CHAPTER 4

EFFECT OF VARIABLE VISCOSITY ON CHEMICALLY REACTING MAGNETO-BLOOD FLOW WITH HEAT AND MASS TRANSFER
4.1 INTRODUCTION:

The study of blood flow through different shape of arteries is of considerable importance in many cardiovascular diseases. The blood flow through an artery considering pulsatile flow in nature has drawn the attention of researchers for a long time due to its pronounced importance in medical science and technology. The study of bio-fluids like blood under the presence of applied magnetic field finds its applications in various upcoming fields like surgical operations, innovative drug targeting, etc. The occurrence of electromagnetic fields during such operations can have impacts on the human circulation system. Furthermore, apparent viscosity was found to be significantly affected by a magnetic field in human blood, which may cause a prospective human health implication in cases of magnetic field. Haik et al. (2001) described a 30% decrease in blood flow rate when subjected to a high magnetic field of 10 T, while, a similar reduction in blood flow rate but at a much smaller magnetic field of 0.002 T was showed by Yadav et al. (2008). Several authors (Das et al., 1994; Makinde and Osalusi, 2006; Misra et al., 2010) have also reported on heat transfer in bio-magneto fluid flows for bio-magnetic convective heat transfer over a stretching surface and bio-magnetic flow and heat transfer in a parallel-plate system. Sharma et al. (2013b) presented a mathematical model for the hydro-magnetic bio-fluid flow in the porous medium with Joule effect. Blood flow in a large blood vessel or artery has an intense influence on the efficiency of thermal therapy.
treatment. Electromagnetic heat, such as micro waves and short waves sends heat up to 2 inches into the muscles and tissues which works best for injuries in muscles, joints and tendons. Furthermore, hyperthermia treatment has been confirmed as effective tool during cancer therapy in recent years. The objective of hyperthermia treatment is to increase the temperature of pathological tissues overhead cytotoxic temperatures (41°C–45°C) without overexposing healthy tissues (Levin et al., 1994). Recently, heat and mass transfer of blood flow considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source discussed by Sharma et al. (2015a). A theoretical analysis of blood flow and heat transfer in a permeable vessel in the presence of an external magnetic field have been discussed by Sinha et al. (2016). Rahbari et al. (2017) discussed the analysis of blood flow with nanoparticles through porous blood vessels in the presence of applied magnetic field using Homotopy Perturbation Method (HPM). There are many important physiological problems those are concerned with the flow of chemically-reacting fluid mixtures. Many biological fluid organisms are examples of such type of mixtures. Since, blood is a complex mixture of plasma, cells, proteins and a variety of other chemicals that is modeled usually in a homogenized sense as a single component of fluid. Blood is balance by a variety of chemical reactions, some that aid its coagulation and others its dissolution. In the literature, the most of studies are based on the constant
physical properties of the fluid flow. However, it can be noted that the physical properties of the fluid may change significantly due to the change in temperature (Herwing et al., 1986). In most of the above mentioned studies, the fluid viscosity is assumed to be constant. However, for many liquids, such as oil, water and blood, the variation in viscosity due to change in temperature is more dominant than other effects on fluid flow. Therefore, it is highly appropriate to consider the effect of temperature-dependent viscosity in momentum and thermal transport processes (El Hakeem et al., 2003; El Hakeem et al., 2004; Ali et al., 2007; Eldabe et al., 2008). Pulsatile hydro-magnetic flow of Newtonian fluid through an inclined porous artery under effect of an external magnetic field with variable magnetic field discussed by Sharma et al. (2015b). Since, about 55% of blood is plasma, which contains 92% water. Therefore, it is necessary to take into account the temperature-dependent viscosity of the fluid to accurately predict the flow and heat transfer rates. However, the published work still lacks the analysis of the blood flow characteristics through an inclined artery with the applied magnetic field in the presence of temperature dependent viscosity. The present model has taken a care for this. Hence, based on the above detailed discussion, the purpose of the present work is to study the effects of variable viscosity on MHD heat and mass transfer blood flow in the presence of viscous dissipation.
4.2 MATHEMATICAL FORMULATION:

Considered the flow of blood in a straight channel of an artery by treating blood as a viscous, homogeneous, incompressible fluid. In the Cartesian coordinate system, the $x$-axis is assumed along the artery in the direction of flow and the $y$-axis is normal to it as shown in Figure below.

![Physical Model of the problem](image)

The pulsatile nature of blood is considered in the study. A constant magnetic field acts perpendicular to the artery. Thus, under above assumptions, the model is governed by the following system of differential equations:

\[
y = H_1 \; u = 0; \; T = T_2; \; C = C_2;
\]

\[
y = 0; \; u = l_z \frac{\delta u}{\delta y}; \; T = T_1 + \delta_T \frac{\delta T}{\delta y}; \; C = C_1 + \delta_C \frac{\delta C}{\delta y}
\]
Continuity Equation

\[ \frac{\partial v}{\partial y} = 0. \]  \hspace{1cm} (4.2.1)

Therefore, \( v = V_0 \) (a constant).

Linear Momentum Equation

\[ \frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \left( \frac{\mu_a}{\rho} \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0^2}{\rho} u \]  \hspace{1cm} (4.2.2)

Energy Equation

\[ \frac{\partial T}{\partial t} + V_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu_a}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p} u^2 \]  \hspace{1cm} (4.2.3)

Concentration Equation

\[ \frac{\partial C}{\partial t} + V_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K'_r (C - C_1) \]  \hspace{1cm} (4.2.4)

The corresponding boundary conditions on the horizontal artery surfaces are given by:

\( y = 0: \ u = 0 \); \( T = T_1 + \delta_T \frac{\partial T}{\partial y} \); \( C = C_1 + \delta_C \frac{\partial C}{\partial y} \)

\( y = H: \ u = 0 \); \( T = T_2 \); \( C = C_2 \)

where \( \mu_a \) is assumed to vary as an inverse linear function of temperature \( T \) (see Lai and Kulacki, 1990), as

\[ \frac{1}{\mu_a} = \frac{1}{\mu_\infty} \left[ 1 + \gamma (T - T_2) \right] = a (T - T_r), \]

where \( a = \gamma / \mu_\infty \) and \( T_r = T_2 - 1 / \gamma \). The values of \( a \) and \( T_r \) are constants and their values depend on the reference state and the thermal property of
the fluid, \( \gamma \). In general, \( a > 0 \) for liquids and \( a < 0 \) for gases. \( \theta_r \) is a constant which is defined by

\[
\theta_r = \frac{(T_r - T_2)}{(T_1 - T_2)} = -\frac{1}{\gamma(T_1 - T_2)}.
\]

It is worth mentioning that for \( \gamma \to 0 \), \( \mu_a = \mu_\infty \), a constant, and \( \theta_r \to \infty \). It is also important to note that \( \theta_r \) is negative for liquids and positive for gases.

\( V_o \) is wall transpiration velocity (\( V = V_o \) at the lower plate and \( V = -V_o \) at the upper plate), where \( u \) is the \( x \)-direction (longitudinal velocity), \( P \) is the hydrodynamic pressure, \( \rho \) is the density of the fluid, \( \tau \) is the dimensional time, \( \sigma \) is the electrical conductivity of the bio-fluid, \( B_0 \) is the transverse magnetic field strength, \( \alpha \) is thermal diffusivity, \( c_p \) is the specific heat capacity of the bio-fluid, \( T \) is the bio-fluid temperature, \( C \) is the species concentration, \( D \) is the mass diffusivity of the species, \( \partial P/\partial X \) denotes longitudinal pressure gradient.

Introducing the following non-dimensional parameters:

\[
U = \frac{u}{V_0}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad \tau = \frac{V_0}{H}t, \quad P^* = \frac{P}{\rho V_0^2},
\]

\[
\theta = \frac{T - T_2}{T_1 - T_2}, \quad Re = \frac{\rho H V_0}{\mu_a}, \quad N_m = \frac{\sigma B_0^2 H}{\rho V_0}, \quad Pr = \frac{\mu}{\rho \alpha}, \quad Ec = \frac{V_0^2}{c_p(T_1 - T_2)}, \quad S_c = \frac{HV_0}{D},
\]

\[
\Phi = \frac{C - C_2}{C_1 - C_2}, \quad d_1 = \frac{\delta C}{H}, \quad d_2 = \frac{\delta_T}{H}, \quad \gamma = \frac{\delta_1}{H}.
\]

Using the expressions of \( \theta = \frac{T - T_2}{T_1 - T_2} \) and \( \theta_r = \frac{(T_r - T_2)}{(T_1 - T_2)} \), \( \mu \) can be written as
\[ \mu = \mu_a \left(1 - \frac{\theta}{\theta_r}\right)^{-1}, \text{ where } \mu_a = \frac{1}{a \theta_r (T_1 - T_2)}. \]

Equations (4.2.2), (4.2.3) and (4.2.4) are reduced to the following non-dimensional form:

**Linear Momentum Equation**

\[
\left(1 - \frac{\theta}{\theta_r}\right)^2 \left( \frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial Y} + \frac{\partial P}{\partial X} \right) = \frac{1}{Re} \left(\left(1 - \frac{\theta}{\theta_r}\right) \frac{\partial^2 U}{\partial Y^2} + \frac{1}{\theta_r} \frac{\partial}{\partial Y} \frac{\partial U}{\partial Y} \right) - \left(1 - \frac{\theta}{\theta_r}\right)^2 \left(N_m U\right)
\]  

(4.2.5)

**Energy Equation**

\[
\left(1 - \frac{\theta}{\theta_r}\right) \left( \frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial Y} \right) = \frac{1}{Pr Re} \left(1 - \frac{\theta}{\theta_r}\right) \frac{\partial^2 \theta}{\partial Y^2} + \frac{Ec}{Re} \left(\frac{\partial U}{\partial Y}\right)^2 + N_m E_c \left(1 - \frac{\theta}{\theta_r}\right) U^2
\]

(4.2.6)

**Concentration Equation**

\[
\frac{\partial \Phi}{\partial \tau} + \frac{\partial \Phi}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 \Phi}{\partial Y^2} - K_r \Phi
\]

(4.2.7)

The transformed boundary conditions now become:

\[
Y = 0 : U = 0; \ \theta = 1 + d_2 \frac{\partial \theta}{\partial Y}; \ \Phi = 1 + d_1 \frac{\partial \Phi}{\partial Y}
\]

\[
Y = 1: U = 0; \ \theta = 0; \ \Phi = 0
\]

(4.2.8)

where \(X\) and \(Y\) are dimensionless coordinates parallel and transverse to the bio-fluid flow respectively, \(U\) is the transformed velocity component in the \(X\)-direction, \(P^*\) is the hydrodynamic pressure (* dropped for convenience in analysis), \(t\) is dimensionless time, \(\theta\) is dimensionless temperature, \(Re\) is a
transpiration Reynolds number, $N_m$ is the hydromagnetic, $P_r$ is the Prandtl number, $E_c$ is the Eckert Number and $S_c$ is the Schmidt number.

As the present problem of fluid flow is pulsatile in nature, therefore the pressure gradient component is decomposed into steady component and the oscillatory component, respectively, as follows:

\[-\frac{\partial P}{\partial X} = \left(\frac{\partial P}{\partial X}\right)_s + \left(\frac{\partial P}{\partial X}\right)_o e^{i\omega t}\] (4.2.9)

To solve the above coupled equations (4.2.5, 4.2.6), the pressure term is redefined as:

\[-\frac{\partial P}{\partial X} = P_s + P_0 \cos(w \cdot t)\]

where $P_s$ and $P_0$ are static pressure component and oscillatory pressure component, respectively.

4.3 METHOD OF SOLUTION:

The governing dimensional equations are non-linear, coupled partial differential equation. In order to solve these equations (4.2.5) - (4.2.7) under the boundary conditions (4.2.8), finite difference scheme has been employed. The region of the flow field is divided into a grid or mesh of lines parallel to Y and t axes to obtain the difference equations. The difference equations at every internal nodal point on a particular $n$-level create a tri-diagonal system of algebraic equations. Such systems of algebraic equations are solved by Thomas algorithm as discussed by Carnahan et al. (1969) using MATLAB.
**4.4 RESULTS AND DISCUSSION:**

MHD bio-fluid flow with variable viscosity has been carried out in previous sections. Numerical calculations for the distribution of velocity, temperature and concentration for various dimensionless has been carried out in order to get physical insight into the problem.

Fig. 4.2 shows the velocity profile $U$ for different values of Reynolds number ($R_e$) at $t = 0.5$. Since, the Reynolds number includes transpiration velocity, hence increase in $R_e$ results in increase of velocity profile ($U$) and peaks attain at the mid of the channel.

![Velocity profile for different values of transpiration Reynolds numbers](image)

Fig.4.2: Velocity profile for different values of transpiration Reynolds numbers
Fig. 4.3: Velocity profile for different values of viscosity parameter

Fig. 4.4: Velocity profile for different values of hydromagnetic parameter
The effect of variable viscosity parameter on velocity profile is shown in Fig. 4.3. It is observed that an increase in viscosity parameter results a decrease in velocity distribution. The effect of hydro-magnetic parameter ($N_m$) is depicted in Fig. 4.4. It is observed that the velocity decreases with increasing $N_m$. This is due to the retarding forces (Lorentz forces) generated by the magnetic field as bio-fluid’s electrical properties have been already confirmed. The effect of velocity slip parameter is shown in Fig. 4.5. By increasing the slip at the wall the velocity increases which is in accordance to the existing results in literature. It is also observed that the velocity is lesser near beginning and ending of the wave but becomes larger in the central region. The velocity profile for different values of Prandtl number is...
shown in Fig. 4.6. It is noted from the figure that velocity decreases as $Pr$ increases.

Fig. 4.6: Velocity profile for different values of Prandtl number

Fig. 4.7: Temperature profile for different values of variable viscosity parameter
Fig. 4.8: Temperature profile for different values of Prandtl number

Fig. 4.9: Temperature profile for different values of thermal slip parameter
The influence of variable viscosity parameter is shown in Fig. 4.7. It is noticed that an increase in viscosity parameter results a decrease in temperature distribution. The effect of Prandtl number ($P_r$) on temperature distribution is shown in Fig. 4.8. The numerical results depict that the temperature decreases with increase in the Prandtl number. Since, $P_r$ denotes the ratio of momentum diffusivity to thermal diffusivity. So, larger $P_r$ fluids ($P_r > 1$) will diffuse momentum faster than heat. The effect of thermal slip parameter ($d_2$) on temperature distribution is shown in Fig. 4.9. It is noted that the temperature profile decreases with increasing thermal slip parameter.

![Concentration profile for different values of chemical reaction parameter (K_r) for d_1=0](image)

Fig. 4.10: Concentration profile for different values of chemical reaction parameter ($K_r$) for $d_1=0$
Fig. 4.11: Concentration profile for different values of Schmidt numbers ($Sc$) for $d_1=0$

The concentration profiles for different values of the chemical reaction parameter $K_r$ is plotted in Fig. 4.10. It is noticed that as increasing values of $K_r$, the concentration distribution across the boundary layer decreases. This demonstrate that diffusion rate can be extremely altered by chemical reaction parameter. Fig. 4.11 shows that dispersion depth of concentration reduces for increasing values of Schmidt number $Sc$. Furthermore, the absolute maximum of $\phi$ is slanted towards the wall with increasing $Sc$. As a result, the concentration profile at the surface decreases with increasing the Schmidt number.
4.5 CONCLUSIONS:

In this chapter, the effects of variable viscosity on heat and mass transfer blood flow in the presence of viscous dissipation has been investigated under the influence of MHD. The erythrocyte slip at the vessel wall and the thermal slip have been duly accounted for. All these considerations have made the study quite close to a real situation. The resulting coupled partial differential equations are solved numerically with the help of MATLAB. The main findings are as follows:

(i) Increasing viscosity parameter serves to reduce flow velocities across the artery.

(ii) By the application of sufficiently strong magnetic field, blood velocity can be diminished. The results presented should be of significant interest to surgeons who usually want to keep the blood flow rate at a desired level during the entire surgical procedure.

(iii) Increasing slip parameter increases the flow velocity across the channel (artery) width, while reverse effect is noticed for temperature distribution with thermal slip parameter.

(iv) Increasing the Prandtl number decreases the temperature profile in the channel i.e. by increasing the Prandtl number $Pr$, it is possible to bring about a reduction in the thermal boundary layer thickness.

(v) Increasing chemical reaction parameter decreases concentration profile.
(vi) Increasing the Schmidt number ($S_c$) decreases concentration profile up to centre of channel, while, reverse effect is observed after that.

Since the study takes into account velocity-slip and also since blood velocity in micro-vessels is low, it bears the potential to furnish some additional information regarding the causes and development of arterial diseases, such as atherosclerosis. Further, the present study will be useful in assessing the accuracy of future experimental/theoretical studies of more complex nature, that may involve greater number of physical parameters.