Chapter 8

Optimization of Multi-Echelon Integrated Inventory Models using Multi-Objective Genetic Algorithm

8.0 Introduction

In this chapter we consider supply chain of supplier, manufacturer, distributor and retailer for multiple items. For Selection of multi players qualitative approach is applied. Thus selection of business players is done on the basis of multiple criteria. In order to optimize total cost of supply chain we use multi-objective genetic algorithm and for visualization of Pareto optimal solutions we use parallel coordinates plot and 3D-RadVis methods. This chapter is divided into two parts.

Model 1 Optimal multi-echelon integrated supply chain selecting best supplier and distributor using multi-objective genetic algorithm
Model 2 Multi-Product Optimization of Supply Chain Using Multi-Objective Genetic Algorithm

8.1 Optimal Multi-Echelon Integrated Supply Chain Selecting Best Supplier and Distributor using Multi-Objective Genetic Algorithm

In this section we have discussed the problem of supplier and distributor selection for an optimal supply chain. Where both selection is done on the basis of multi-criteria like offer price, limited supply and storage capacity, delivery time,
geographic location, quality etc. On the basis of these multi-criteria we have formulated multi-objective mathematical model. We have optimized this model using multi-objective Genetic algorithm and visualized by parallel coordinates plot. In the end, numerical example is carried out to justify the feasibility of the model.

8.1.1 Problem description

The problem of selecting the supply partners for an organization is given in present model. The question is to select the best supplier and best distributor to find the optimal total cost of the entire supply chain. The pictorial representation of present model is given below in Figure 8.1. This model is for p items.

![Figure 8.1: Present model](image)

Price (include transportation cost), quality, delivery time and supplier supply capacity for each items are used to evaluate best supplier. While price, distribution area, delivery time and storage capacity are taken into consideration to select best distributor.


8.1.2 Notations and Assumptions

8.1.2.1 Notations

\( i = 1, 2, \ldots, p \)  Index of items
\( j = 1, 2, \ldots, m \)  Index of candidate suppliers
\( k = 1, 2, \ldots, n \)  Index of candidate distributors
\( D_i \)  Demand of item \( i \)
\( P_i \)  Processing price of manufacturer for item \( i \)
\( T_k \)  Transportation cost for distributor \( k \) to retailer
\( P_{ij} \)  Price from supplier \( j \) to manufacturer to supply item \( i \)/unit ($)
\( P_{ik} \)  Price from manufacturer to distributor \( k \) to receive item \( i \)/unit ($)
\( C_{ij} \)  Supply capacity of supplier \( j \) to supply item \( i \)
\( C_{ik} \)  Storage capacity of distributor \( k \) to store item \( i \)
\( q_{ij} \)  Defective quality of supplier \( j \) when supplying item \( i \)
\( Q_i \)  Acceptable quality for item \( i \)
\( A_{ik} \)  Outside distribution area of distributor \( k \) when distribute item \( i \)
\( A_i \)  Acceptable outside distribution area for item \( i \)
\( l_{ij} \)  Late delivery of supplier \( j \) when supply item \( i \)
\( L_i \)  Acceptable delivery for item \( i \)
\( l_{ik} \)  Late delivery of distributor \( k \) when distribute item \( i \)
\( L_i \)  Acceptable delivery for item \( i \)
\( TC \)  Total cost for item \( i \)
\( PUC \)  Total purchasing cost for item \( i \)
\( PRC \)  Total processing cost for item \( i \)
\( TRC \)  Total manufacturer to distributor transportation cost
\( MIC \)  Total manufacturer inventory carrying cost
\( DIC \)  Total distributor inventory carrying cost
\( RIC \)  Total retailer inventory carrying cost
\( INC \)  Total inventory carrying cost for system

8.1.2.2 Assumptions

- Demand of customer is deterministic.
- Supplier’s supply capacity of each item is limited.
• Supplier selection is done on the base of quality and delivery performance.

• Distributor selection is done on the base of distributor coverage area and delivery performance.

• Distributor’s storage capacity of each item is limited.

• Transportation cost per item from supplier to manufacturer and manufacturer to distributor are included into price.

• Inventory carrying cost at any player of supply chain remains fixed.

8.1.3 Multi-echelon inventory model

Here we want to minimize the total cost of supply chain for different items so our objective function of the mathematical model is given below

\[
TC = PUC + PRC + TRC + INC
\]  

(8.1)

The basic costs involved as below.

**Purchasing cost:**

Purchasing cost is defined as follow

\[
PUC = \sum_{i} \sum_{j} x_{ij} p_{ij}
\]  

(8.2)

Where \( x_{ij} \) = order quantity of \( i^{th} \) item from \( j^{th} \) supplier

**Processing cost:**
Here we take constant processing cost for different items.

\[
PRC = \sum_i x_i P_i
\]  
(8.3)

where \( x_i = \sum_j x_{ij} \)

**Transportation cost:**

Transportation from distributor \( k \) to retailer is given below

\[
TRC = \sum_i \sum_k y_{ik} T_k
\]  
(8.4)

Where \( y_{ik} = \) order quantity of \( i^{th} \) item from manufacturer to distributor \( k \)

**Inventory carrying cost:**

Here we take fix carrying cost per item for any player of supply chain

\[
INC = xMIC + yDIC + yRIC
\]  
(8.5)

Where \( x = \sum_i x_i; \ y = \sum_i y_i \)

The constraints are involved in present model are the following

All the items customer demand must be fulfill by supplier.

\[
\sum_j x_{ij} \geq D_i
\]  
(8.6)

Quality supply by supplier to manufacturer is less than or equal to supply capacity of supplier.

\[
x_{ij} \leq C_{ij}
\]  
(8.7)
Aggregate quality supply by supplier to manufacturer must be acceptable

\[ \sum_{j} x_{ij}q_{ij} \leq Q_iD_i \quad (8.8) \]

Aggregate delivery time taken by supplier to manufacturer must be acceptable

\[ \sum_{j} x_{ij}l_{ij} \leq L_iD_i \quad (8.9) \]

All the items customer demand must be fulfill by distributor.

\[ \sum_{k} y_{ik} = D_i \quad (8.10) \]

Quality supply by manufacturer to distributor is less than or equal to storage capacity of distributor.

\[ y_{ik} \leq C_{ik}^\prime \quad (8.11) \]

Aggregate distribution area covered by distributor must be acceptable

\[ \sum_{k} y_{ik}A_{ik}^\prime \leq A_i^\prime \quad (8.12) \]

Aggregate delivery time taken distributor to retailer must be acceptable

\[ \sum_{k} y_{ik}l_{ik}^\prime \leq L_i^\prime D_i \quad (8.13) \]
So, for best supplier selection we have following objective function and constraints

\[
\min f = \sum_{i} \sum_{j} x_{ij} P_{ij} \tag{8.14}
\]

subject to

\[
\sum_{j} x_{ij} \geq D_i; x_{ij} \leq C_{ij};
\]

\[
\sum_{j} x_{ij} q_{ij} \leq Q_i D_i; \sum_{j} x_{ij} l_{ij} \leq L_i D_i
\]

And for best distributor selection we have following objective function and constraints

\[
\min g = \sum_{i} \sum_{k} y_{ik} P'_{ik} \tag{8.15}
\]

subject to

\[
\sum_{k} y_{ik} \geq D_i; y_{ik} \leq C'_{ik};
\]

\[
\sum_{k} y_{ik} A'_{ik} \leq A'_{i}; \sum_{k} y_{ik} l'_{ik} \leq L'_{i} D_i
\]

Where \(x_{ij}\) and \(y_{ik}\) are decision variables.

8.1.4 Multi-objective Genetic algorithm

Genetic algorithm (GA) start with set of population, called Chromosomes. In iterations of GA, selection of individuals is done on the basis of fitness score of chromosomes for crossover and mutation to produce new chromosomes. Thus new set of population is generated for next iteration. The iterations are continuing until a solution with desired tolerance is not achieved. Since GA generates multiple solutions, it goes well with MOP as MOP require it to identify non-interior solutions.
When we use GA for MOP selection of matting pair create an issue. For resolve this issue different researcher use different methods like give different random weights to objective, separated population into subpopulation etc. Here we separate population into subpopulation on the base of items.

We have m suppliers and n distributors. So in multi objective GA total chromosomes for suppliers and distributors are $2^m$ and $2^n$ respectively. We convert multi object problem on the base of different items. Thus allocating items for best suppliers are different $p$ linear programs.

$$\min f = \sum_i x_{ij}P_{ij} \quad (8.16)$$
$$\text{subject to } \sum_j x_{ij} \geq D_i; x_{ij} \leq C_{ij};$$
$$\sum_j x_{ij}q_{ij} \leq Q_iD_i; \sum_j x_{ij}l_{ij} \leq L_iD_i$$

And allocating items for best distributors are different $p$ linear programs.

$$\min g = \sum_i y_{ik}P'_{ik} \quad (8.17)$$
$$\text{subject to } \sum_k y_{ik} \geq D_i; y_{ik} \leq C'_{ik};$$
$$\sum_k y_{ik}A'_{ik} \leq A'_i; \sum_k y_{ik}l'_{ik} \leq L'_iD_i$$

8.1.5 Numerical examples and sensitivity analysis

Consider supply chain with 3 suppliers, 1 manufacturer, 3 distributor and 1 retailer

$D = 50; L_i = Q_i = A'_i = L'_i = 3\%; P_1 = 10(\$/unit); P_2 = 8(\$/unit); P_3 = 9(\$/unit); T_1 = 1.2(\$/unit); ;$
\[ T_2 = 1.45 ($/\text{unit}) ; \quad T_3 = 1.5 ($/\text{unit}) ; \quad MIC = 2 ($/\text{unit}) ; \quad DIC = 4 ($/\text{unit}) ; \quad RIC = 1.5 ($/\text{unit}) \]

Supplier and distributor information are given in Table 8.1 and Table 8.2 respectively.

### Table 8.1: Supplier Information

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Price($)</td>
<td>8</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Supplier capacity</td>
<td>180</td>
<td>120</td>
<td>80</td>
</tr>
<tr>
<td>Quality (%)</td>
<td>0.1</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>Late delivery(years)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Table 8.2: Distributor Information

<table>
<thead>
<tr>
<th>Distributor</th>
<th>Distributor 1</th>
<th>Distributor 2</th>
<th>Distributor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Price($)</td>
<td>20</td>
<td>28</td>
<td>19</td>
</tr>
<tr>
<td>Supplier capacity</td>
<td>280</td>
<td>120</td>
<td>480</td>
</tr>
<tr>
<td>Distributor area (%)</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Late delivery(years)</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Supplier selection:**

Using multi objective GA in Matlab14a we get set of optimal solutions. Those show in Table-1(Appendix-D)

Using parallel co-ordinate method we visualize appropriate optimal solution from Pareto-optimal front. That visualize by Figure 8.2. As shown in Figure 8.2 X-axis represent solutions and Y-axis represent related total cost \( f_1 \), \( f_2 \) and \( f_3 \) for
Figure 8.2: Visualization of Pareto-optimal front to select supplier

From Table 1 (Appendix-D) and Figure 8.2 it is clear that $36^{th}$ solution is best because it minimize the cost and full fill the required demand. So manufacturer order quantity from different supplier’s are given in Table 8.3 and manufacturer pay total 1330 ($).
Table 8.3: Optimal order quantity by manufacturer to supplier to minimize total cost

<table>
<thead>
<tr>
<th></th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Item 2</td>
<td>33</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>Item 3</td>
<td>38</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

**Distributor selection:** Using multi objective GA in Matlab14a we get set of optimal solutions. Those Pareto-optimal front solutions are given in Table 2.(Appendix-D)

Using parallel co-ordinate method we visualize appropriate optimal solution from Pareto-optimal front. That visualize by Figure 8.3. As shown in Figure 8.3 X-axix represent solutions and Y-axix repesent related total cost $g_1$, $g_2$ and $g_3$ for the three items.

From Table 2 (Appendix-D) and Figure 8.3 it is clear that $60^{th}$ solution is best because it minimize the cost and full fill the required demand. So manufacturer supply quantity to different distributors are given in Table 8.4 and for that he pay total 3150 ($).

Table 8.4: Optimal order quantity from manufacturer to distributor to minimize total cost

<table>
<thead>
<tr>
<th></th>
<th>Distributor 1</th>
<th>Distributor 2</th>
<th>Distributor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>30</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Item 2</td>
<td>10</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>Item 3</td>
<td>17</td>
<td>13</td>
<td>20</td>
</tr>
</tbody>
</table>
So total cost for supply chain is
\[ TC = PUC + PRC + TRC + INC = 4055.2(\$) \]

### 8.2 Multi-Product Optimization of Supply Chain Using Multi-Objective Genetic Algorithm

#### 8.2.1 Pictorial representation of model

Due to globalization of the world economy, options are available everywhere. The selection of supply players at different stage is present here. We optimize the total cost of entire supply chain by choosing appropriate supply players. The problem description of present model is given below in Figure 1. This model is for finite
(t) items.

Manufacturer selects best supplier on the base of price, transportation cost, quality, delivery time and supplier supply capacity for each items. Manufacturer has finite(n) plants. He select plant on the base of production cost, transportation cost, quality, production time and production capacity of each item. Evaluate best distributor on the base of purchase cost, transportation cost, distribution coverage area, delivery time and storage capacity. While purchase cost, transportation cost, customer selection, delivery time and storage capacity are taken into consideration to select best retailer.

8.2.2 Notations and Assumptions

8.2.2.1 Notations
\begin{align*}
i &= 1, 2, \ldots, t \quad \text{Index of items} \\
j &= 1, 2, \ldots, m \quad \text{Index of candidate suppliers} \\
k &= 1, 2, \ldots, n \quad \text{Index of plants} \\
\lambda &= 1, 2, \ldots, p \quad \text{Index of candidate distributors} \\
\nu &= 1, 2, \ldots, q \quad \text{Index of candidate retailers} \\
D_i &\quad \text{Demand of item } i \\
P_{ij} &\quad \text{Price from supplier } j \text{ to manufacturer to supply item } i \text{ /unit ($)} \\
P_{ik} &\quad \text{Processing cost of plant } k \text{ for item } i \text{ /unit ($)} \\
P_{i\lambda} &\quad \text{Purchase cost of distributor } \lambda \text{ from manufacturer to receive item } i \text{ /unit ($)} \\
P_{i\nu} &\quad \text{Purchase cost of retailer } \nu \text{ from distributor to receive item } i \text{ /unit ($)} \\
T_{ijk} &\quad \text{Transportation cost from supplier } j \text{ to plant } k \text{ for item } i \text{ /unit ($)} \\
T_{ik\lambda} &\quad \text{Transportation cost from plant } k \text{ to distributor } \lambda \text{ for item } i \text{ /unit ($)} \\
T_{i\lambda\nu} &\quad \text{Transportation cost from distributor } \lambda \text{ to retailer } \nu \text{ for item } i \text{ /unit ($)} \\
C_{ij} &\quad \text{Supply capacity of supplier } j \text{ to supply item } i \\
C_{ik} &\quad \text{Production capacity of plant } k \text{ to produce item } i \\
C_{i\lambda} &\quad \text{Storage capacity of distributor } \lambda \text{ to store item } i \\
C_{i\nu} &\quad \text{Storage capacity of retailer } \nu \text{ to store item } i \\
q_{ij} &\quad \text{Defective quality of supplier } j \text{ when supplying item } i \\
q_{ik} &\quad \text{Defective quality of plant } k \text{ when supplying item } i \\
a_{i\lambda} &\quad \text{Outside distribution area of distributor } \lambda \text{ when distribute item } i \\
u_{i\nu} &\quad \text{Unsatisfied customer from retailer } \nu \text{ for item } i \\
Q_{ij} &\quad \text{Acceptable defective quality from supplier } j \text{ for item } i \\
Q_{ik} &\quad \text{Acceptable defective quality from plant } k \text{ for item } i \\
A_{i\lambda} &\quad \text{Acceptable outside distribution area form distributor } \lambda \text{ for item } i \\
U_{i\nu} &\quad \text{Acceptable Unsatisfied customer from retailer } \nu \text{ for item } i \\
l_{ij} &\quad \text{Late delivery of supplier } j \text{ when supply item } i \\
l_{ik} &\quad \text{Late delivery of plant } k \text{ when supply item } i \\
l_{i\lambda} &\quad \text{Late delivery of distributor } \lambda \text{ when supply item } i \\
l_{i\nu} &\quad \text{Late delivery of retailer } \nu \text{ when supply item } i \\
L_{ij} &\quad \text{Acceptable late delivery form supplier } j \text{ for item } i \\
L_{ik} &\quad \text{Acceptable late delivery form plant } k \text{ for item } i \\
L_{i\lambda} &\quad \text{Acceptable late delivery form distributor } \lambda \text{ for item } i \\
L_{i\nu} &\quad \text{Acceptable late delivery form retailer } \nu \text{ for item } i \\
T_{RC_{ijk}} &\quad \text{Total transportation cost form supplier } j \text{ to plant } k \text{ for item } i \text{ /unit ($)} \\
T_{RC_{ik\lambda}} &\quad \text{Total transportation cost form plant } k \text{ to distributor } \lambda \text{ for item } i \text{ /unit ($)} \\
T_{RC_{i\lambda\nu}} &\quad \text{Total transportation cost form distributor } \lambda \text{ to retailer } \nu \text{ for item } i \text{ /unit ($)}
\end{align*}
8.2.3 Assumptions

- Manufacturer contain finite number of plants.
- Plants production capacity of each item is limited.
- Plant’s selection is done on the base of quality and production time.
- Retailer’s storage capacity of each item is limited.
- Retailer selection is done on the base of customer satisfaction and delivery performance.
- Inventory carrying cost at any player of supply chain remains fixed.

8.2.4 Multi-echelon inventory model

The total cost for entire supply chain is derived as below. We want to minimize by selecting best business players at different stage

\[ TC = PUC + PRC + TRC + INC \]  

(8.18)

The basic costs involved as below.
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8.2. Multi-Product Optimization...

**Purchasing cost:**

Purchasing cost is defined as follow

\[ PUC = \sum_i \sum_j x_{ij} P_{ij} \]  \hspace{1cm} (8.19)

Where \( x_{ij} \) = order quantity of \( i^{th} \) item from \( j^{th} \) supplier

**Processing cost:**

Total processing cost for different plants is defined as follow.

\[ PRC = \sum_i \sum_k y_{ik} P_{ik} \]  \hspace{1cm} (8.20)

Where \( y_{ik} \) = Order quantity of \( i^{th} \) item produce by \( k^{th} \) plant

**Transportation cost:**

Transportation from distributor \( k \) to retailer is given below

\[ TRC = \sum_i (TRC_{ijk} + TRC_{ik\lambda} + TRC_{i\lambda v}) \]  \hspace{1cm} (8.21)

Where \( TRC_{ijk} = \sum_k y_{ik} T_{ijk} \) and \( y_{ik} \) = order quantity of \( i^{th} \) item produce by plant \( k \)

\( TRC_{ik\lambda} = \sum_\lambda z_{i\lambda} T_{ik\lambda} \) and \( z_{i\lambda} \) = order quantity of \( i^{th} \) item form manufacturer to distributor \( \lambda \)

\( TRC_{i\lambda v} = \sum_v w_{iv} T_{i\lambda v} \) and \( w_{iv} \) = order quantity of \( i^{th} \) item from distributor to retailer \( k \)

**Inventory carrying cost:**

Here we take fix carrying cost per item for any player of supply chain
\[ INC = yMIC + zDIC + wRIC \]  
(8.22)

Where \( y = \sum_i y_i \); \( z = \sum_i z_i \); \( w = \sum_i w_i \)

The constraints are involved in present model are the following:

All the items customer demand must be fulfill by supplier.

\[ \sum_j x_{ij} \geq D_i \]  
(8.23)

Quality supply by supplier to manufacturer is less than or equal to supply capacity of supplier.

\[ x_{ij} \leq C_{ij} \]  
(8.24)

Aggregate quality supply by supplier to manufacturer must be acceptable

\[ \sum_j x_{ij}q_{ij} \leq Q_iD_i \]  
(8.25)

Aggregate delivery time taken by supplier to manufacturer must be acceptable

\[ \sum_j x_{ij}l_{ij} \leq L_iD_i \]  
(8.26)

All the items customer demand must be fulfill by manufacturer.

\[ \sum_k y_{ik} \geq D_i \]  
(8.27)
Product produce by each plant is less than or equal to production capacity of plants

\[ y_{ik} \leq C_{ik} \quad (8.28) \]

Aggregate quality produce by plant must be acceptable

\[ \sum_k y_{ik} q_{ik} \leq Q_{ik} D_i \quad (8.29) \]

Aggregate delivery time taken by plant to distributor must be acceptable

\[ \sum_k y_{ik} l_{ik} \leq L_{ik} D_i \quad (8.30) \]

All the items customer demand must be fulfill by distributor.

\[ \sum_\lambda z_{i\lambda} \geq D_i \quad (8.31) \]

Quality supply by manufacturer to distributor is less than or equal to storage capacity of distributor.

\[ z_{i\lambda} \leq C_{i\lambda} \quad (8.32) \]

Aggregate distribution area covered by distributor must be acceptable

\[ \sum_\lambda z_{i\lambda} a_{i\lambda} \leq A_{i\lambda} D_i \quad (8.33) \]
Aggregate delivery time taken distributor to retailer must be acceptable

\[ \sum_{\lambda} z_{i\lambda} l_{i\lambda} \leq L_{i\lambda} D_i \]  
(8.34)

All the items customer demand must be fulfill by retailer.

\[ \sum_{\nu} w_{iv} \geq D_i \]  
(8.35)

Quality supply by distributor to retailer is less than or equal to storage capacity of retailer.

\[ w_{iv} \leq C_{iv} \]  
(8.36)

Aggregate customer satisfaction must be acceptable

\[ \sum_{\nu} w_{iv} u_{iv} \leq U_{iv} D_i \]  
(8.37)

Aggregate delivery time taken by retailer to customer must be acceptable

\[ \sum_{\nu} w_{iv} l_{iv} \leq L_{iv} D_i \]  
(8.38)

So, for best supplier selection we have following objective function and constraints
\[
\text{min } f_1 = \sum_i \sum_j x_{ij} p_{ij} \tag{8.39}
\]

subject to \[ \sum_j x_{ij} \geq D_i; x_{ij} \leq C_{ij}; \]
\[ \sum_j x_{ij} q_{ij} \leq Q_i D_i; \sum_j x_{ij} l_{ij} \leq L_i D_i \]

For best plant selection we have following objective function and constraints

\[
\text{min } f_2 = \sum_i \sum_k y_{ik} p_{ik} \tag{8.40}
\]

subject to \[ \sum_k y_{ik} \geq D_i; y_{ik} \leq C_{ik}; \]
\[ \sum_k y_{ik} q_{ik} \leq Q_{ik} D_i; \sum_k y_{ik} l_{ik} \leq L_{ik} D_i \]

For best distributor selection we have following objective function and constraints

\[
\text{min } f_3 = \sum_i \sum_{\lambda} z_{i\lambda} p_{i\lambda} \tag{8.41}
\]

subject to \[ \sum_{\lambda} z_{i\lambda} \geq D_i; z_{i\lambda} \leq C_{i\lambda}; \]
\[ \sum_{\lambda} z_{i\lambda} a_{i\lambda} \leq A_{i\lambda} D_i; \sum_{\lambda} z_{i\lambda} l_{i\lambda} \leq L_{i\lambda} D_i \]

For best retailer selection we have following objective function and constraints
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8.2. Multi-Product Optimization...

\[
\begin{align*}
\min f_4 &= \sum_i \sum_\nu z_{i\nu k} P_{i\nu} \\
&\text{subject to } \sum_\nu w_{i\nu} \geq D_i; w_{i\nu} \leq C_{i\nu}; \\
&\sum_\nu w_{i\nu} u_{i\nu} \leq U_{i\nu} D_i; \sum_\nu z_{i\nu l_{i\nu}} \leq L_{i\nu} D_i
\end{align*}
\]

(8.42)

Where \(x_{ij}, y_{ik}, z_{i\lambda}\) and \(w_{i\nu}\) are decision variables.

8.2.5 Computational algorithm

8.2.5.1 Multi-objective GA

Total cost function for all items and individual cost function are considered to be fitness function and minimized using below mentioned algorithm. MATLAB 14a is used for running iterations.

1. Set numerical values for different parameters except for decision variables \(x_{ij}, y_{ik}, z_{i\lambda}\) and \(w_{i\nu}\) in the fitness function.

2. Start with an initial population of 20 chromosomes.

3. Rank the chromosomes on the basis of their fitness score.

4. Chromosomes with good fitness score will get entry in mating pool.

5. Perform stochastic uniform crossover for reproduction. Crossover fraction is considered 0.8 and 2-Elites are considered at each generation.

6. Again rank members of new generation on the basis of their fitness function and select members which can create next generation
7. Perform step 3 and step 4 till absolute difference between two successive members is negligible i.e $|x_{i+1} - x_i| < \varepsilon (\varepsilon = 10^{-5})$

8.2.5.2 3D-RadVis visualization technique

For N Pareto front solution of M objectives

1. Compute
   \[ x = \frac{\sum_{j=1}^{M} f_{Norm,cos}(\theta_j)}{\sum_{j=1}^{M} f_{Norm}} \]
   \[ y = \frac{\sum_{j=1}^{M} f_{Norm,sin}(\theta_j)}{\sum_{j=1}^{M} f_{Norm}} \]
   where \( f_{Norm} = \frac{f_i(x) - \min(f_i(x))}{\max(f_i(x)) - \min(f_i(x))} \)

2. \( px = x + 1 \)

3. \( py = y + 1 \)

4. Find normal vector perpendicular to the extreme point
   \[ n = norm(z); \text{ where } z \text{ is hyper plane.} \]

5. Calculate \( c = n.z_1 \)

6. for \( i = 1 to N \) find \( d = \frac{abs(f_i.n - c)}{|n|} \)

7. Finally we convert 3D-RadVis \( R = [x, y, d] \)

8.2.6 Numerical examples

Consider supply chain with 3 suppliers, 1 manufacturer with 3 plants, 3 distributor and 3 retailer

\( D(i) = 50; Q_{iv} = Q_{ij} = Ai\lambda = Q_{ik} = L_{ij} = L_{ik} = L_{iv} = L_{i\lambda} = 3\% / \text{unit} ; MIC = 2(\$/\text{unit}); DIC = 4(\$/\text{unit}); RIC = 1.5(\$/\text{unit}) \)
suppliers, plants, distributors and retailers related other information are given in Table 8.5, 8.6, 8.7 and 8.8 respectively.

### Table 8.5: Supplier Information

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1  2  3</td>
<td>1  2  3</td>
<td>1  2  3</td>
</tr>
<tr>
<td>Price($)</td>
<td>6  4  5</td>
<td>6  5  4.5</td>
<td>7  5  4</td>
</tr>
<tr>
<td>Supplier capacity</td>
<td>80 90 90</td>
<td>50 70 60</td>
<td>80 70</td>
</tr>
<tr>
<td>Quality(%)</td>
<td>0.1 0.2 0.2</td>
<td>0.2 0.1 0.1</td>
<td>0.2 0.3</td>
</tr>
<tr>
<td>Late delivery(month)</td>
<td>0.1 0.3 0.3</td>
<td>0.2 0.05 0.05</td>
<td>0.3 0.1</td>
</tr>
</tbody>
</table>

### Table 8.6: Plant Information

<table>
<thead>
<tr>
<th>Plants</th>
<th>Plant 1</th>
<th>Plant 2</th>
<th>Plant 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1  2  3</td>
<td>1  2  3</td>
<td>1  2  3</td>
</tr>
<tr>
<td>Production cost($)</td>
<td>2.4 3.6 4.6</td>
<td>2.8 4.2 4.5</td>
<td>3.2 4.1 5</td>
</tr>
<tr>
<td>Transportation cost($)</td>
<td>0.4 0.4 0.2</td>
<td>0.3 0.4 0.1</td>
<td>0.1 0.2 0.1</td>
</tr>
<tr>
<td>Production capacity</td>
<td>60 90 70</td>
<td>80 70 60</td>
<td>80 70</td>
</tr>
<tr>
<td>Quality(%)</td>
<td>0.25 0.1 0.2</td>
<td>0.2 0.1 0.1</td>
<td>0.2 0.15</td>
</tr>
<tr>
<td>Production time(months)</td>
<td>0.1 0.2 0.2</td>
<td>0.15 0.15 0.05</td>
<td>0.25 0.1</td>
</tr>
</tbody>
</table>
Chapter 8. Optimization of Multi-Echelon...

8.2. Multi-Product Optimization...

Table 8.7: Distributor information

<table>
<thead>
<tr>
<th>Distributors</th>
<th>Distributor 1</th>
<th>Distributor 2</th>
<th>Distributor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Purchase price($)</td>
<td>10.4</td>
<td>9.6</td>
<td>11.1</td>
</tr>
<tr>
<td>Transportation cost($)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Storage capacity</td>
<td>70</td>
<td>80</td>
<td>70</td>
</tr>
<tr>
<td>Coverage area(%)</td>
<td>0.1</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>Late delivery (month)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 8.8: Retailer Information

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Purchase price($)</td>
<td>12.4</td>
<td>11.2</td>
<td>14.1</td>
</tr>
<tr>
<td>Transportation cost($)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Storage capacity</td>
<td>60</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Customer no satisfaction (%)</td>
<td>0.01</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>Late delivery (month)</td>
<td>0.01</td>
<td>0.2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Supplier selection:

Using multi objective GA and 3D-RadVis visualization technique in Matlab14a we get set of optimal solutions. Those show in Table 3 (Appendix E) and Figure 8.5 As shown in Figure 8.5 at z=0 we get minimum total cost of all three items.

From Table 3 (Appendix E) and Figure 8.5 it is clear that 2nd solution is best because it minimize the cost and full fill the required demand. So manufacturer order quantity from different supplier’s are given in Table 8.9.
Plant selection: Using multi objective GA and 3D-RadVis visualization technique in Matlab14a we get set of optimal solutions. Those show in Table 4(Appendix E) and Figure 8.6. As shown in Figure 3 at z=0 we get minimum total cost of all three items.

From Table 4 (Appendix E) and Figure 8.6 it is clear that 18th solution is best because it minimize the cost and full fill the required demand. So for production manufacturer choose quantity for different plants are given in Table 8.9.
Figure 8.6: Visualization of Pareto-optimal front representation to select plant

**Distributor selection**

Using multi objective GA and 3D-RadVis visualization technique in Mat-lab14a we get set of optimal solutions. Those show in Table 5(Appendix E) and Figure 4. As shown in Figure 4 at z=0 we get minimum total cost of all three items.

From Table 5 (Appendix E) and Figure 8.7 it is clear that 57th solution is best because it minimize the cost and full fill the required demand. So manufacturer supply different quantity to distributor are given in Table 8.9.
Retailer Selection: Using multi objective GA and 3D-RadVis visualization technique in Matlab14a we get set of optimal solutions. Those show in Table 6(Appendix E) and Figure 8.8. As shown in Figure 8.7 at z=0 we get minimum total cost of all three items.

From Table 6 (Appendix E) and Figure 8.8 it is clear that 13th solution is best because it minimize the cost and full fill the required demand. So distributor supply quantity from different retailer’s are given in Table 8.9.
8.3 Conclusions

In this we propose three operation research techniques multi-objective GA, Parallel Co-ordinate Plots and 3D-RadVis. Theses models consider different con-
Conflict parameters to select supplier, plant, distributor and retailer so single objective is converted function to multi-objective function item wise. We optimized multi-objective problem using GA. In model (8.1) we visualize Pareto optimal front by Parallel Co-ordinate Plots and in model (8.2) by 3D-RadVis technique using Matlab14a. It is cleared by graphical representation given in numerical example 3D-RadVis technique give better result. Due to Multi-objective GA and 3D-RadVis even handle more number business players give solution within few minutes. From results and numerical example it shows that it fulfill customer’s demand under constrain and minimize total supply chain cost by selecting appropriate supply chain partners.
Conclusion and Future scope