CHAPTER 5

RADIATION EFFECT ON MHD BLOOD FLOW THROUGH A TAPERED POROUS STENOSED ARTERY WITH THERMAL AND MASS DIFFUSION
5.1. INTRODUCTION

The study of blood flow through arteries is of great importance in various cardiovascular diseases. The poor circulation of blood in our body due to blockage in arteries is a main cause of health risks. Arteries carry oxygenated blood with nutrients from heart to each cell of the body, in circulatory system of the human body. Blood is a dense red liquid circulating in the blood vessels and it has a strong nurturing effect on the human body which serves as one of the elementary substances constituting the human body. The experimental and theoretical studies of blood flow through the circulatory system of living mammals, have been the topic of scientific research in last few decades. The huge amount of literature is presented on the subject of arterial blood flow, few of them are (Shukla et al., 1980; Higashi et al., 1993; Panja and Sengupta, 1996) but less attentions have been focused to study the heat and mass transfer on MHD blood flow through tapered stenosed arteries. Blood flow characteristics through stenosed artery in the presence of multi-stenosis is discussed by Chakravarty and Sannigrahi (1999). The analysis of blood flow dynamics problems attracts interest due to the various proposed applications in medical sciences and bioengineering. The study of bio-fluids under the presence of magnetic field with dissipation finds its applications in various forthcoming fields like innovative drug targeting, surgical operations etc. Haik et al. (2001) described a 30% decrease in rate of blood flow when applied to a high magnetic field of 10 T,
while, an analogous reduction in the rate of blood flow but at a much smaller magnetic field of 0.002 T was shown by Yadav et al. (2008). Sharma et al. (2013b) presented a mathematical model for the hydro-magnetic bio-fluid flow in the porous medium with Joule effect. A theoretical analysis of blood flow and heat transfer in a permeable vessel in the presence of an external magnetic field have been discussed by Sinha et al. (2016). Shit and Roy (2016) studied the effects of induced magnetic field on blood flow through a constricted channel, and demonstrated that increasing the values of magnetic field reduces the velocity of the blood flow at the center. Rahbari et al. (2017) analysed blood flow containing nanoparticles through porous blood vessels in the presence of magnetic field using Homotopy Perturbation Method. Blood flow in a large blood vessel has a reflective influence on the efficiency of thermal therapy treatment. Electromagnetic heat, such as micro waves and short waves sends heat up to 2 inches into the muscles and tissues which works best for muscles, injuries in joints, and tendons. Moreover, hyperthermia treatment has been confirmed as effective during cancer therapy in recent years. The objective of hyperthermia treatment is to raise the temperature of pathological tissues above cytotoxic temperatures (41–45°C) without overexposing healthy tissues (Levin et al., 1994). Heat and mass transfer of blood flow considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source discussed by Sharma et al. (2015a). Sinha and Shit
(2015) have investigated the combined effects of thermal radiation and MHD with heat transfer blood flow through a capillary. The effect of radiation on inclined arterial blood flow through a non-Darcian porous medium with magnetic field discussed by Sharma et al., (2015b).

There are many important physiological problems those are concerned with the flow of chemically reacting fluid. Many biological fluid organisms are examples of such mixtures. It is known that, the blood is a complex mixture of plasma, cells, proteins and a variety of other chemicals that is exhibited typically in a homogenized sense as a single component of fluid. Blood is balance by a variety of chemical reactions, some that aid its coagulation and others its dissolution. Gnaneswara (2014) have reported the impacts of chemical reaction and thermal radiation on hydromagnetic convective boundary layer slip flow. Sharma and Gaur (2017) studied the effect of variable viscosity on chemically reacting magneto-blood flow with heat and mass transfer. Recently, MHD third order blood flow in an irregular channel through a porous medium with both homogeneous and heterogeneous reactions discussed by Gnaneswara (2017).

The relations between the fluxes and the driving potentials are more intricate in nature when heat and mass transfer occur simultaneously in a moving fluid. It is noted that the mass transfer due to temperature gradient is called the Soret effect, while, the heat transfer due to concentration effect is called the Dufour effect. The Soret and Dufour effects were recently found to be of
order of considerable magnitude so that it cannot be neglected (Eckert and Drake, 1974). The Soret and Dufour effects on MHD unsteady mixed convection flow for a radiative heat generating fluid with chemical reaction is analysed by Sharma et al. (2012) and Sharma et al. (2014). Soret and Dufour effects on the Casson fluid flow with MHD over a stretched surface discussed by Hayat et al., (2012). Recently, Hayat et al., (2017) discussed Soret and Dufour effects on MHD peristaltic transport of Jeffrey fluid in a curved channel.

The motivation for any mathematical analysis of physiological fluid flows is to have a better understanding of the particular flow being modeled. If there is resemblance between the results obtained from the theoretical analysis and experimental and clinical data, then the mechanism of flow can at least be explained. A correct mathematical study can help to explain the major contributing factors in many flows in the human body because peristalsis is evident in many physiological flows. Whenever, comparing the results between mathematical model and experimental data, it is appropriate that the data obtained from experimental research be as close as possible to the actual physiological parameter those being analyzed. In spite of all these studies, the Soret and Dufour effects on MHD blood flow through stenosed artery with thermal radiation and chemical reaction have received a little attention. Therefore, the main object of the present analysis is to study the importance of Soret and Dufour effect in the presence of thermal radiation in
blood flow. Newtonian model of blood flow is taken for this study. Chemical reaction is also studied for the blood flow in the presence of mild stenosis in porous tapered artery.

5.2 MATHEMATICAL FORMULATION:

Let \((r, \theta, z)\) be the cylindrical polar coordinate system with \(\bar{u}\) and \(\bar{w}\) are the radial and axial velocity components in the \(\bar{r}\) and \(\bar{z}\) directions, respectively, with \(r = 0\) as the axis of symmetry of the tube. Consider the Newtonian viscous, incompressible fluid of viscosity \(\mu\) and density \(\rho\) in a tube having a length of \(L\). It is also assumed that the temperature and concentration to the wall of the tube are \(\bar{T}_0\) and \(\bar{C}_0\), respectively. Both temperature and concentration Symmetry condition is employed at the centre of the tube.

The geometry of the stenosis is defined as Mekheimer and El Kot (2008)

\[
h(z) = d(z)[1 - \eta(b^{n-1}(z - a) - (z - a)^{n})], \quad a \leq z \leq a + b
\]

\[= d(z), \text{ otherwise}\]  \hspace{1cm} (5.2.1)

with \(d(z) = d_0 + \varepsilon z\), where \(d(z)\) is the radius of the segment of tapered artery in the stenotic region, \(b\) is the length of the stenosis, \(d_0\) is the radius of the non-tapered artery in the non-stenotic region, \(\varepsilon\) is the tapering parameter, \(n(\geq 2)\) is referred as the shape parameter (the symmetric stenosis occurs for \(n = 2\)) which is determining the shape of the constriction profile and shows
its location (as shown in Fig. 5.1) as discussed in Mekheimer and El Kot (2008). The parameter $\eta$ is defined as

$$\eta = \frac{\delta^* n^{\frac{1}{n-1}}}{d_0 b^n (n - 1)}$$  \hspace{1cm} (5.2.2)

where $\delta^*$ denotes maximum height of the stenosis and located at

$$z = a + \frac{b}{n^{\frac{1}{n-1}}}$$  \hspace{1cm} (5.2.3)

Fig. 5.1: Physical Model of the problem

The magnetic Reynolds number is assumed very small so that the induced magnetic field can be negligible in comparison with the applied magnetic field for slightly conducting fluid. It is also assumed that the electric field is absent due to absence of applied voltage. The fluid properties are assumed to be constant except in the body force term the influence of the density variation with temperature and concentration. A first-order homogeneous chemical reaction is considered to take place in the flow.
The governing equations of the steady, incompressible, Newtonian fluid with energy and mass concentration are given as:

\[
\frac{\partial \ddot{u}}{\partial \ddot{r}} + \frac{\ddot{u}}{\ddot{r}} + \frac{\partial \dot{w}}{\partial \dot{z}} = 0 \tag{5.2.4}
\]

\[
\rho \left[ \ddot{u} \frac{\partial \ddot{u}}{\partial \ddot{r}} + \ddot{w} \frac{\partial \ddot{u}}{\partial \ddot{z}} \right] = -\frac{\partial \ddot{p}}{\partial \ddot{r}} + \frac{\partial}{\partial \ddot{r}} \left[ 2\mu \frac{\partial \ddot{u}}{\partial \ddot{r}} \right] + \frac{2\mu}{\ddot{r}} \left( \frac{\partial \ddot{u}}{\partial \ddot{r}} - \ddot{u} \right) + \frac{\partial}{\partial \ddot{z}} \left[ \mu \left( \frac{\partial \ddot{u}}{\partial \ddot{z}} + \frac{\partial \dot{w}}{\partial \ddot{r}} \right) \right] \tag{5.2.5}
\]

\[
\rho \left[ \ddot{u} \frac{\partial \dot{w}}{\partial \ddot{r}} + \ddot{w} \frac{\partial \dot{w}}{\partial \dot{z}} \right] = -\frac{\partial \ddot{p}}{\partial \dot{z}} + \frac{1}{\ddot{r}} \frac{\partial}{\partial \ddot{r}} \left[ \mu \ddot{r} \left( \frac{\partial \ddot{u}}{\partial \ddot{r}} + \frac{\partial \dot{w}}{\partial \ddot{r}} \right) \right] - \sigma_1 \mu_0^2 H_0^2 \ddot{w} - \frac{\mu \ddot{w}}{k_1} \tag{5.2.6}
\]

\[
\rho c_p \left[ \ddot{u} \frac{\partial \ddot{T}}{\partial \ddot{r}} + \ddot{w} \frac{\partial \ddot{T}}{\partial \dot{z}} \right] = \frac{k}{\ddot{r}} \frac{\partial}{\partial \ddot{r}} \left( \ddot{r} \frac{\partial \ddot{T}}{\partial \ddot{r}} \right) + \left( \frac{\partial \ddot{w}}{\partial \ddot{r}} \right)^2 + \frac{\partial q_r}{\partial \dot{z}} + \frac{D_m K_T}{C_m} \left( \frac{\partial^2 \ddot{C}}{\partial \ddot{r}^2} + \frac{1}{\ddot{r}} \frac{\partial \ddot{C}}{\partial \ddot{r}} + \frac{\partial^2 \ddot{C}}{\partial \dot{z}^2} \right) \tag{5.2.7}
\]

\[
\left( \ddot{u} \frac{\partial \ddot{C}}{\partial \ddot{r}} + \ddot{w} \frac{\partial \ddot{C}}{\partial \dot{z}} \right) = D \left( \frac{\partial^2 \ddot{C}}{\partial \ddot{r}^2} + \frac{1}{\ddot{r}} \frac{\partial \ddot{C}}{\partial \ddot{r}} + \frac{\partial^2 \ddot{C}}{\partial \dot{z}^2} \right) + \frac{D K_T}{T_m} \left( \frac{\partial^2 \ddot{T}}{\partial \ddot{r}^2} + \frac{1}{\ddot{r}} \frac{\partial \ddot{T}}{\partial \ddot{r}} + \frac{\partial^2 \ddot{T}}{\partial \dot{z}^2} \right) \tag{5.2.8}
\]

The corresponding boundary conditions are given by:

\[
\frac{\partial \dot{w}}{\partial \dot{r}} = 0, \quad \frac{\partial \ddot{T}}{\partial \dot{r}} = 0, \quad \frac{\partial \ddot{C}}{\partial \dot{r}} = 0 \quad \text{at} \quad \dot{r} = 0
\]

\[
\ddot{w} = \ddot{T} = \ddot{C} = 0 \quad \text{at} \quad \ddot{r} = h(\ddot{z}) \tag{5.2.9}
\]

where \(d(\ddot{z})[1 - \eta(b^n - (\ddot{z} - a) - (\ddot{z} - a)^n)]\).

Above equations are converted into dimensionless form by introducing the following dimensionless parameters:
\[ r = \frac{r}{d_0}, \quad z = \frac{z}{b}, \quad u = \frac{b\bar{u}}{u_0\delta}, \quad w = \frac{\bar{w}}{u_0}, \quad Re = \frac{\rho b u_0}{\mu}, \quad M = \sigma_1 \mu_m H_0 \left( \frac{\sigma_1}{\mu} \right), \]

\[ K = \frac{k_1}{d_0}, \quad E_c = \frac{u_0^2}{c_p T_0}, \quad P_r = \frac{\mu c_p}{k}, \quad R = \frac{k^* k_{\infty}}{4\sigma^*_T^3}, \quad S_c = \frac{\mu}{\rho D_m}, \quad S_r = \frac{D_m K_F \bar{p}_T}{\mu T_m c_v}, \quad D_F = \frac{D_m K_F \bar{c}_0}{c_s c_p T_0}, \quad K_r = \frac{k_1 \bar{u}}{u_0}, \quad \theta = \frac{(\bar{T} - T_0)}{T_0}, \quad \sigma = \frac{\bar{c} - \bar{c}_0}{\bar{c}_0}\]

and by taking additional conditions (Mekheimer and El Kot, 2008)

\[ Re \frac{\delta^* n^{\frac{1}{n-1}}}{b} \ll 1 \]  
(5.2.10)

\[ \frac{d_0 n^{\frac{1}{n-1}}}{b} \approx O(1). \]  
(5.2.11)

For the case of mild stenosis \( \frac{\delta^*}{d_0} \ll 1 \), the dimensionless equations take the following form

\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \]  
(5.2.12)

\[ \frac{\partial p}{\partial r} = 0, \]  
(5.2.13)

\[ \frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \frac{\partial w}{\partial r} \right) \right) - (M^2 + \frac{1}{K}) w, \]  
(5.2.14)

\[ \left( 1 + \frac{4}{3K} \right) \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\partial p}{\partial r} \right) \right] + E_c Pr \left( \left( \frac{\partial w}{\partial r} \right)^2 \right) + D_F \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \sigma}{\partial r} \right) \right) = 0 \]  
(5.2.15)

\[ \frac{1}{S_c} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right) + S_r \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right) - K_r \sigma = 0, \]  
(5.2.16)

with boundary conditions
\[ \frac{\partial w}{\partial r} = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad \frac{\partial \sigma}{\partial r} = 0 \quad \text{at} \ r = 0 \]

\[ w = 0, \ \theta = 0, \ \sigma = 0 \ \text{at} \ r = h(z) \quad (5.2.17) \]

where \( h(z) = (1 + \varepsilon z)[1 - \eta_1((z - \vartheta) - (z - \vartheta)^n)] \),

\[ \vartheta \leq z \leq \vartheta + 1, \ \varepsilon = \tan(\varphi), \ \varphi \] is called tapered angle of artery and for non-tapered artery \( (\varphi = 0) \), the diverging tapering \( (\varphi > 0) \) converging tapering \( (\varphi < 0) \) (Mekheimer and El Kot, 2008), and

\[ \delta^* = \frac{\delta}{a}, \ \vartheta = \frac{a}{b}, \ \eta_1 = \frac{\delta n^2}{(n-1)}, \quad (5.2.18) \]

5.3 SOLUTION OF THE PROBLEM:

The governing dimensional equations are non-linear, coupled partial differential equation. To find the solution of above equations, Finite difference method is applied. First order derivatives are discretized by forward difference method and second order derivatives are discretized by central difference method. Variation in radial direction is taken by varying \( i \) and in axial direction by varying \( j \). Step size is chosen as 0.001. Following discretized equations are obtained.

\[
\left( \frac{\partial p}{\partial z} \right)_{i,j} = \frac{1}{r_{ij}} \frac{w_{i+1,j} - w_{i,j}}{\Delta r} + \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{(\Delta r)^2} - \left( M^2 + \frac{1}{K} \right) w_{i,j} 
\]

\[ (5.3.1) \]

\[
(1 + \frac{4}{3R}) \left( \frac{1}{r_{ij}} \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta r} + \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta r)^2} \right) + E_c \Pr \left( \frac{(w_{i+1,j} - w_{i,j})^2}{\Delta r} \right) +
\]

\[
D_f \left( \frac{1}{r_{ij}} \frac{\sigma_{i+1,j} - \sigma_{i,j}}{\Delta r} + \frac{\sigma_{i+1,j} - 2\sigma_{i,j} + \sigma_{i-1,j}}{(\Delta r)^2} \right) = 0 
\]

\[ (5.3.2) \]
\[
\frac{1}{S_c} \left( \frac{\sigma_{i+1,j} - \sigma_{i,j}}{\Delta r} + \frac{\sigma_{i+1,j} - 2\sigma_{i,j} + \sigma_{i-1,j}}{(\Delta r)^2} \right) + S_T \left( \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta r} \right) = 0
\]

These discretized equations are converted into algebraic equations by taking varying \(i\) from 1 to \(h(z)\) and \(j\) is varied from 0 to 1. Algebraic equations form tridiagonal matrix at every \(j\) step which are solved by Thomas Algorithm in MATLAB.

### 5.4 RESULTS AND DISCUSSION:

Blood flow characteristics are carried out for a specific set of values of the different physical parameters involved in the model analysis and are represented graphically. Various effects on Newtonian incompressible blood flow in presence of magnetic field in porous diverging tapered artery are observed. In order to get physical insight into the problem, the numerical calculations for the axial velocity, temperature and concentration profiles for different parameters have been carried out. Values chosen for numerical solutions are \(z = 0.5, n = 2, \vartheta = 0, \varepsilon = 0.005, \delta^* = 0.6\) and other dimensionless parameters are varied to understand blood flow behavior.

In order to verify the accuracy of present results, we have compared our results with earlier reported results for velocity considering Magnetic and Porous effects to be zero taking symmetric artery from \(r = -1\) to \(r = 1\). The comparisons are found to be in good agreement, as shown in Table 5.1.
### Table 5.1. Comparison of Results

<table>
<thead>
<tr>
<th>$r$</th>
<th>$w$ (Present results)</th>
<th>$w$ (Naddem et. al., 2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.8</td>
<td>0.302</td>
<td>0.303</td>
</tr>
<tr>
<td>-0.6</td>
<td>0.538</td>
<td>0.535</td>
</tr>
<tr>
<td>-0.4</td>
<td>0.700</td>
<td>0.703</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.809</td>
<td>0.802</td>
</tr>
<tr>
<td>0.2</td>
<td>0.809</td>
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<tr>
<td>1.0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**VELOCITY PROFILES:**

The velocity profile for different values of $M$, $K$ and $\varepsilon$ are shown in Figures (5.2-5.4). Velocity is found to decrease with increase in magnetic field, shown in Figure 5.2, as magnetic field introduces Lorentz force in electrically conducting fluid. This force acts against blood flow which reduces velocity with increase in magnetic field. These results are qualitatively agreed with the expectations; as magnetic field exerts retarding force on the natural convection flow. It is also observed that
velocity increases with increase in $K$ as shown in Figure 5.3. Physically, the effect of porous medium on the boundary layer growth is significant due to the increase in the thickness of the thermal boundary layer. As anticipated, an increase in the permeability of the porous medium results to the increase in the flow of fluid through the artery. The resistance of the medium may be neglected when the holes (pores) of the porous medium become large. Variation due to change in tapering angle is shown in Figure 5.4, and it is seen that converging artery has higher axial velocity as compared to non-tapered and diverging artery.

![Fig. 5.2: Velocity Profile for Varying $M$](image-url)
Fig. 5.3: Velocity Profile for Varying $K$

Fig. 5.4: Velocity Profile for Varying $\varepsilon$

TEMPERATURE PROFILES:

Temperature profile for different values of $M, R, Pr, K, Df, Br(=Ec*Pr), Sr$ are shown in Figures (5.5 to 5.11). Figure 5.5 shows the change in temperature when $Br$ is varied. It is found that increase in $Br$ value increases temperature which also means there is increase in temperature with increase in Eckert number as $Br = Ec*Pr$. Variation of temperature with Dufour
number is shown in Fig. 5.6. It is found that increase in Dufour effect increases temperature. Fig. 5.7 shows the variation of temperature with $K$ and it is found that as $K$ increases temperature of blood increases. It is observed from Fig. 5.8 that temperature of blood in presence of stenosis decreases with increase in magnetic field. It is also observed that increase in Prandtl number increase the temperature of blood as shown in Fig. 5.9. It is also observed that temperature increases with increase in radiation parameter $R$ as shown in Fig. 5.10. The radiation acts as a heat source within the blood, therefore the arterial blood temperature (and hence also that along the centerline) should gradually increase with increasing radiation parameter (dosage). Fig. 5.11 shows variation of temperature with $Sr$. Changes are very small and temperature increases with increase in $Sr$.

![Temperature Profile for Varying $Br$](image)

**Fig. 5.5: Temperature Profile for Varying $Br$**
Fig. 5.6: Temperature Profile for Varying $Df$

Fig. 5.7: Temperature Profile for Varying $K$

Fig. 5.1: Temperature Profile for Varying $M$
Fig. 5.9: Temperature Profile for Varying $Pr$

Fig. 5.10: Temperature Profile for Varying $R$

Fig. 5.11: Temperature Profile for Varying $Sr$
CONCENTRATION PROFILES:

Concentration profile for different values of $Sc$, $Sr$, $Br$, $M$ and $Kr$ are shown in Figures (5.12-5.16). It is analyzed that with an increase in Schmidt number ($Sc$), Brickmann number ($Br$), and Soret number ($Sr$) concentration profile decreases, while, reverse effect is noted for magnetic field ($M$) and chemical reaction parameter $Kr$. The concentration decreases as the Schmidt number increases. This is due to the fact that the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

![Fig. 5.12: Concentration Profile for Varying $Br$](image)

![Fig. 5.13. Concentration Profile for Varying $Kr$](image)
Fig. 5.14: Concentration Profile for Varying $M$

Fig. 5.15: Concentration Profile for Varying $Sc$

Fig. 5.16: Concentration Profile for Varying $Sr$
5.5 CONCLUSIONS:

MHD blood flow through a stenosed artery with Soret and Dufour effects in the presence of thermal radiation has been studied. The resulting system of equations were solved numerically using explicit finite difference scheme. In order to determine the accuracy and validate the present methodology, results are compared with the available results existing in the literature. The effects of the thermal radiation, the magnetic field, Soret and Dufour parameters on the stenosis geometry have been found throughout the region and the results obtained are of medical interest and clinical applications. The study shows that, for patients having thermal radiation therapy, the resistance of blood flow due to stenosis and magnetic field is reduced by increasing thermal radiation absorption.

- The temperature profile is increased with increasing Radiation parameter & Dufour number (0.12%)
- It is observed that presence of stenosis greatly effects the flow of blood in artery.
- Magnetic field has significant effect on blood flow in artery.
- The concentration profile is decreased with increases in Dufour number (or decrease in Soret number).
- Variation due to change in tapering, it is seen that converging artery has higher axial velocity as compared to non-tapered and diverging.