Chapter IV

Digital Halftoning Techniques Based on Error Diffusion Filters: Technical Background and Performance Evaluation

4.1 Preamble

This chapter provides a technical background on both the digital halftoning techniques based on ED filters such as Floyd-Steinberg, Jarvis and Stucki and the halftoned image quality assessment metrics. It also concerns with the assessment of the existing standard ED filters to find out the best ED filter for halftoning process to be used in the subsequent chapter, i.e., to be used in the upcoming developed scheme of HVC. Thus, performance evaluation by particular techniques and image quality metrics as $MSE$, $PSNR$, $AD$, $MD$, $MAE$ and $UQI$ are offered in order to determine the best ED filter for halftoning process.

4.2 Technical Background

In image processing, it is always required to print various color quantized images, and this causes the loss of the image details. The non-available colors in the palette of halftoned image are approximated by a diffusion of available palette’s colored pixels. Such diffusion is perceived by human eyes as a mixture of the colors via it. To overcome such a problem, halftoning is used to create a visible pattern into the image that the diffusion can be perceived as a pattern not as a mixture of the colors. In this regard, a blue noise halftone pattern is proved to be the slightest hideous and confusing pattern [67].

Blue noise dithering patterns are generated by ED and ordered dithering techniques. However, the ordered dithering technique is able to generate a blue noise dithering without any propensity to debase into areas with artifacts. This means that the ED is a better technique, for halftoned images with relatively few colors can be distinguished by a dotted appearance or characteristic graininess [68].
In general, the idea behind halftoning is that pixels are grouped together in small blocks and various patterns of binary pixels are used as one gray level pixel. By using a simple threshold operation, it is easy to quantize a gray level image into a binary one only through mapping its upper half into white and the other one into black. Yet, the simple threshold operation’s drawback as the quality of resultant image is very poor, and accordingly, it is essential to improve its quality by using the technique of dithering by distributing the quantized error.

Dithering technology is used to regenerate the grayscale image with binary value to be ready to perform different applications related to dithered images like HVC [69]. However, this section is interested in concerned with providing a brief survey on the digital halftoning and its methods, focusing on ED filters as they are appropriate to the current study.

4.2.1 Digital Halftoning

Digital halftoning is considered as a pre-processing method for converting different types of an image into a binary image in order to perform some applications. It is purposely a practical way of noise used to diffuse the quantization error randomly. It refers to the conversion process of a continuous-tone image of 256 levels into a pattern of black and white image elements for reproduction by a binary display device which can choose either to print or not to print the dots, and in both cases, the direct rendition of the tones is impossible [70],[71]. In digital halftoning, the original and the halftoned images look the same when being observed by HVS from distance.

To put it clearly, the HVS acts as a low-pass filter which blurs these printed and not printed dots together to create the illusion of continuous shades of gray. According to HVS, to produce highest quality images, maintain sharp edges, and other fine details, the randomly arranged and isolated dots should properly be distributed. Meantime, certain display devices have not an ability to reproduce the isolated dots systematically from dot to another. Thus, producing printing artifacts extremely degrade the visual quality.

Therefore, the principal goal of studying halftoning is to determine the optimal distribution of dots for those devices to produce the patterns in an efficient computation manner [72]. The main issues in picking out a halftoning technique are the image quality
and the amount of computation. The halftone image quality depends on its spatial resolution and the halftone patterns severity.

A model-based halftoning is illustrated in figure (4.1) that exploits models of the display device and visual perception to produce high quality images [73].

**Figure (4.1):** The block diagram of model-based halftoning.

In this model, the continuous-tone image after being halftoned can be displayed through the display model in media or HVS model. For first model, such example of media is toner or ink on paper, and the common rendering devices for such media are printers.

For HVS model, the eye acts as a spatial low-pass filter that blurs the rendered pixel pattern due to which the halftoned image is perceived as a continuous-tone image.

Though all halftoning techniques rely on understanding the properties of the display device and human vision, the goal of such a model is to exploit explicit models of the display device and the HVS to maximize the displayed images quality [73].

### 4.2.2 Digital Halftoning Methods

In the decade that followed the declaring of digital halftoning, many technical improvements are proposed to address the blue-noise creation. Common types of digital halftoning are presented as follows:

1. **Ordered Halftoning**

   In ordered dithering or so called blue-noise dither array halftoning, a small mask matrix is generated as a threshold matrix in order to compare with pixel-wise of gray level
image. After the comparing process, the binary image with shade information will be generated [69].

In ordered halftoning, different gray scale ranges are also presented by chosen binary pixels in square grids form. The pattern corresponding to a specific square grid is selected to the suitable gray level. Separately, the calculation of each grid depends on the surrounding ones.

Being a point process operation, blue-noise dither array halftoning is far less computationally complex than error-diffusion, making it a viable technique for commercial printing applications whose image sizes are orders of magnitude larger than those associated with desktop printers [72].

In contrast, the quality of the final halftoned image is reduced to have some characteristic diagonal artifacts. It is not easy to work with free-form and arbitrary palettes in ordered halftoning method [74]. Now, the error-diffusion algorithm itself has also undergone some major improvements over the past two decades [72].

2. Error Diffusion Halftoning

For improving the performance of diffusion in an image, ED is an appropriate standard technique with an ability to reduce the pattern noise and remove the affection of the boundary and ‘black-hole’ [75]. For this reason, it is considered as a best technique to convert a continuous-tone (grayscale) image into a binary one which produces high quality of images with reducing the computation cost.

To put it differently, ED is seen as a standard technique among the existing halftoning methods. This is due to its simplicity and efficiency in halftoning a grayscale image. Moreover, it provides halftone shared images with quite good quality. The mechanism of error diffusion is to diffuse the error at each pixel of an image. To do this, the quantization error is filtered and feedback to the input. The error filter process diffuses the quantization error on one pixel away to the neighboring gray pixels. In nature, the ED noise is of high frequency or “blue noise”, and, for human vision, it can provide pleasing halftone images [76].
Figure (4.2) shows that the binary ED diagram where the \((m,n)^{th}\) pixel of the input grayscale image is indicated by the \(f(m,n)\), the sum of input pixel value and the past diffused error is indicated by the \(d(m,n)\) while the output quantized pixel value by \(g(m,n)\) [77].

**Figure (4.2):** The block diagram of Error Diffusion.

Generally, the first main component of the ED is the threshold block \(t(m,n)\). The \(g(m,n)\) can be given as:

\[
g(m,n) = \begin{cases} 
1 & \text{if } d(m,n) \geq t(m,n) \\
0 & \text{otherwise}
\end{cases} \tag{4.1}
\]

The value of \(t(m,n)\) is computed depending on the value of \(d(m,n)\). The second component is error filter \(h(k,l)\) whose input is the difference between \(d(m,n)\), \(g(m,n)\), and can be indicated by \(e(m,n)\). Finally, \(d(m,n)\) can be mathematically presented as:

\[
d(m,n) = f(m,n) - \sum_{k,l} h(k,l) e(m-k,n-l) \tag{4.2}
\]

### 4.2.3 Error Diffusion Halftoning Filters

The common ED filters which used to produce halftoned images are presented in details as follows:
A. Floyd-Steinberg Halftoning Filter

Floyd and Steinberg [67] have proposed an ED filter with the idea to keep track of the error and called it Floyd-Steinberg filter. Figure (4.3(a)) shows the process of Floyd-Steinberg filter.

At each step for input grayscale image, the value of the current pixel $J(i, j)$ is taken and represented by an integer value between 0 and 255, then compared with typical threshold value ($t=128$). The output pixel $I(i, j)$ is to be black (value = 0) if $J(i, j) > t$, otherwise; it is white (value = 1). The diffusion matrix of distributed error fractions to four adjacent pixels is shown in (figure 4.3(b)).

![Diagram of Floyd-Steinberg filter process](image)

**Figure (4.3):** a) Process of Floyd-Steinberg filter, b) Floyd-Steinberg diffusion matrix.
The computed difference between the $J(i, j)$ value and the $t$ value is considered as an error. The number of elements to which error is diffused remains as it is. The only difference is in the fraction of error which is distributed among these elements. The amount of error in this algorithm which is distributed to each neighbor is $7/16$ of the error to the right pixel, $5/16$ to the lower pixel, $3/16$ to the diagonal left pixel, and $1/16$ to the diagonal right pixel. In brief, the pixel currently being processed ($J(i, j)$) communicates with four neighboring pixels. These amounts of error distributed to the neighboring pixels reduce the contrast loss [78].

B. Jarvis Halftoning Filter

Jarvis et al. [79] have introduced another ED filter that diffuses an error to the twelve neighboring pixels instead of four pixels in Floyd-Steinberg halftoning algorithm. Figure (4.4(a)) shows the diffusion matrix of distributed error fractions to twelve pixels.

Assume that the address of current pixel is (1,4), then the amount of each error which is distributed to the neighboring pixels is as following:

$$(1,5) \frac{7}{48}, (1,6) \frac{5}{48}, (2,2) \frac{3}{48}, (2,3)\frac{5}{48}, (2,4) \frac{7}{48}, (2,5) \frac{5}{48},$$

$$(2,6)\frac{3}{48}, (3,2) \frac{1}{48}, (3,3)\frac{3}{48}, (3,4) \frac{5}{48}, (3,5) \frac{3}{48}, (3,6) \frac{1}{48}.$$

C. Stucki Halftoning Filter

Stucki [80] has proposed an ED filter that diffuses the error to the twelve neighboring pixels as shown in figure (4.4(b)). The difference between Jarvis algorithm and Stucki algorithm is the fraction added to the neighboring pixels which is forty-two.
4.2.4 Halftoned Image Quality Assessment Metrics

In the past, many metrics for the evaluating image quality were introduced. The objective metrics of image quality assessment can be categorized on the basis of their applications into general purpose and application-specific image quality assessment metrics [81].

The metrics of image quality assessment are those metrics which are designed intentionally to be used for the quality assessment in a variety of different applications. They have no assumptions about a specific distortion type. Usually, they are designed using common features of HVS. For example, image compression, transmission, halftoning; watermarking, restoration and enhancement are some of the largest application areas of the
image quality assessment techniques. Here are some of common metrics of image quality assessment:

### 4.2.4.1 Error Sensitivity Based Metrics

In this part, some of the common visual quality metrics which are based on the sensitivity of error are explained as follows:

1. **MSE**

   The average of the square of the “errors” between the original and processed image is measured by mean square error. The error amount can be computed by the sum of difference between the original and halftoned image.

2. **PSNR**

   The peak signal to noise is the ratio which can be computed between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. PSNR is usually expressed in terms of the logarithmic decibel.

   \[ \text{PSNR} = 10 \log_{10} \frac{P_{max}}{P_{noise}} \]

   \( P_{max} \) is the maximum possible power of a signal, and \( P_{noise} \) is the power of corrupting noise.

   \( MSE \) and its counterpart \( PSNR \) are among the earliest and most popular metrics used to compare the quality of the processed images [82], [83], [84]. The popularity of these metrics comes from a set of features such as their simplicity in calculation, clearness of physical meanings, and mathematical appropriation in the context of optimization. These metrics are expressed as shown in Eq. (3.1) & (3.2), respectively.

3. **AD**

   The average error between the input image and the halftoned image is measured by the average difference metric. This metric is expressed as:

   \[ AD = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (h_{ij} - h'_{ij}) \]  

4. **MD**

   The MD metric is used to measure the average error between the original and the halftoned image while the maximum error is measured by the maximum difference metric. This metric is expressed as:

   \[ MD = \max_{i,j} \left| h_{ij} - h'_{ij} \right| \]
\[
MD = \max |h_{ij} - h'_{ij}|
\]  

(4.4)

5. MAE

To measure how convergence of the predictions and eventual outcomes, mean absolute error metric is used which mathematically expressed as:

\[
MAE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} |h_{ij} - h'_{ij}|
\]  

(4.5)

Where \(h_{ij}\) and \(h'_{ij}\) denote the pixels value of the original and halftoned image, respectively.

4.2.4.2 Structural Similarity Based Metrics

The idea behind the metrics of structural similarity is the high suitability of extract structural information from the visual scenes by HVS. Therefore, an evaluation metrics should be used to provide a good approximation to perceptual image quality to measure the structural similarity (or distortion).

A main question that should be answered to convert the general structural similarity principle into specific image quality assessment algorithms is how to define the structural distortions. The following method uses different definitions for the structural distortions.

1. Universal Quality Index UQI:

As a substitutability of the traditional methods of error summation, a UQI metric was introduced by \textit{Wang and Bovik} [85]. This metric has a set of features such as easy to calculate and appropriate to various applications of an image processing. Any image distortion in the UQI approach can be modeled as a combination of three factors. These factors are loss of correlation, luminance distortion, and contrast distortion. Suppose that an original image is \(x\) and the output distorted image is \(y\), so the loss of correlation is valued by the correlation coefficient which measures the linear correlation degree between \(x\) and \(y\). The luminance and contrast are estimated by mean and standard deviation, respectively. Mathematically, the UQI is expressed as shown in Eq. (3.3).
The dynamic range of $UQI$ is $[-1, 1]$. The best result is 1 which is achieved when $x$ is exactly equal to $y$.

Generally, the listed metrics such as $MSE$, $PSNR$, and structural similarity ($UQI$) use the source image and halftone image in order to measure the quality [86]. A more detailed review of an image quality metrics can be found in [87], [88], [89], [90], [91].

4.3 Experimental Results and Performance Analysis

This section presents the experimental results of applying the three discussed ED filter on different images, and the performance analysis in to get a clear view of the filter that achieves the best results.

4.3.1 Experimental Results

Two experiments are conducted and implemented on the grayscale Lena image of the size of 512×512 and the grayscale Baboon image of the size of 256×256 as shown in figure (4.5 (a)) & (4.6 (a)), respectively. In both experiments, the three different filters are applied on the said images separately which resulted in generating three halftoned images in each experiment as being observed in figure (4.5 (b), (c), and (d)) and (4.6 (b), (c), and (d)).

However, the three different filters of ED diffuse the error quantization to the neighboring pixels to produce enhanced halftoned images. The Jarvis filter is able to produce most visually enhanced halftoned image. This is due to the fact that it diffuses the error to twelve neighboring pixels and the divisor in which the weights are distributed is also the largest among all the filters with forty-eight. Yet, it takes more time to diffuse the error.
Figure (4.5): Simulation results of the first experiment, a) Lena image, b) halftoned image using Floyd-Steinberg filter, c) halftoned image using Jarvis filter, and d) halftoned image using Stucki filter.

In Stucki filter with weight of forty-two comes to be computationally efficient in term of time and the amount of diffusion of quantization error. Therefore, the results produced by the Stucki filter are better than those obtained by the Floyd-Steinberg filter.
4.3.2 Performance Analysis through Statistical Analysis

It has been pointed out that the Jarvis filter has an ability to provide a halftoned image with better visual quality as comparing to other filters. This analysis qualitatively dependent on the more diffused error to large number of neighboring pixels.

Moreover, statistical analysis through different visual quality metrics has been done to prove quantitatively that the more diffused error to large number of neighboring pixels leads to better quality of halftoned images.

As shown in tables ((4.1) & (4.2)), the results are completely close with no much of visual difference as compared the halftoned images obtained by applying different filters based on mathematical metrics such as $MSE$, $PSNR$, $AD$, $MD$, $MAE$ and $UQI$. 

Figure (4.6): Simulation results of the second experiment, a) Baboon original image, b) halftone image using Floyd-Steinberg filter, c) halftone image using Jarvis filter, and d) halftone image using Stucki filter.
Regarding the values of PSNR and UQI in tables (4.1 & 4.2), it can be observed that the best halftoned images can be obtained by using filters that diffuse the error to more neighboring pixels. Here, Jarvis filter presents the best results and the Stucki filter is better than Floyd-Steinberg filter. While calculating PSNR, it can be found that the higher the PSNR is, the better the quality of the halftoned image is.

Moreover, more error is diffused, the better visual quality is obtained. From UQI, it is very clear that with higher UQI, the halftoned image looks like the original image.

**Table (4.1):** Visual quality metrics comparison results of ED Filters; Floyd-Steinberg, Jarvis, and Stucki applied on Lena image 512×512 in first experiment.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Error Diffusion Filters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Floyd-Steinberg</td>
</tr>
<tr>
<td>MSE</td>
<td>0.044495</td>
</tr>
<tr>
<td>PSNR</td>
<td>5.343</td>
</tr>
<tr>
<td>AD</td>
<td>108</td>
</tr>
<tr>
<td>MD</td>
<td>244</td>
</tr>
<tr>
<td>MAE</td>
<td>108</td>
</tr>
<tr>
<td>UQI</td>
<td>0.0029592</td>
</tr>
</tbody>
</table>
Table (4.2): Visual quality metrics comparison results of ED Filters; Floyd-Steinberg, Jarvis, and Stucki applied on Baboon image 256×256 in second experiment.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Error Diffusion Filters</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Floyd-Steinberg</td>
</tr>
<tr>
<td>MSE</td>
<td>5161.6</td>
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<td>PSNR</td>
<td>4.8103</td>
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<tr>
<td>AD</td>
<td>47</td>
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<tr>
<td>MD</td>
<td>125</td>
</tr>
<tr>
<td>MAE</td>
<td>47</td>
</tr>
<tr>
<td>UQI</td>
<td>0.0018989</td>
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</table>

The comparison results can be clearly presented through the graphical diagrams in figure (4.7).
b) MAE - MD - AD

<table>
<thead>
<tr>
<th>Method</th>
<th>MAE</th>
<th>MD</th>
<th>AD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floyd-Steinberg</td>
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<td>244</td>
<td>108</td>
</tr>
<tr>
<td>Jarvis</td>
<td>107</td>
<td>244</td>
<td>107</td>
</tr>
<tr>
<td>Stucki</td>
<td>107</td>
<td>244</td>
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</table>

c) PSNR

<table>
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<th>Jarvis</th>
<th>Stucki</th>
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<tbody>
<tr>
<td>PSNR</td>
<td>5.343</td>
<td>5.3739</td>
<td>5.3432</td>
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d) Development of Secret Sharing Schemes for Visual Cryptography

![UQI Chart]

<table>
<thead>
<tr>
<th>Floyd-Steinberg</th>
<th>Jarvis</th>
<th>Stucki</th>
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<tbody>
<tr>
<td><strong>UQI</strong></td>
<td>0.0029592</td>
<td>0.0030928</td>
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e) MSE Chart

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<tbody>
<tr>
<td><strong>MSE</strong></td>
<td>5161.6</td>
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Development of Secret Sharing Schemes for Visual Cryptography

f) MAE - MD - AD

<table>
<thead>
<tr>
<th></th>
<th>Floyd-Steinberg</th>
<th>Jarvis</th>
<th>Stucki</th>
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<tr>
<td>MAE</td>
<td>47</td>
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<table>
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<th>Floyd-Steinberg</th>
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<td>PSNR</td>
<td>4.8095</td>
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<td>4.8113</td>
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</table>

Development of Secret Sharing Schemes for Visual Cryptography

92
h)

![UQI graph]

**Figure (4.7):** Graphical representation of visual quality metrics, a) - d) results of metrics of all filters applied on Lena image 512×512 in first experiment. e) - h) Results of metrics of all filters applied on Baboon image 256×256 in second experiment.

### 4.4 Chapter Summary

In this chapter, different standard filters of ED have been applied to convert a grayscale image to halftone image. The comparison of these filters is made on the basis of various metrics such as \(MSE\), \(AD\), \(MD\), \(MAE\) between original image and halftoned image together with the \(PSNR\) and \(UQI\).

On the basis of experimental results and performance evaluation, it is concluded that the ED filters have a low complexity and the halftoned images have a better quality. Consequently, it is obvious that the more diffusion of error to the neighboring pixels proves to be a better filter. However, its algorithms become slower as shown in Jarvis and Stucki filters. In spite of that, it can be observed that these filters give clean and sharp output, which helps to offset the slow processing time.

Thus, the Jarvis filter as a better filter is to be used in the subsequent chapter as a preprocessing stage of HVC.