triggered single phase inverter
Figure 6.21 Output voltage of the hardware model with square gate pulse

Figure 6.22 Harmonic spectrum of the hardware model with square gate pulse
Table 6.3  Harmonic analysis of single phase square pulse triggered inverter circuit

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Harmonic Order</th>
<th>Magnitude (% of Fundamental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>33.32</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>20.02</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>14.31</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>11.15</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>9.14</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>7.75</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>6.73</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>5.95</td>
</tr>
<tr>
<td>10</td>
<td>19</td>
<td>5.34</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>4.85</td>
</tr>
<tr>
<td>12</td>
<td>23</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Thus from the Table 6.3, it is observed that as the order of the harmonic increases the magnitude of the harmonic content expressed as a percentage of the fundamental decreases. Also from the spectrum analysis it is observed that the magnitudes harmonic order of 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th}, 11\textsuperscript{th} etc., are higher and they are detrimental to the operation of critical equipments. Thus if these order of harmonics can be eliminated, it is possible to operate the critical equipments with better performance. In the subsequent sections the selective harmonic elimination of the harmful harmonic levels using computed PWM waveforms are discussed in detail.

6.5.2   Elimination of the First Two Odd Harmonics

For eliminating first two lower order harmonics three values of notch angles need to be calculated; one for fundamental component and the
other two for the first two lower order harmonics 3rd and 5th. Using the mathematical technique discussed in the previous section, the final notch angles are found to be $\alpha_1 = 16.708^\circ$, $\alpha_2 = 46.483^\circ$ and $\alpha_3 = 51.429^\circ$, where $\alpha_1$, $\alpha_2$, $\alpha_3$ are the notch angles. These notch angles calculated in degrees then converted to seconds, which is given by

$$\alpha_{sec} = \left(\frac{\alpha_{\text{degrees}}}{180}\right) * T \text{ Seconds}$$

(6.41)

In this case the output frequency is 50 Hz and hence $T = 0.02$ seconds. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.23 and 6.24 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.25 and 6.26 respectively. Figure 6.27 shows the comparative results of the individual harmonic distortions.

Figure 6.23 Switching waveforms and output voltage waveforms for N=3
Figure 6.24  Harmonic spectrum of the output voltage (for N=3)

Figure 6.25  Switching waveforms and output voltage waveforms of the hardware model for N=3

Figure 6.26  Harmonic spectrum of the output voltage of the hardware model (N=3)
Figure 6.27 Comparative results of % harmonic distortion (N=3)

From the harmonic spectrum it can be concluded that the 3\textsuperscript{rd} and 5\textsuperscript{th} harmonics are eliminated and their magnitudes are very close to zero.

6.5.3 Elimination of the First Three Odd Harmonics

To eliminate the first three odd lower order harmonics the four notching angles are obtained from the developed program. The values of the notching angles are: \( \alpha_1 = 14.514^\circ \), \( \alpha_2 = 38.786^\circ \), \( \alpha_3 = 44.607^\circ \) and \( \alpha_4 = 89.551^\circ \). These notching angles are converted into seconds and directly given to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.28 and 6.29 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.30 and 6.31 respectively. Figure 6.32 shows the comparative results of the individual harmonic distortions. From the harmonic spectrum it is found that the lower order harmonics i.e. 3\textsuperscript{rd}, 5\textsuperscript{th} and 7\textsuperscript{th} are eliminated and their magnitudes are very close to zero.
Figure 6.28 Switching waveforms and output voltage waveforms for N=4

Figure 6.29 Harmonic spectrum of the output voltage (for N=4)
Figure 6.30  Switching waveforms and output voltage waveforms of the hardware model for N=4

Figure 6.31  Harmonic spectrum of the output voltage of the hardware model (N=4)
Figure 6.32  Comparative results of % harmonic distortion (N=4)

6.5.4  Elimination of the First Four Odd Harmonics

For eliminating the first four odd lower order harmonics the five notching angles are obtained from the developed program. The values of the notching angles are: \( \alpha_1 = 12.171^\circ \), \( \alpha_2 = 30.77^\circ \), \( \alpha_3 = 36.876^\circ \), \( \alpha_4 = 60.983^\circ \) and \( \alpha_5 = 62.643^\circ \). These notching angles are converted into seconds and directly give to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.33 and 6.34 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.35 and 6.36 respectively. Figure 6.37 shows the comparative results of the individual harmonic distortions. Based on the harmonic spectrum, it is found that the lower order harmonics i.e. 3\(^{rd}\), 5\(^{th}\), 7\(^{th}\), and 9\(^{th}\) are eliminated.
Figure 6.33 Switching waveforms and output voltage waveforms for N=5

Figure 6.34 Harmonic Spectrum of the Output Voltage (for N=5)
Figure 6.35  Switching waveforms and output voltage waveforms of the hardware model for N=5

Figure 6.36  Harmonic spectrum of the output voltage of the hardware model (N=5)

Figure 6.37  Comparative results of % harmonic distortion (N=5)
6.5.5 Elimination of the First Five Odd Harmonics

For eliminating the first five odd lower order harmonics the six notching angles are obtained from the developed program. The values of the notching angles are: \( \alpha_1 = 10.896^\circ \), \( \alpha_2 = 26.867^\circ \), \( \alpha_3 = 32.985^\circ \), \( \alpha_4 = 53.799^\circ \), \( \alpha_5 = 56.111^\circ \) and \( \alpha_6 = 89.829^\circ \). These notching angles are converted into seconds and directly give to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.38 and 6.39 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.40 and 6.41 respectively. Figure 6.42 shows the comparative results of the individual harmonic distortions. Based on the harmonic spectrum, it is found that the lower order harmonics i.e. 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th} and 11\textsuperscript{th} are eliminated.

![Switching waveforms and output voltage waveforms for N=6](image_url)
Figure 6.39 Harmonic spectrum of the output voltage (for N=6)

Figure 6.40 Switching waveforms and output voltage waveforms of the hardware model for N=6

Figure 6.41 Harmonic spectrum of the output voltage of the hardware model (N=6)
Elimination of the First Six Odd Harmonics

For eliminating the first six odd lower order harmonics the seven notching angles are obtained from the developed program. The values of the notching angles are: $\alpha_1= 9.576^{\circ}$, $\alpha_2= 22.952^{\circ}$, $\alpha_3= 28.868^{\circ}$, $\alpha_4= 45.737^{\circ}$, $\alpha_5= 48.567^{\circ}$, $\alpha_6= 68.159^{\circ}$ and $\alpha_7= 68.902^{\circ}$. These notching angles are converted into seconds and directly give to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.43 and 6.44 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.45 and 6.46 respectively. Figure 6.47 shows the comparative results of the individual harmonic distortions.
Figure 6.43 Switching waveforms and output voltage waveforms for N=7

Figure 6.44 Harmonic spectrum of the output voltage (for N=7)

Figure 6.45 Switching waveforms and output voltage waveforms of the hardware model for N=7
Based on the harmonic spectrum, it is found that the lower order harmonics i.e. 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th}, 11\textsuperscript{th} and 13\textsuperscript{th} are eliminated.

6.5.7 Elimination of the First Seven Odd Harmonics

For eliminating the first seven odd lower order harmonics the eight notching angles are obtained from the developed program. The values of the notching angles are: $\alpha_1 = 8.748^\circ$, $\alpha_2 = 20.620^\circ$, $\alpha_3 = 26.351^\circ$, $\alpha_4 = 41.218^\circ$, $\alpha_5 = 44.321^\circ$, $\alpha_6 = 61.904^\circ$, $\alpha_7 = 63.043^\circ$ and $\alpha_8 = 89.917^\circ$. These notching angles
are converted into seconds and directly give to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.48 and 6.49 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.50 and 6.51 respectively. Figure 6.52 shows the comparative results of the individual harmonic distortions.

Figure 6.48 Switching waveforms and output voltage waveforms for N=8

Figure 6.49 Harmonic spectrum of the output voltage (for N=8)
Figure 6.50  Switching waveforms and output voltage waveforms of the hardware model for N=8

Figure 6.51  Harmonic spectrum of the output voltage of the hardware model (N=8)

Figure 6.52  Comparative results of % harmonic distortion (N=8)
Based on the harmonic spectrum, it is found that the lower order harmonics i.e. 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th} and 15\textsuperscript{th} are eliminated.

6.5.8 Elimination of the First Eight Odd Harmonics

For eliminating the first eight odd lower order harmonics the nine notching angles are obtained from the developed program. The values of the notching angles are: $\alpha_1=7.895^\circ$, $\alpha_2=18.294^\circ$, $\alpha_3=23.748^\circ$, $\alpha_4=36.523^\circ$, $\alpha_5=39.791^\circ$, $\alpha_6=54.610^\circ$, $\alpha_7=56.135^\circ$, $\alpha_8=72.466^\circ$ and $\alpha_9=72.859^\circ$. These notching angles are converted into seconds and directly give to the gate drive circuitry of the inverter.

The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the simulation model are shown in Figures 6.53 and 6.54 respectively. The switching waveforms and output voltage waveforms and the harmonic spectrum of the output voltage of the hardware model are shown in Figures 6.55 and 6.56 respectively. Figure 6.57 shows the comparative results of the individual harmonic distortions.

![Figure 6.53 Switching waveforms and output voltage waveforms for N=9](image-url)
Figure 6.54 Harmonic Spectrum of the output voltage (for N=9)

Figure 6.55 Switching waveforms and output voltage waveforms of the hardware model for N=9

Figure 6.56 Harmonic spectrum of the output voltage of the hardware model (N=9)
Based on the harmonic spectrum, it is found that the lower order harmonics i.e. 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th}, 15\textsuperscript{th} and 17\textsuperscript{th} are eliminated.

### 6.5.9 Variation in the Fundamental Voltage for Different Modulation Indexes

The proposed simulated and hardware model can be tested with various modulation index values varied from 0.1 to 1 for 3<N<9. Whenever the modulation index varies for a particular value of N correspondingly the fundamental RMS is also varied. The variation of the fundamental RMS voltage for N=3 with modulation index varies from 0.1 to 1 is shown in Figures 6.58(a-j) and 6.59(a-j) for both simulated and hardware models. Figure 6.60 represents the comparison of simulation and hardware results for fundamental RMS voltage. The simulated results are very close to the hardware results.
Figure 6.58 (Continued)
Figure 6.58  Harmonic spectrum with the variation in fundamental RMS voltage for N=3 with 0.1<M<1 (Simulation)
Figure 6.59 (Continued)
Figure 6.59  Harmonic spectrum with the variation in fundamental RMS voltage for N=3 with 0.1<M<1 (Hardware Model)

Figure 6.60 Comparison of simulation and hardware results
6.6 CONCLUSION

A simple and effective, minimization technique to solve the selective harmonic elimination using computed PWM control method for single phase voltage source inverters has been discussed in this chapter. By solving the harmonic equations, the values of notch angles are obtained, which control the switching instants of the PWM wave. The switching angles obtained by these methods to eliminate harmonics are not as accurately as obtained through the computed PWM technique. The PWM wave generated by the computed PWM technique takes less time when compared to the sine-triangular comparison method. They do not involve any ‘trial and error’ procedures and the PWM pulse can triggered accurately at the desired values of the notch angles.

The computed method was applied to developed inverter with 3<N<9 the modulation index of 1.0. The selective harmonic elimination was carried out effectively for all the values of N. The same thing can be extended for other values of modulation index values. The variation of the fundamental RMS voltage for the modulation index value between 0.1 and 1.0 for N=3 is shown and it was found that it is decreasing as the modulations decreases. All the simulation results were upheld with hardware results.