Chapter 3

Optimal Integrated Inventory Policy for Deteriorating Units under Selling Price Dependent Demand when Holding Cost is Capacity Utilization Dependent

3.0 Introduction

In this chapter we have developed an integrated manufacturer-retailer inventory model with selling price dependent demand. The production rate is finite and proportional to the demand rate and credit period is linked to the order quantity. Items in inventory are subject to time dependent deterioration which follows Weibull distribution. We optimize the individual as well as joint total profit per unit time. Finally, numerical examples and sensitivity analysis are presented to validate the proposed model.

3.1 Notations and Assumptions

3.1.1 Notations

- Inventory Parameters for Manufacturer
  
  $A_m$  Set up costs($)
  $C_m$  Production cost / unit($)
  $h_m$  Holding cost / unit / annum($)
  $O_{mp}$  Manufacturer opportunity cost / $ / unit time

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3.1. Notations and Assumptions

- Production rate (and known) $P$
- Fix credit period offered by manufacturer to retailer (years) $M$
- Manufacturer’s Profit / time($) $\pi_m$
- Capacity utilization ($\rho < 1$ and known) $\rho$

- Inventory Parameters for Retailer

  - Ordering cost($) $A_r$
  - Retailer opportunity cost / $$/unit time $O_{rp}$
  - Retailer’s order quantity per order $Q$
  - Replenishment time (Years) $T$
  - Number of shipment $n$
  - Holding cost for retailer($) $h_r$
  - Selling price/unit time($) $S$
  - Interest earned/unit/annum($) $I_e$
  - Time dependent deterioration of Inventory follows Weibull distribution $\theta(t)$
  - Retailer’s demand dependent on selling price (For convenience simply is used in place of $D(S)$) $D(S)$
  - Order quality to qualify for offer of trade credit (Years) $Q_d$
  - The time length when $Q_d$ units are depleted to zero $T_d$
  - Profit / time unit for retailer($) $\pi_r$
  - Joint total profit per unit time($) $\pi$
  - Purchase cost per unit($) $C_r$

- Necessary condition for different Inventory parameters

  $P > D; S > C_r > C_m; \rho = \frac{D}{P}; \rho < 1$

3.1.2 Assumptions

1. In this model of supply chain a single manufacturer and retailer under single item is in consideration.

2. Demand rate $D(S)$ for retailer is dependent on selling price; Demand is negatively related to selling price; thus demand rate is defined as $D(S) =$
For convenience, $D$ is used in place of $D(S)$

3. Replenishment rate is instantaneous and no shortages allowed.

4. Items in inventory are subject to time dependent deterioration. Deterioration follow Weibull distribution; $\theta(t) = \alpha \beta t^{\beta-1}$ where $0 < \alpha < 1$ is shape parameter and $\beta (\beta \geq 1)$ is scale parameter. Deteriorated items can neither be repaired nor be replaced during the cycle time.

5. The retailer qualifies for trade credit offer if order is equal to or larger than quantity $Q_d$ by the manufacturer otherwise the retailer must use cash on delivery strategy.

6. During the credit period, the retailer sells the items and use sales revenue to earn interest at the rate of $I_e$. At the end of the permissible delay period, the retailer pays the purchasing cost to the manufacturer and incurs an opportunity cost at the rate of $I_b$ for the items in stock.

3.2 Model Formulation

In this section, the authors formulate an integrated inventory model with selling price sensitive demand, where the delay in payments is only permitted if the order quantity is greater than or equal to a predetermined quantity. The inventory holding cost in our model contains two components: unit holding cost and interest charge. The unit holding cost relates to the actual ownership of the goods and includes storage and maintenance expenses, which is accounted on a per-unit-of-inventory basis. The interest charge is considered on the money value of the inventory on hand.
3.2.1 Manufacturer’s Total Profit per Unit Time

For the manufacturer, the total profit per unit time is composed of sales revenue, setup cost, holding cost and opportunity cost. These components are evaluated as following:

(a) Sales revenue: The total sales revenue per unit time is \( (C_r - C_m)Q_T \) (See Appendix A for computation of \( Q \))

(b) Setup cost: The manufacturer produces \( nQ \) units in one production run. The cycle length is \( \frac{nQ}{D} = nT \)

Therefore, the setup cost per unit time is

\[
SC_m = \frac{A_m}{nT} \quad (3.1)
\]

(c) Holding cost: The manufacturer’s inventory holding cost per unit time can be calculated by subtracting the retailer’s accumulated inventory level from the manufacturer’s accumulated inventory level. Hence, the manufacturer’s average inventory per unit time is given by

\[
HC_m = nQ\left(\frac{Q}{P} + (n - 1)\frac{Q}{D}\right) - \frac{n^2Q^2}{2P} - \frac{Q^2}{D}(1 + 2 + ... + (n - 1))\frac{Q}{P}
\]

\[
= \frac{Q}{2P}[n(n - 1)(P - D) + D]
\]

\[
= \frac{Q}{2}[n(n - 1)(1 - \rho) + \rho]
\]

where \( \rho = \frac{D}{P} \) and \( Q = \int_0^T Q(T)dt \)

So the holding cost per unit time is

\[
HC_m = DC_m(h_m + O_{mp})[(n - 1)(1 - \rho) + \rho]\left[\frac{T^2}{2} + \frac{\alpha \beta}{\beta + 2}\right] \quad (3.2)
\]
Note that a similar derivation in manufacturer’s average inventory using a manufacturing lot size of units can be found in Joglekar (1988).

(d) Opportunity cost: If the retailer orders up to $Q_d$ or more, then a credit period $M$ will be offered. Under this situation, when $T \geq T_d$, the delay in payment is permissible and corresponding opportunity cost per unit time is

$$OC_{mp} = \frac{C_m O_{mp} Q_m}{T}$$ \hspace{1cm} (3.3)

Conversely when $T < T_d$, no opportunity cost will occur since a delay in payments is not permitted.

Consequently, manufacturer net profit per unit is sales revenue minus the total relevant cost, which can be expressed as following

$$\pi_m(n) = \begin{cases} 
\pi_{m1}(n) & T < T_d \\
\pi_{m2}(n) & T \geq T_d 
\end{cases}$$ \hspace{1cm} (3.4)

where

$$\pi_{m1}(n) = (C_r - C_m) \frac{Q}{T} - SC_m - HC_m; \quad T < T_d \hspace{1cm} (3.5)$$

$$\pi_{m2}(n) = (C_r - C_m) \frac{Q}{T} - SC_m - HC_m - OC_{mp}; \quad T \geq T_d \hspace{1cm} (3.6)$$

### 3.2.2 Retailer’s Profit Per Unit Time

(a) Sales revenue: The total sales revenue per unit time is $(S - C_r) \frac{Q}{T}$ (See Appendix A for computation of $Q$)
(b) Ordering cost: The ordering cost per unit time is

\[ O_{cr} = \frac{A_{r}}{nT} \]  \hspace{1cm} (3.7)

(c) Holding cost: The retailer’s holding cost (excluding interest charges) per unit time is

\[ HC_{r} = \frac{h_{r}D}{T} \left( \frac{T^2}{2} + \frac{\alpha \beta T^{\beta + 2}}{(\beta + 1)(\beta + 2)} \right) \]  \hspace{1cm} (3.8)

(d) Opportunity cost: Based on the length T, M and T_d the following cases occur.

(i) \(0 < T < T_d\)  
(ii) \(T_d < T \leq M\)  
(iii) \(T_d \leq M \leq T\)  
(iv) \(M \leq T_d \leq T\)

The cases (iii) and (iv) are similar shown in Figure (3.1).

![Figure 3.1: Opportunity cost for case \(T_d \leq M \leq T\) and \(M \leq T_d \leq T\)](image)
\[ OC_{rp} = \begin{cases} \frac{C_r O_p Q}{T} & 0 < T < T_d \\ 0 & T_d < T \leq M \\ \frac{C_r O_p D}{T} \left[ \frac{T^2}{2} + \frac{\alpha \beta (T^{\beta+2} + M^{\beta+2})}{(\beta+1)(\beta+2)} + \frac{\alpha T M (T^{-\beta} - M^{\beta})}{(\beta+1)} \right] & T_d \leq M \leq T \leq M \leq T \end{cases} \] (3.9)

(e) Interest earned: Same as opportunity cost interest earned per unit time in all four cases as follows. Interest earned in the case of \( T_d \leq T \leq M \) and \( T_d \leq M \leq T \) is shown in Figure (3.2) and Figure (3.3) respectively.

**Figure 3.2: Interest earned by retailer when \( T_d \leq T \leq M \)**

\[ IE_r = \begin{cases} 0 & 0 < T < T_d \\ \frac{S_{Ir}}{T} \left[ \int_0^T Q(t) dt + Q(M - T) \right] & T_d \leq T \leq M \\ \frac{S_{Ir}}{T} \left[ \int_0^M Q(t) dt \right] & T_d \leq M \leq T \leq T \end{cases} \] (3.10)
Figure 3.3: Interest earned by retailer when $T_d \leq M \leq T$

Hence, retailer’s total profit per unit time is

$$
\pi_r(S, T) = \begin{cases} 
\pi_{r1}(S, T) & 0 < T < T_d \\
\pi_{r2}(S, T) & T_d \leq T \leq M \\
\pi_{r3}(S, T) & T_d \leq M \leq T \text{ or } M \leq T_d \leq T
\end{cases}
$$

(3.11)

Where

$$
\pi_{r1}(S, T) = (S - C_r) \frac{Q}{T} - O_{cr} - HC_r - \frac{C_r O_{rp} Q}{T} 
$$

(3.12)

$$
\pi_{r2}(S, T) = (S - C_r) \frac{Q}{T} - O_{cr} - HC_r + \frac{SL_c}{T} \left[ \int_0^T Q(t) dt + Q(M - T) \right] 
$$

(3.13)

$$
\pi_{r3}(S, T) = (S - C_r) \frac{Q}{T} - O_{cr} - HC_r - \frac{C_r O_{rp} D}{T} \left[ \frac{T^2}{2} + \frac{\alpha \beta (T^{\beta + 2} + M^{\beta + 2})}{(\beta + 1)(\beta + 2)} + \frac{\alpha TM (T^{\beta} - M^{\beta})}{(\beta + 1)} \right] + \frac{SL_c}{T} \left[ \int_0^M Q(t) dt \right]
$$

(3.14)
3.2.3 Joint total profit per unit time

\[ \pi(S, n, T) = \begin{cases} 
\pi_m(n) + \pi_r(S, T) & 0 < T < T_d \\
\pi_m(n) + \pi_r(S, T) & T_d \leq T \leq M \\
\pi_m(n) + \pi_r(S, T) & T_d \leq M \leq T \quad \text{or} M \leq T_d \leq T 
\end{cases} \]  \quad (3.15)

3.3 Algorithm

1. Set all the parametric values for the mathematical model.
2. Fix \( Q_d \) qualifying quantity to get trade credit.
3. Calculate \( T_d \) from \( Q_d \).
4. Set \( n = 1 \).
5. Compute optimal \( S, T \) form \( \frac{\partial \pi_j}{\partial S} = 0 \) and \( \frac{\partial \pi_j}{\partial T} = 0 \); \( j = 1, 2, 3 \).
6. Find corresponding \( j = 1, 2, 3 \).
7. Take \( n = n + 1 \).
8. Repeat step 4 to 6 until
\[ \pi(n - 1, S(n - 1), T(n - 1)) \leq \pi(n, S(n), T(n)) \geq \pi(n + 1, S(n + 1), T(n + 1)) \]
9. Once optimal \( n^* \) is calculated, \( S^*, T^* \) are calculated then optimal order quantity \( Q^* \) could be calculated.
3.4 Numerical Example

1. Consider one numerical example

\[ a = 100, b = 0.5, \alpha = 0.1, M = 0.1 \text{ (years)}, \beta = 1.5, \delta_e = 8\%, A_m = \$400, A_r = \$50, C_m = \$8, C_r = \$20, h_m = 50\%, h_r = 50\%, O_{mp} = \$6, O_{rp} = \$7 \]

Table 3.1: Optimal solution under different \( Q_d \)

<table>
<thead>
<tr>
<th>( Q_d )</th>
<th>( Q^* )</th>
<th>( n^* )</th>
<th>( S^* ) ($)</th>
<th>( T^* ) (Years)</th>
<th>Profit ($)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>63.13</td>
<td>1</td>
<td>35</td>
<td>0.75</td>
<td>503.72</td>
<td>1137.21</td>
</tr>
<tr>
<td>30</td>
<td>63.13</td>
<td>2</td>
<td>35</td>
<td>0.75</td>
<td>413.53</td>
<td>1173.19</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>3</td>
<td>34</td>
<td>0.69</td>
<td>699.12</td>
<td>1173.25</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>4</td>
<td>35</td>
<td>0.7</td>
<td>654.12</td>
<td>1191.27</td>
</tr>
<tr>
<td>120</td>
<td>90.07</td>
<td>5</td>
<td>35</td>
<td>0.75</td>
<td>590.23</td>
<td>1194.73</td>
</tr>
<tr>
<td>150</td>
<td>60</td>
<td>4</td>
<td>34</td>
<td>0.7</td>
<td>658.72</td>
<td>1107.56</td>
</tr>
<tr>
<td>180</td>
<td>60</td>
<td>4</td>
<td>32</td>
<td>0.7</td>
<td>667.7</td>
<td>937.09</td>
</tr>
</tbody>
</table>

The optimal shipments and ordering units with retailer, manufacturer and joint profit for different values of \( Q_d \) are exhibited in Table-3.1. Joint and individual profits corresponding to different \( Q_d \) is shown graphically in Figure 3.4

Figure 3.4: Order quality \( Q_d \) towards profit
From Table 3.1, it is seen that manufacturer, retailer and joint profit increase in $Q_d$ and then further increase in pre-specified units lower their profits. So optimal order quantity $Q^*$ is approximately equal to $Q_d$ when $Q_d \in (60, 90)$. Thus manufacturer is advised to set threshold which is effective if threshold is set by the manufacturer is too high, the retailer will be reluctant to order a quantity greater than threshold to take advantage of delayed payment. From numerical example 60 units gives maximum joint profit.

2. Consider the data given in Example 1 we study the effect of delayed payment for $Q_d = 60$ units.

<table>
<thead>
<tr>
<th>$M$ (Years)</th>
<th>$Q^*$</th>
<th>$n^*$</th>
<th>$S^*$ ($)</th>
<th>$T^*$ (Years)</th>
<th>Manufacturer</th>
<th>Retailer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>60</td>
<td>3</td>
<td>34</td>
<td>0.69</td>
<td>699.12</td>
<td>1173.25</td>
<td>1872.37</td>
</tr>
<tr>
<td>0.2</td>
<td>60</td>
<td>3</td>
<td>35.34</td>
<td>0.76</td>
<td>729.79</td>
<td>1266.45</td>
<td>1996.27</td>
</tr>
<tr>
<td>0.3</td>
<td>60</td>
<td>3</td>
<td>35.3</td>
<td>0.78</td>
<td>782.8</td>
<td>1342.2</td>
<td>2125.01</td>
</tr>
<tr>
<td>0.4</td>
<td>60</td>
<td>3</td>
<td>35.26</td>
<td>0.71</td>
<td>835.82</td>
<td>1417.96</td>
<td>2253.78</td>
</tr>
<tr>
<td>0.5</td>
<td>60</td>
<td>3</td>
<td>35.18</td>
<td>0.72</td>
<td>888.83</td>
<td>1493.72</td>
<td>2382.55</td>
</tr>
</tbody>
</table>

Table 3.2 clearly shows that longer credit period increases individual as well as joint profit of the supply chain.

### 3.5 Sensitivity Analysis

In this Example we carry out sensitivity analysis to find the critical inventory parameters the change in manufacture, retailer and joint profit is studied by varying inventory parameters as -20%, -10%, 10% and 20%. The results are shown in Fig-
Figure 3.5: Sensitivity of parameters towards joint profit

Further, retailer’s and manufacturer’s profit is calculated in this section.

Table 3.3: Optimal solution of Integrated and Non Integrated scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Retailer</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non Integrated</td>
<td>Ordering quantity-52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Retail price- $ 37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Shipments = 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time = 0.72 years</td>
<td>Total annual profit = $1100.47</td>
</tr>
<tr>
<td></td>
<td>Total annual profit = $507.78</td>
<td>Total annual profit = $507.78</td>
</tr>
<tr>
<td>Integrated</td>
<td>Ordering quantity-60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Retail price = $35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total shipments = 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time = 0.69 years</td>
<td>Total annual profit = $1137.21</td>
</tr>
<tr>
<td></td>
<td>Total annual profit = $699.12</td>
<td>Total annual profit = $699.12</td>
</tr>
</tbody>
</table>

3.5.1 Observations

Retailer’s Profit = (Retailer’s integrated total annual profit) \times 
\left(\frac{\text{Retailer’s independent total annual profit}}{\text{Retailer’s independent total annual profit} + \text{Manufacturer’s independent total annual profit}}\right)
Manufacturer’s Profit = \((\text{Manufacturer’s integrated total annual profit}) \times \\
\left( \frac{\text{Manufacturer’s independent total annual profit}}{\text{Retailer’s independent total annual profit} + \text{Manufacturer’s independent total annual profit}} \right) \)

= 498.00

It is very clearly shown in Table 3.3 that integrated model gives reasonably better profit for both retailer and supplier compare to non-integrated model. Hence results also indicate that integrated supply chain model is beneficial for all members in comparison with isolated or non-integrated model.

3.6 Conclusions

In this chapter, an integrated model for time dependent deteriorated stock is proposed. Model provides algorithm to calculate optimal number of shipments for joint profit maximization. Joint profit for both retailer and manufacture is calculated and analyzed with individual profits. In this study demand is sales price dependent. This type of model helps to provide long term sustainable supply chain. Observations from numerical data clearly show that manufacturer - retailer joint profit and individual profits both increases in case of integrated model then non-integrated model.