CHAPTER 5

Simulated Characteristics of a Ge$_{1-x}$Sn$_x$/Ge Multiple Quantum Well Heterojunction Phototransistor with Ge$_{1-x}$Sn$_x$ Alloy as Base

5.1 Introduction

Silicon (Si) based photodetectors (Masini et al., 2001) and optical sensors (Valette et al., 1990) find widespread applications in different areas (Chuang, 1995). However, Si-based photodetectors cannot be used in important telecommunication windows around 1.33 and 1.55 µm or in the C and L bands due to incompatibility of its band gap (Basu, 1997). Though Ge has no such limitation, the large lattice mismatch between Ge and Si requires that the thickness of Ge layers grown directly on Si should be below the critical layer thickness in order to avoid misfit dislocations (Ashu & Matthai, 1991). Moreover, due to the indirect nature of the band gap of Ge and Si, their absorption coefficients are quite low (Deen and Basu, 2012).

The successful growth of group IV semiconductor alloys like GeSn (Lieten et al., 2013) and SiGeSn (Bauer et al., 2003) on Si substrate has changed the scenario during the past decade (Moontragoon et al., 2012). It is now possible to grow strain-free layers of the alloy on virtual substrates and to achieve direct band gap alloy materials leading to large absorption coefficients (D’Costa et al., 2009). In general, the absorption coefficient ($\alpha$) of a material can be calculated using $\alpha = 4\pi k/\lambda$, where, $k$ is the extinction coefficient and $\lambda$ is the wavelength of incident light. Some research works have examined the use of GeSn alloy as a photodetector material in the C and L bands and detectors working in the photoconductive mode (Roucka et al., 2008) as well as in p-i-n configuration have been fabricated (M. Oehme et al., 2012; Su et al., 2011). p-i-n photodiodes (M. Oehme et al., 2012; D. Zhang et al., 2013) and avalanche photodiodes (APDs) (Yuan Dong et al., 2015) are widely used as optical detectors in the telecommunication systems (Deen and Basu, 2012), operating at wavelengths 1.33 µm and 1.55 µm (T. Lee et al., 1981). The APD has internal gain but possesses excess noise (Hakim et al., 1990), which degrades the signal-to-noise ratio (SNR). Heterojunction phototransistors (HPTs) have inherent optical gain like conventional
APDs but are free from excess noise (Scavennec et al., 1983), and in addition, their growth is compatible with existing photodetector technology (Sorianello et al., 2015). Because of these advantages, a number of researchers (Helme & Houston, 2007; Park & Jang, 2010; Refaat et al., 2006) considered HPTs as a useful alternative to APDs. Studies on HPTs have taken into consideration of GaAs and AlGaAs system (Khan et al., 2011), other III-V compounds and their alloys (Park & Jang, 2010; Tan et al., 2005) as well as alloys of group IV semiconductors (W. Wang et al., 2017).

Since, GeSn alloys grown on a Si platform exhibit high absorption coefficients (D’Costa et al., 2009), it seems worthwhile to examine the potential of HPTs based on GeSn for use in C and L bands of the optical communication window. Proper alloying of GeSn can increase absorption up to 10 and 20 times than pure Ge at operating wavelength of 1.55 µm (C-band) and 1.620 µm (L-band), respectively (D’Costa et al., 2009). In a literature work (Basu et al., 2014) the terminal currents, photo generated currents, current and optical gains and responsivity of Ge/GeSn/Ge HPTs was reported at 1.55 µm and compared with the available theoretical and experimental values for InGaAs/InP HPTs reported earlier (Frimel & Roenker, 1997b, 1997a). It was found that the performance of HPTs with GeSn base was almost comparable with HPTs having InGaAs base for lower base doping and base thickness. In a more recent work (G. E. Chang et al., 2016), it has been demonstrated using the refined band structure (calculated by $k \cdot p$ perturbation) that GeSn based HPTs can cover the C band as well as wavelengths up to 2.2 µm. Further, GeSn based HPT has also been demonstrated experimentally with photodetection beyond 2003 nm (W. Wang et al., 2016). In continuation with this work, it was reported that approximately 10 times enhancement in optical response was achieved in comparison to conventional GeSn based photodetectors (W. Wang et al., 2017).

For enhanced modulation and emission properties, multiple quantum well (MQW) structures are the favorable one now a days due to higher carrier and optical confinement (Huang et al., 2017). In quantum well (QW) heterostructures a narrow band gap semiconductor material (well) is sandwiched between two higher band gap material (barrier). Application of electric field to the QW substantially changes the optical absorption spectrum which are near to the band gap energy (Dutta, 1993). Because of these properties, they are being used in various heterostructure based photonic devices such as, heterojunction phototransistors, p-i-n photodetectors, lasers, light emitting diodes etc. (Dou et al., 2016; Ghetmiri et al., 2017; Huang et al., 2017;
M Oehme et al., 2014). HPT with MQW based on Si-GeSi (Pei et al., 2003; Y. Zhang et al., 2008) and InAs/InGaAs (Fukano et al., 2007) material systems have been reported for application in IR spectral region. MQW HPT structure incorporating SiGeSn (barrier)/GeSn(well) has been reported recently (Chakraborty et al., 2017). Application of MQW structure in p-i-n photodetector having Ge(barrier)/GeSn(well) has been demonstrated experimentally (Huang et al., 2017). Till now, no other research work has been reported on the application of MQW in GeSn based HPT.

The use of MQW structures as the active layer can enhance the absorption coefficient by quantum-confinement effects, therefore further enhancing the photoresponses. In this chapter, simulation and analysis of three-terminal (3T) Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge multiple-quantum-well (MQW) heterojunction phototransistors (HPTs) is carried out. The intended photodetection wavelength range is short-wave infrared (SWIR) spectral region i.e., 1100-2500 nm. The active layer incorporates Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge MQW structures between the base-collector junctions pseudomorphically grown on Ge-on-Si substrates, providing compatibility with CMOS platforms for low-cost and scalable manufacturing technology. Introducing Sn into the Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge MQW structures can effectively reduce the direct bandgap, thereby considerably extended the photodetection range in the SWIR spectral range with improved absorption efficiencies. The current gain in terms of barrier/well thicknesses, number of periods of the MQW structures, doping concentrations of the layers for the HPT structures are calculated. The calculation results show that absorption coefficient can be considerably enhanced by increasing the Sn content in the Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge MQW active layer, and the current gain can be considerably enhanced by optimizing the HPT structures. With the enhanced absorption coefficient and current gain, high-responsivity photodetection can be achieved in the SWIR spectral range, making this device very promising for a wide range of SWIR photodetection applications.

5.2 Device Structure and Layer Parameters

Figure 5.1(a) shows 3-terminal n-p-n Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge MQW HPT device structure. A 400-nm-thick, fully strain-relaxed Ge buffer is adopted as the VS to reduce defects during fabrication on Si substrate. A 200-nm-thick n\textsuperscript{+} sub-collector is then grown. A 200-nm-thick Ge\textsubscript{1-x}Sn\textsubscript{x} layer is then used as the collector. Subsequently, Ge\textsubscript{1-x}Sn\textsubscript{x}/Ge MQW with N periods are then deposited as the active layer for photon absorption. A 50-nm-
thick Ge$_{1-x}$Sn$_x$ layer is chosen as the base, followed by an n-type Ge emitter. The material compositions, conductivity types, thicknesses, and doping concentrations of the various layers are given in table 5.1. (The thickness of $i^{th}$ layer is denoted by $t_i$ and the doping concentration is denoted by $N_i$ ($P_i$) for n-type (p-type) material, and the subscripts for the emitter, base, well, barrier, collector, sub-collector are denoted by e, b, w, br, c, sc, respectively.) Since the thicknesses of the layers are small, here the entire HPT stack is assumed to be pseudomorphic to the underlying Ge VS. In this way, the Ge layers are strain-free and the Ge$_{1-x}$Sn$_x$ layers are subjected a compressive strain of $\varepsilon = -0.143 x$. The device structure is designed as a normal-incidence HPT with a double-mesa structure with a SiO$_2$ antireflection and passivation layer in which light is incident on the base. The simulated band diagram is schematically shown in figure 5.1(b). The base thickness and doping concentration have an important role in maintaining reasonable emitter injection efficiency. In this model, to improve base transport factor, a base thickness of 50 nm is considered with a medium base doping. Three different values of Sn concentration (x) 3%, 6% and 9% are considered in device analysis and simulations.
Figure 5.1 (a) Device structure used in simulation study (b) simulated band-diagram for device

Table 5.1: Layers thickness and doping concentrations of the various layers

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Thickness ( t ) (nm)</th>
<th>Doping Concentration (cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Emitter</td>
<td>n-doped Ge</td>
<td>100 nm</td>
<td>( 2 \times 10^{17} )</td>
</tr>
<tr>
<td>2 Base</td>
<td>p-doped Ge(_{1-x})Sn(_x)</td>
<td>50 nm</td>
<td>( 8 \times 10^{17} )</td>
</tr>
<tr>
<td>3 Multiple Quantum Well</td>
<td>Ge(_{1-x})Sn(_x) (well)</td>
<td>10 nm</td>
<td>Undoped</td>
</tr>
<tr>
<td></td>
<td>Ge (barrier)</td>
<td>15 nm</td>
<td></td>
</tr>
<tr>
<td>4 Collector</td>
<td>n-doped Ge(_{1-x})Sn(_x)</td>
<td>200 nm</td>
<td>( 2 \times 10^{16} )</td>
</tr>
<tr>
<td>5 Sub collector</td>
<td>n-Ge</td>
<td>200 nm</td>
<td>( 5 \times 10^{17} )</td>
</tr>
<tr>
<td>6 Virtual Substrate</td>
<td>Ge</td>
<td>400 nm</td>
<td>-</td>
</tr>
<tr>
<td>7 Substrate</td>
<td>Si</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The principle of operation of the device in brief is as follows. Electron-hole-pairs (EHPs) are first created in the base and absorption regions of MQW on application of incident light. Initial photocurrent is generated due to electrons, which are swept across the collector region. Further, increasing forward bias increases the electron flow from emitter to collector. This process contributes towards increase in collected photocurrent, which in turn enhances the quantum efficiency. Figure 5.2 shows the flow chart of device simulation using COMSOL Multiphysics.
5.3 Theory

5.3.1 Current Gain

The base-emitter junction is forward biased and base-collector junction is reverse biased for gain calculation of present n-p-n HPT structure. This biasing allows flow of large electron current from emitter to collector region, which significantly enhances the photocurrent at collector region with improvement in external quantum efficiency. In present study, two-dimensional (2D) six-layered structure of HPT, which contains a MQW having 3-periods of QWs, is used for simulation. Ge$_{1-x}$Sn$_x$ alloy as collector and base region along with different periods of MQW (Ge/Ge$_{1-x}$Sn$_x$ (barrier/well)) are modelled. Light signal with optical power of 1µW is assumed to incident normally on base, as indicated in figure 5.1(a).
Finite element method (FEM) using COMSOL Multiphysics is used in this work to evaluate the current gain of the device. Recombination mechanisms, such as, trap assisted and Auger recombinations (Shockley-Read-Hall domain trapping model), are included in the analysis. Some of the equations are as follows:

\[
\frac{1}{q} \nabla \cdot J_n = -U_n \tag{5.1(a)}
\]

\[
\frac{1}{q} \nabla \cdot J_p = U_p \tag{5.1(b)}
\]

where, \( U_n \) and \( U_p \) are the generation and/or recombination rates, \( J_n \) and \( J_p \) are the current densities. Correspondingly, the electron and hole concentrations can be solved from,

\[
J_n = \left[ qn\mu_n \nabla E_c + qn \cdot D_n \nabla \ln(T_I) \right] + \mu_n k_B T_I \left( \frac{n}{N_c} \right) \nabla n \tag{5.2(a)}
\]

\[
J_p = \left[ qp\mu_p \nabla E_v - qp \cdot D_p \nabla \ln(T_I) \right] - \mu_p k_B T_I \left( \frac{p}{N_v} \right) \nabla p \tag{5.2(b)}
\]

where, \( n \) and \( p \) are the electron and hole concentrations, respectively, \( \mu_n \) and \( \mu_p \) are mobilities of electrons and holes respectively, \( D \) is diffusion constant, \( N_c \) and \( N_v \) are density-of-states for electrons in the conduction band and holes in the valence band, respectively, \( k_B \) is Boltzmann constant, \( T_I \) is the operating temperature, and \( \nabla \ln(T_I) \) denotes small change in logarithmic value of initial temperature \( T_I \) during conduction process (flow of charge carriers). Here, \( G = F_{G/2}(\eta) - F_{-G/2}(\eta) \), with \( F_j(\eta) \) representing the Fermi-Dirac integral (Chuang, 1995). After the current components are calculated, the total current can be obtained via \( J_{\text{total}} = J_n + J_p \). The current gain can be determined from the ratio of collector current (\( I_c \)) to the base current (\( I_b \)).

\[
\beta = \frac{I_c}{I_b} \tag{5.3}
\]

5.3.2 Strained Electronic Band Structures

The energy bands of the strained Ge_{1-x}Sn_{x}/Ge heterostructures are lined up using model-solid theory and deformation potential theory (Chuang, 1995; Walle, 1989). The unstrained direct bandgap energy of GeSn can be calculated as follows,

\[
E_g^{T}(\text{Ge}_{1-x}\text{Sn}_x) = (1-x)E_g^{T}(\text{Ge}) + xE_g^{T}(\text{Sn}) - x(1-x)b_t \tag{5.4}
\]
where $E_c^\Gamma(\text{Ge}) = 0.8\text{eV}$ and $E_g^\Gamma(\text{Sn}) = -0.413\text{eV}$ are the direct band energies of Ge and Sn, respectively, $b_t = 2.42\text{ eV}$ is the bowing parameter (Huang et al., 2017). The calculated composition-dependent band edges (in units of eV) of the various bands for pseudomorphic Ge$_{1-x}$Sn$_x$ on Ge, relative to the valence band maximum of bulk Ge, can be expressed as follows:

$$E_c^\Gamma(\text{Ge}_{1-x}\text{Sn}_x) = 0.8 - 1.395x + 1.721x^2$$

(5.5)

$$E_{hh}^\Gamma(\text{Ge}_{1-x}\text{Sn}_x) = 1.337x + 0.049x^2$$

(5.6)

$$E_{lh}^\Gamma(\text{Ge}_{1-x}\text{Sn}_x) = 0.071x + 2.185x^2$$

(5.7)

$$E_{so}^\Gamma(\text{Ge}_{1-x}\text{Sn}_x) = -0.2885 + 0.031x - 2.20x^2$$

(5.8)

where $E_c^\Gamma$, $E_{hh}^\Gamma$, $E_{lh}^\Gamma$, and $E_{so}^\Gamma$ denotes the $\Gamma$-conduction band, heavy-hole (HH) band, light-hole (LH) band, and spin-orbital (SO) band, respectively. Figure 5.3 shows Sn-composition-dependent band edges of the various bands for pseudomorphic Ge$_{1-x}$Sn$_x$ on Ge. As the Sn content increases, the $\Gamma$-conduction band edge shifts to lower energies due to large bowing effect and negative bandgap (effective value) of $\alpha$-Sn (Chang et al., 2016). For the valence band, the HH and LH bands shift to higher energies because compressive strained are induced in the layer on increasing Sn concentration while the SO band shifts to lower energies. These results suggest that the bandgap energy is reduced with increase in Sn content. In addition, the HH band is pushed above the LH band, and their energy separation increases with increase in Sn content. As a result, the top valence band is HH-like. After various bands are lined up, the band structures for pseudomorphic Ge$_{1-x}$Sn$_x$ on Ge can be calculated using a multi-band $k \cdot p$ method to obtain the dispersion relations for electrons in the $\Gamma$-conduction band ($E_c^\Gamma(k)$), and the holes in the $m^{th}$ valence band (Chang et al., 2016). On the other hand, for the pseudomorphic Ge$_{1-x}$Sn$_x$/Ge MQW structures, the sub-band energy and wave functions can be calculated using effective mass theory under the axial approximation.
5.3.3 Optical Absorption Coefficient

Having the calculated band structures, the direct-band gap inter-band absorption coefficient for the Ge$_{1-x}$Sn$_x$ base and collector fully-strained to the Ge VS, taking into account a Lorentzian lineshape, can be calculated as follows (Chang et al., 2016; Chuang, 1995):

$$\alpha_{\text{bulk}}(\hbar \omega) = \frac{\pi \hbar e^2}{n_c \epsilon_0 c m_0^* \hbar \omega} \sum_a \frac{2d_k}{(2\pi)^2} |e \cdot p_{\alpha}|^2 \frac{\gamma/(2\pi)}{[E_{\alpha,\Gamma}(k) - E_{\alpha}(k) - \hbar \omega]^2 + \left(\gamma/2\right)^2}$$  \hfill (5.9)

where $n_c$ is the background refractive index, $c$ is the speed of light in free space, $\epsilon_0$ is the permittivity of free space, $\hbar$ is the Planck constant, $e$ is the elementary charge, $m_0$ is the free electron mass, $|e \cdot p_{\alpha}|^2 = m_0 E_p^2 / 6$ is the moment matrix element with $E_p$ being the optical energy, $\gamma$ is the full-width-at-half-maximum (FWHM) of the Lorentzian line shape, and the summation $m$ is carried out to consider for all direct inter-band transition from the valence band to the $\Gamma$-conduction band.

For the Ge$_{1-x}$Sn$_x$/Ge MQW, the direct-band inter-band absorption coefficient can be calculated with the calculated sub-band energies and wave functions. Both discrete and continuous-states contribute to the direct-gap absorption. The continuous-states absorption coefficient under the axial approximation, taking account the Sommerfeld enhancement factor $S(k_\Gamma)$ and the Lorentzian broadening function, can be calculated as follows (Chang & Chang, 2012; Chuang, 1995):
\[ \alpha_\omega(h\omega) = \frac{\pi e^2}{n_c e_0 \varepsilon_m^2 \omega} \frac{1}{k_w} \sum_{\nu=1}^{N\nu} \sum_{m} \int \frac{k d k}{2\pi} S(k) M_{\nu m}(k) \frac{\gamma / (2\pi)}{[E_\nu(k) - E_m^\nu(k) - h\omega]^2 + (\gamma / 2)^2} \] (5.10)

\[ S(k) = \frac{\kappa_0}{1 + e^{-\kappa_0/k_w}} \] (5.11)

where \( E_x = \hbar^2 k_x^2 / (2m_x) \) is the reduced effective mass, \( R_x \) is the exciton Rydberg energy, \( k_x \) is the magnitude of the wave vector parallel the plane of MQWs, \( \sigma = \nu U(L) \) refers to the upper (lower) block of the \( J=4 \) Luttinger-Kohn Hamiltonian for the valence band, \( E_n^\nu \) is the \( n^{th} \) electron subband energy in the \( \Gamma \)-conduction band, \( E_m^\nu \) is the \( m^{th} \) hole subband energy in the valence band, and \( M_{\nu m} \) is the average squared moment matrix element. For the discrete-states contributions, considering only the 1s-excitons, the discrete-states direct-gap absorption coefficient can be calculated as follows (Chang & Chang, 2012; Chuang, 1995):

\[ \alpha_{ex}(h\omega) = \frac{\pi e^2}{n_c e_0 \varepsilon_m^2 \omega \tau_e} \left[ \frac{1}{2\pi} \sum_{\nu=1}^{N\nu} \sum_{m} \frac{1}{\lambda^2} [\vec{e} \cdot \vec{p}_m]^2 |I_{\nu m}|^2 \frac{\gamma / (2\pi)}{[E_x - \hbar\omega]^2 + (\gamma / 2)^2} \right] \] (5.12)

where \( E_z = E_n^\nu - E_m^\nu + E_{ex} \) is the exciton transition energy, \( E_{ex} \) is the exciton binding energy, \( \Lambda \) is a variation parameter, \( |\vec{e} \cdot \vec{p}_m|^2 \) is the squared moment matrix and it is value for TE-polarization is \( \frac{3}{2} M_b^z \left( \frac{1}{2} M_b^z \right) \) for HH (LH) excitons. The variation parameter \( \Lambda \) and the exciton binding energy \( E_{ex} \) are calculated using the variation method. After the discrete-states and continuous-states absorption are obtained, the total absorption coefficient can be obtained as \( \alpha_{tot}(h\omega) = \alpha_\omega(h\omega) + \alpha_{ex}(h\omega) \).

### 5.3.4 Responsivity

Having the absorption coefficients of the layers and current gain of the HPT structure, the responsivity of the HPT structure can be calculated as follows (Chang et al., 2016):

\[ R(h\omega) = \frac{e\beta(1-R_b)}{h\omega} \left[ \left( 1 - e^{-\eta_b} \right) + e^{-\eta_b} \left[ 1 - e^{-(X+1)\varepsilon_b-N_{a_h}L_w} \right] + e^{-\eta_b} \times e^{-(X+1)\varepsilon_b-N_{a_h}L_w} \left( 1 - e^{-\eta_b} \right) \right. \\
\left. + e^{-\eta_b} \times e^{-(X+1)\varepsilon_b-N_{a_h}L_w} \times e^{-\eta_b} \times \left\{ 1 - e^{-\varepsilon_b L_w} \right\} \right] \] (5.13)
Here, $e$ is the elementary charge; $\hbar \omega$ is the photon energy; $\alpha_b$, $\alpha_{br}$, $\alpha_w$, $\alpha_c$ and $\alpha_{sc}$ are the absorption coefficients of the base, barrier, well, collector, and sub-collector, layers respectively. $t_b$, $t_{br}$, $t_w$, $t_c$ and $t_{sc}$ are the thicknesses of the base, barrier, well, collector, and sub-collector, respectively. $L_{sc}^P$ is the hole diffusion length for sub-collector (Ge), and $R_B$ is the reflectivity of the entire HPT structure. $R_B$ can be calculated numerically using the transfer matrix method (Chuang, 1995) where the wavelength-dependent refractive-indices of the materials are considered (H. Tran et al., 2016).

5.4 Results and Discussions

Figure 5.4 shows the absorption spectra for pseudomorphic Ge$_{1-x}$Sn$_x$ on Ge with different Sn contents. Generally, for Ge ($x=0\%$), the absorption coefficient is small at low photon energies (Chang et al., 2016), and then considerably increases at certain photon energy, corresponding to the direct bandgap energy. With an increase in Sn content, the absorption edge shifts to longer wavelengths because of the bandgap shrinkage. Therefore, the absorption coefficient in the SWIR region is considerably enhanced by increasing the Sn content in the material.

![Figure 5.4 Absorption coefficient variation with incident photon energy/wavelength for pseudomorphic Ge$_{1-x}$Sn$_x$ on Ge](image)

In order to compare the gain of various structure, figure 5.5 represents gain variation with applied base emitter voltage for same doping concentration of all regions. 3\% Sn concentration is considered in Ge$_{1-x}$Sn$_x$. For bulk structure without quantum well, minimum current gain is obtained. As, quantum well structure enhances the light
absorption at the base, a significantly high and stable current gain is obtained in case of MQW HPT structure as compared with that of structure without quantum well. Inset of figure 5.5 shows difference in current gain values for single and MQW structures.

Figure 5.5 Current gain (β) variation (for Sn=3%) with applied base emitter voltage for different HPT structures (bulk, single QW and MQW of 3 periods) with P_in=1 µW at λ=1.55 µm and constant collector emitter voltage (V_CE=1 V)

Recombination at space charge region and surface recombination at the exposed layers (emitter and/or base) can be easily studied by Gummel plot (Sridhara et al., 1998). This plot depicts the variation of collector (I_C) and base (I_B) currents with base emitter voltage (V_BE). As, the work is focused on low biasing voltages, the value of V_BE is considered up to 0.7 V. For Gummel plot, fixed values of collector emitter voltage (V_CE=1 V) and base collector voltage (V_BC=0 V) are taken. Gummel plot for HPTs having 3 and 6% Sn concentration (GeSn base) is shown in figure 5.6 (a) and (b), respectively. These are the cases for 3% and 6% Sn concentration in GeSn base, collector and well regions with applied incident power of 1 µW. The sudden reversal in base current at around 0.145 V and 0.147 V for 3% and 6% Sn concentration in GeSn base, respectively can be explained on the basis of negative base current generated (excess holes) due to optical absorption at the base-collector region (Sridhara et al., 1998). Increasing the applied V_BE dominate the electrical current over photo-generated base current. Thus, a peak in gain plot is observed which is shown in figure 5.7(b), which shows variation of current gain with applied V_BE. The dip in base current will be shifted towards higher V_BE by increasing the applied optical power as
shown in figure 5.6(c), the shift has been experimentally demonstrated in case of InGaP/GaAs HPT (C. Wu et al., 2015).

Figure 5.6 Variation of (a) base and collector currents with applied base emitter voltage for 3% Sn and (b) base and collector currents with applied base emitter voltage for 6% Sn composition in GeSn (c) base (I_B) currents with V_BE for different optical power of incident light for MQW HPT with 3% Sn concentration in GeSn.
Collector current increases on increasing the Sn concentration as shown in figure 5.7(a). On increasing $V_{BE}$, $I_C$ increases and follows a saturation level from $V_{BE} = 0.5$ V. As observed from the figure 5.7(b), there is increase in gain on increasing Sn concentration from 3% to 6%. Further increasing Sn concentration up to 9% slightly decreases the current gain for initially assumed parameters. This reduction in current gain is attributed to the significantly increased base current. As the direct band gap property of GeSn alloy can be seen when $x \geq 0.08$, thus preferred Sn concentration would be between 6% and 10% for stable performance of the proposed structure.

Figure 5.7 Variation of (a) collector currents with base emitter voltage for MQW HPTs with 3%, 6% and 9% Sn concentration in GeSn (b) Current gain ($\beta$) variation with applied base emitter voltage for Ge$_{1-x}$Sn$_x$/Ge MQW HPTs with different Sn concentration in GeSn
5.4.1 Effects of Base and Subcollector Doping Concentrations on Current Gain

In this section, the effect of base and subcollector doping concentrations on current gain are discussed for different Sn concentrations considering three periods of QWs at the base. Taking all the parameters constant (as in table 5.1) with fixed base thickness of 50 nm, the variation of current gain with base doping concentration (from $P_b = 5 \times 10^{17} \text{ cm}^{-3}$ to $1 \times 10^{19} \text{ cm}^{-3}$) is shown in figure 5.8(a). Figure 5.8(a) shows that there is a sudden decrease in current gain on increasing the base doping concentration for 3% Sn concentration. There is also a gradual decrease in the current gain in case of 6% of Sn concentration. For high Sn concentration (9%) there is a slight decrease in the current gain. In heavily doped base, impurity scattering causes reduction in hole minority diffusion length and carrier lifetime (Conklin et al., 1995). High base resistance will be developed for low base doping which can affect oscillation frequency. Therefore, $P_b = 5 \times 10^{18} \text{ cm}^{-3}$ is chosen for optimizing the current gain of the HPT structure.

Keeping all the parameters same as in table 5.1, the variation of current gain with subcollector doping level is shown in figure 5.8(b). It is evident that there is no such significant increase in the current gain on increasing subcollector doping. This observation is attributed to the fact that the current gain is dominated by emitter injection efficiency and base transmission efficiency, and thus less dependent on the subcollector. As a result, the effect of subcollector is almost negligible in current gain and quantum efficiency. However, a low doping concentration in subcollector has a positive effect on breakdown voltage of subcollector region and on frequency performance of the device (Y. Zhang et al., 2008). Therefore, $N_{sc} = 5 \times 10^{17} \text{ cm}^{-3}$ is chosen for optimizing the current gain of the HPT structures. In addition, the collector and sub-collector regions can have a thickness of 120 nm each to make layer relaxation less likely without lowering the current gain.
4.2 Effects of Well and Barrier Thickness on Current Gain

In this section the effect of well and barrier layer width on current gain is discussed. For low Sn concentration (3%), current gain is nearly independent of GeSn well layer thickness as depicted in figure 5.9(a). On increasing Sn concentration to 6%, there is a gradual decrease in the current gain. Higher Sn concentration causes much decrease in the current gain value. As shown in figure 5.9(b), current gain is nearly independent of Ge barrier layer thickness. For both the conditions, other parameters are taken as shown in table 5.1 with three periods of MQW.
5.4.3 Effect of Multiple Periods (N) of QW on Current Gain

Current gain dependency on periods of QWs is shown in figure 5.10. The observation is that, there is increase in the value of current gain from three to five periods of MQW. This increase in current gain is attributed to the increased photon absorbing path that increases the collector current. However, further increase in number of QWs leads to a decrease in current gain because the base current increases more significantly than collector current. It is also interesting to note that for periods greater than three, there is increase in current gain with increasing the Sn concentration. This is because of the enhanced absorption coefficient that increases the collector current. Thus, a five period MQW structure with 9% Sn concentration appears to yield the highest current gain. However, the total thickness of the total periods of MQWs should be under the limit
of critical thickness for strain balanced structure and for optimum performance. Furthermore, it should be noted that the current gain of the optimized Ge$_{1-x}$Sn$_x$ HPTs are considerably larger than the values for HPTs without optimization (W. Wang et al., 2017). Thus, it is crucial to optimize the HPT structure in order to achieve the highest current gain for high sensitivity detection.

Figure 5.10 Current gain variation as a function of periods N of MQW for a fixed doping concentration of emitter, base and collector

5.4.4 Reflectivity of the Device Structure

To achieve high responsivity, it is crucial to reduce the reflectivity of the HPT structure to allow more photons for entering the HPTs, thus contributing to photocurrents. This can be achieved to employ an SiO$_2$ antireflection layer on the base (G. E. Chang et al., 2016) with an optimal thickness. We then calculate the reflectivity spectra ranging from 1000 to 2400 nm as a function of the thickness of the SiO$_2$ antireflection layer ($t_{SiO_2}$) for the Ge$_{1-x}$Sn$_x$/Ge MQW HPT. The results are displayed in figure 5.11. For x=3%, as shown in figure 5.11(a), the calculated reflectivity spectra show clear oscillation peaks, which are attributed to the interference between the layers. For $t_{SiO_2} = 50$ nm, the reflectivity remains a high value of ~0.35. As $t_{SiO_2}$ increases, the reflectivity is reduced, and reach the minimal values in the SWIR range at $t_{SiO_2} = 50$ nm. As $t_{SiO_2}$ increases further, the reflectivity increases again. These results indicate that the optimal thickness of the SiO$_2$ antireflection layer is determined to be $t_{SiO_2} = 300$ nm. The same results are obtained for high Sn contents, as shown in figure
5.11(b) and (c). Therefore, the optimal value of $t_{\text{SiO}_2} = 300 \text{ nm}$ will be used for the calculation of responsivity.

![Reflectivity spectra of Ge$_{1-x}$Sn/Ge MQW HPTs](image)

Figure 5.11 Reflectivity spectra of Ge$_{1-x}$Sn/Ge MQW HPTs for different thickness of SiO$_2$ antireflection layer and Sn concentrations, where, $N=5$, $t_{br}=15$ nm and $t_w=10$ nm

### 5.4.5 Responsivity and Quantum Efficiency

Figure 5.12(a) shows the responsivity spectra (governed by eq. (5.13)) for different Sn concentration. The corresponding internal quantum efficiency (IQE) spectra are calculated using $\text{IQE} = 1240 \times R/\lambda$, where, $R$ is the responsivity (A/W) and $\lambda$ is the free-space wavelength (in nm), shown in figure 5.12(b). For $x = 3\%$, the responsivity gradually decreases with increasing wavelengths, and then reaches cutoff at $\lambda_c = 1720 \text{ nm}$. This indicates that the photodetection range covers the entire fibre-optic communication band (1260-1675 nm). In addition, compared to typical responsivity of 0.15 A/W for conventional GeSn-based photodetectors in the telecommunication bands (Y. H. Peng et al., 2014), the GeSn/Ge MQW HPT exhibits a much larger responsivity of 5-10 A/W attributed to the high current gain. These results indicate the Ge$_{1-x}$Sn$_x$/Ge MQW HPT can find important applications for fiber-optical communications. As the Sn content increases further, the bandgap energy is reduced, extending the absorption edge toward longer wavelengths. Therefore, the photodetection range is further extended to longer wavelengths beyond 1900 nm for $x>6\%$. In addition, increasing the Sn content also leads to higher current gains. As a result, the responsivity also increases considerably. These results suggest Ge$_{1-x}$Sn$_x$/Ge MQW HPTs can achieve high-responsivity photodetection in the infrared range. Together with the unique advantages of CMOS compatibility, Ge$_{1-x}$Sn$_x$/Ge MQW
HPTs can be a potential candidate for their application in communication band and at near/mid infrared region.

Figure 5.12 Variation of (a) responsivity and (b) internal quantum efficiency with wavelength for optimized Ge$_{1-x}$Sn$_x$/Ge MQW HPTs for different Sn concentrations for five periods of MQW.

Furthermore, in the current design, the entire HPT stack is grown on Ge VS. As a result, significant strain could accommodate in the Ge$_{1-x}$Sn$_x$ layers, thereby limiting the critical thickness and increasing the defect density especially when the Sn content is high. This situation can be considerably softened by using the recently developed graded GeSn VS technology (Margetis et al., 2018).
5.5 Conclusion

The design and the performance optimization of Ge$_{1-x}$Sn$_x$/Ge MQW HPTs is presented in this chapter. Detailed analysis is performed on the strained electronic band structures, optical absorption coefficient, current gain in terms of the thicknesses and doping concentration of the layers. Further, responsivity and internal quantum efficiency are also calculated. The results show that the base doping concentration, the well width, the Sn composition, and the number of quantum wells have great influences on the current gain. Optimizing the HPT structures can lead to current gain higher than 100, thereby considerably enhance the responsivity. In addition, increasing the Sn content allows for significantly shrinking of the band gap, thus extending the photodetection range in the SWIR range. On the basis of the CMOS compatibility, high current gain, and extended photodetection range, we conclude that the proposed Ge$_{1-x}$Sn$_x$/Ge MQW HPT can achieve high-responsivity SWIR photodetection range for a wide range of important applications.