CHAPTER-2
LITERATURE REVIEW

PREAMBLE

This chapter is the discussion of the previous work by other researchers. At the beginning of the chapter earlier studies on the effects of solids concentration and wall effects on the settling behaviour of suspensions are discussed. The drag-coefficient particles Reynolds number relationships proposed by earlier workers are presented and their recommendations are discussed.

Next part of the chapter includes discussion on rheological properties of the suspensions, the rheological behaviours and different rheological models. Further effects of system parameters like solids concentration, particle size and temperature on suspension viscosity are discussed. Different dispersants are also discussed. The chapter concludes with the discussion on applications of artificial neural networks in various fields of chemical engineering.

2.1 Effect of Solids concentration on suspension settling

When the concentration of solids is high, the distance between particles becomes so small that the particles come into contact with one another. As a result, the settling process occurs differently and Stokes law fails to describe the settling of particles beyond infinite dilution. Hindered settling is a term used to describe behavior at higher concentrations where sedimentation rates are largely related to concentration rather than to particle size \(1\). In hindered settling particles are crowded and surrounding particles interfere with the motion of individual particles, typically creating a slower-moving mixture than would normally be expected. Upward displacement of the liquid is also more prominent under hindered settling conditions. Settling rates in multiparticle systems are influenced by particle interaction effects, which consist of collision among particles and non-hydrodynamic interactions.

In the concentrated suspensions of significant solid volume fraction \(\phi\), as the suspension settles, it replaces equivalent volume of liquid. If suspensions settle with their settling velocity \(u_m\) under the action of gravity, the displaced liquid moves
upward. The liquid velocity \((u_w)\) depends on the settling velocity of solids. According to continuity, the relative motion between solid particles and liquid can be presented as follows (Equation 2.1):

\[
\phi u_m + (1 - \phi) u_w = 0 \tag{2.1}
\]

\[
u = u_m - u_w \tag{2.2}
\]

Where, \(u\) is the relative velocity between solids particles and the dispersing medium and is the pertinent settling velocity as given below in Equation 2.3:

\[
u = \frac{u_m}{(1 - \phi)} \tag{2.3}
\]

For sedimentation phenomena, dimensional analysis suggests that,

\[
\frac{u}{u_\infty} = f[(1 - \phi), (\text{Re}_p)] \tag{2.4}
\]

Where, \(u_\infty\) is the free falling velocity of the particle alone in the same fluid and the container as the sedimentation of uniform spheres and \(\text{Re}_p\) is the particle Reynolds number. Richardson and Zaki \cite{2} extensively studied sedimentation of solid-liquid suspensions of spherical particles; they investigated the dependency of the settling velocity on the solids volume fraction. Their results were summarized with the relationship known worldwide as the Richardson-Zaki equation. Their relationship (Equation 2.5) is a power law of the void ratio \((1 - \phi)\).

\[
u = u_\infty (1 - \phi)^n \tag{2.5}
\]

The parameter \(n\) was found to be a function of the flow regime, expressed by the particle Reynolds number \(\text{Re}_p\), and of the particle to column diameter ratio. Richardson and Zaki \cite{2} have given the following relationships (Equations 2.6 to 2.9) for the exponent; \(n\) as a function of particle diameter \((d_p)\) to cylinder diameter \((D)\) ratio and particle Reynolds number \(\text{Re}_p\):

\[
n = 4.65 + 19.5\left(\frac{d_p}{D}\right) \quad \text{for} \quad \text{Re}_p < 0.2 \tag{2.6}
\]

\[
n = 4.36 + 17.6\left(\frac{d_p}{D}\right)^{-0.03} \quad \text{for} \quad 0.2 < \text{Re}_p < 1 \tag{2.7}
\]

\[
n = 4.45 + \text{Re}_p^{-0.1} \quad \text{for} \quad 1 < \text{Re}_p < 500 \tag{2.8}
\]

\[
n = 2.39 \quad \text{for} \quad 500 < \text{Re}_p < 7000 \tag{2.9}
\]
When the ratio of the particle to the container diameter is small, n has a value of about 4.6 \[^3\], value of n between 4.65 and 5.1 was reported by Barnea and Mizrahi \[^4\], Capes \[^5\] obtained the reduced values of n between 3 and 3.5. Turian et al \[^6\] have suggested the alternative scheme of maintaining the value of n to be near to 4.68. Recently Koo \[^7\] reported the value of n as 5.5 for settling of mono-dispersed suspensions. When particles of mixed sizes settle, inter-particle interactions are further compounded as the faster settling, larger particle approach and overtake the slower, smaller particles leaving then in their wake of trailing displaced liquid. Garside and Al Dibouni \[^8\], Cheng \[^9\] and Baldock \[^10\] confirmed the decreasing behaviour of n with the particle Reynolds number.

### 2.2 Wall Effects on suspension settling

When particles settle in a quiescent liquid in a given container, the presence of the container wall at a finite distance from the settling particle exerts retarding effect on it. This phenomenon due to presence of solid boundary at a finite distance is known as wall effect. Wall effect is commonly reported as a function of particle to container diameter ratio \((d_p/D)\). The wall effect becomes significant at higher values of \(d_p/D\). Garside and Al-Dibouni \[^8\] and Selim et al \[^11\] examined wall effects on sedimentation particles. Settling of single spherical particle and multisized particle mixture in Newtonian fluid was exhaustively investigated. It was found that the correction for wall effects gives measurably better correlation of free falling velocity. For single particle terminal velocity Garside and Al-Dibouni \[^8\] recommended the following wall effect correlations:

\[
\frac{u_x}{u} = \left[ \left( 1 - 0.475 \frac{d_p}{D} \right) \left( 1 - \frac{d_p}{D} \right) \right]^4 \quad \text{for } \text{Re}_p < 0.2 \quad (2.10)
\]

\[
\frac{u_x}{u} = \left( 1 - 2.35 \frac{d_p}{D} \right) \quad \text{for } 0.2 < \text{Re}_p < 10^3 \quad (2.11)
\]

\[
\frac{u_x}{u} = \frac{1}{\left( \frac{d_p}{D} \right)^{8/2}} \quad \text{for } 10^3 < \text{Re}_p < 3 \times 10^3 \quad (2.12)
\]
Lali et al.\textsuperscript{[12]} studied settling of spherical particles of glass and stainless steel in aqueous solution of CMC in the concentration range of 0.2 to 2 weight %, the settling velocities were measured in nine different cylinders in the range of 9 - 200 mm diameter. It was found that the wall effect factor is function of both the particle to container diameter ratio and the particle Reynolds number. Di Felice and Parodi\textsuperscript{[13]} have demonstrated that the container wall diameter influences the falling velocity only for dilute systems, which is in contrast with a widely used, established empirical relationships. In creeping flow regime Francis\textsuperscript{[14]} proposed the following relationship for the wall effects:

\[
\frac{u}{u_\infty} = \left[ \frac{1 - \frac{d_p}{D}}{0.475d_p} \right]^{-4} \tag{2.13}
\]

This relationship is simpler than those proposed by Clift et al\textsuperscript{[15]}. Di Felis\textsuperscript{[16]} proposed the following equations (2.14) and (2.15) for wall effects correction for the terminal settling velocity in viscous and inertial flow regimes respectively:

\[
\frac{V_t}{U_\infty} = \left[ \frac{1 - \frac{d_p}{D}}{0.33d_p} \right]^{2.7} \tag{2.14}
\]

\[
\frac{u}{u_\infty} = \left[ \frac{1 - \frac{d_p}{D}}{0.33d_p} \right]^{-0.85} \tag{2.15}
\]

### 2.3 Drag coefficient particle Reynolds number relationship

The $C_D$-$Re_p$ relationship (Equation 2.16) was first proposed by Stokes\textsuperscript{[18]}; he obtained the solution to a general equation of motion for a single sphere settling in a fluid by assuming the total drag as viscous drag. The relationship is the well known Stokes law, applicable for $Re_p < 1$.

\[
C_D = \frac{24}{Re_p} \tag{2.16}
\]
Gilbert et al.\textsuperscript{[19]} proposed a three parameter equation (Equation 2.3.2) to correlate the drag coefficient and the particle Reynolds number. They introduced a third parameter in addition to the two parameters given by Stokes.

\[ C_D = X \text{Re}_p^Y + Z \quad (2.17) \]

A five parameter model (Equation 2.18) suggested by Clift and Gauvin\textsuperscript{[15]} incorporated the corrections to the Stokes law; these additional parameters were obtained as a function of particle Reynolds number.

\[ C_D = \frac{24}{\text{Re}_p}(1 + A \text{Re}_p^B) + \frac{C}{1 + D \text{Re}_p^E} \quad (2.18) \]

Tourton and Levenspiel\textsuperscript{[20]} introduced two equations (Equations 2.19 and 2.20) to correlate the drag coefficient and particle Reynolds number for falling spheres. These two equations are similar to the original three parameter model by Gilbert et al.\textsuperscript{[19]}, and five parameter model by Clift and Gauvin\textsuperscript{[15]}.

\[ C_D = 27.2 \text{Re}_p^{0.827} + 0.427 \quad \text{for } \text{Re}_p < 2 \times 10^5 \quad (2.19) \]

\[ C_D = \frac{24}{\text{Re}_p}(1 + 0.173 \text{Re}_p^{0.657}) + \frac{0.413}{1 + 16300 \text{Re}_p^{-1.07}} \quad \text{for } \text{Re}_p < 2.6 \times 10^6 \quad (2.20) \]

Another five parameter equation obtained by Khan and Richardson\textsuperscript{[21]} is as given below:

\[ C_D = (2.25 \text{Re}_p^{0.31} + 0.36 \text{Re}_p^{0.06})^{3.45} \quad \text{for } \text{Re}_p < 3 \times 10^5 \quad (2.21) \]

Turton and Levenspiel\textsuperscript{[20]} tested their experimental data with three parameter equation (Equation 2.19) and five parameter equation (Equation 2.20), as shown in the following Figures 2.1 and 2.2. The results show that the three parameter equation predict the drag coefficients value well for low particle Reynolds number where as the predictions show significant error for the higher value of particle Reynolds number. The five-parameter equation, on the other hand, is capable of showing a minimum C\textsubscript{D} value, and it can be seen from Figure 2.2 that this equation correlates the data extremely well throughout the whole particle Reynolds number range. Equation 2.20 has the added advantage of converging to the Stokes equation at low particle Reynolds number; it is a better correlation for prediction of the drag coefficient values in the sub critical regime (\text{Re}_p < 2 \times 10^5).
Figure 2.1: Comparison between experimental and predicted values of drag coefficient of spheres using three parameter model[20]

A four parameter general drag coefficient-Reynolds number relationship proposed by Haider and Levenspiel [22] is given below (Equation 2.22)

\[
C_D = \frac{24}{\text{Re}_p} (A + \text{Re}_p^B) + \frac{C}{1 + \text{Re}_p^D} 
\]

(2.22)

The experimental data used to find the best values of the four parameters was same as used by Turton and Levenspiel [20]. The final equation obtained for the drag coefficient is as given below (Equation 2.23):

\[
C_D = \frac{24}{\text{Re}_p} (0.1806 + \text{Re}_p^{0.6459}) + \frac{0.4251}{6880.95} \frac{1}{1 + \text{Re}_p} 
\]

(2.23)

The drag coefficient values predicted by this equation were compared with the experimental values and earlier predictions by Turton and Levenspiel [20] with the three parameter equation (Equation 2.21) and five parameter equation (Equation 2.22) as discussed earlier. The drag coefficient values calculated by four parameter model as depicted in Figure 2.3 were found more accurate.
An explicit equation for particle settling velocities in solid-liquid systems was presented by Zigrang and Sylvester \[23\]. This equation can be used to predict the settling velocity of particles in liquid-solid suspensions; this correlation gives the terminal velocity of a single spherical particle in a liquid as given below (Equation 2.24):

\[
u_s = \frac{\left(\sqrt{14.51 + 1.83d_s^{3/2}} - 3.81\right)^2}{d_s}
\]  
(2.24)

Where, \(d_s\) and \(u_s\) are dimensionless diameter and dimensionless terminal velocity of sphere respectively.

\[
d_s = \left(\frac{3}{4C_DRe_p^2}\right)^{\frac{1}{3}}
\]  
(2.25)

\[
u_s = \left(\frac{4Re_p}{3C_D}\right)^{\frac{1}{4}}
\]  
(2.26)
Figure 2.3: Comparison between experimental and predicted values of drag coefficient of spheres using four parameter model

Turton and Clark [24] proposed the following correlation (Equation 2.27) to represent a very simple and accurate way to evaluate the terminal velocity of a sphere of known dimensions and density in a known fluid.

\[
C_0 = \frac{\frac{4}{3} \pi d_{\text{ sph }}^3 (\rho_{\text{ s}} - \rho_f)}{u_e \rho_f} = \left( \frac{d_{\text{ sph}} u \rho_f}{\mu} \right)^{1.214} + \frac{C}{\left( \frac{1}{R_e} \right)^{0.824}} + \frac{D}{\left( \frac{1}{R_e} \right)^{0.321}}
\]

This correlation cannot predict the velocity of a particle driven through a fluid by a force other than its own weight without further manipulation. There are number relationships available in the literature for the drag coefficient as a function of particle Reynolds number; some of them are reported in the following Table 2.1
Table 2.1: Drag coefficient-Reynolds number relationships

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Equation</th>
<th>Range of Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perry and Chilton [25]</td>
<td>$C_D = \frac{24}{\text{Re}_p}$</td>
<td>$\text{Re}_p &lt; 0.3$</td>
</tr>
<tr>
<td></td>
<td>$C_D = \frac{18.5}{\text{Re}_p^{0.6}}$</td>
<td>$0.3 &lt; \text{Re}_p &lt; 1000$</td>
</tr>
<tr>
<td></td>
<td>$C_D = 0.44$</td>
<td>$1000 &lt; \text{Re}_p &lt; 20000$</td>
</tr>
<tr>
<td>Cheng [9]</td>
<td>$C_D = \frac{24}{\text{Re}_p} \left(1 + 0.27 \text{Re}_p\right)^{0.43} + 0.47(1 - \exp(-0.04 \text{Re}_p^{0.38}))$</td>
<td>$\text{Re}_p &lt; 100$</td>
</tr>
<tr>
<td>Flemmer and Banks [26]</td>
<td>$C_D = \frac{24}{\text{Re}_p} 10^E$</td>
<td>$\text{Re}_p &lt; 3 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>Where,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E = 0.261\text{Re}_p^{0.369} - 0.105\text{Re}<em>p^{0.431} - \frac{0.124}{1 + (\log</em>{10}\text{Re}_p)^2}$</td>
<td></td>
</tr>
<tr>
<td>Saha et al [27]</td>
<td>$C_D = \frac{18.5}{\text{Re}_p^{0.75}}$</td>
<td>$1 &lt; \text{Re}_p &lt; 1000$</td>
</tr>
<tr>
<td>Choncha and Almendra [28]</td>
<td>$C_D = 0.28 \left(1 + \frac{9.06}{\text{Re}_p^{0.5}}\right)^2$</td>
<td>$1 &lt; \text{Re}_p &lt; 1000$</td>
</tr>
<tr>
<td>Madhav and Chhabra [29]</td>
<td>$C_D = \frac{24}{\text{Re}_p} \left(1 + 0.0604\text{Re}_p^{0.529}\right)$</td>
<td>$0.095 &lt; \text{Re}_p &lt; 400$</td>
</tr>
<tr>
<td>Brown and Lawler [30]</td>
<td>$C_D = \frac{24}{\text{Re}_p} \left(1 + 0.15\text{Re}_p\right)^{0.681} + \frac{0.407}{1 + 8710\text{Re}_p^{1}}$</td>
<td>$\text{Re}_p &lt; 2 \times 10^5$</td>
</tr>
</tbody>
</table>

2.4 Flow curve modeling

In order to describe the flow curves mathematically, several rheological models have been developed. Mechanistic approaches are, however, difficult to apply because of the diversity of the properties responsible for the rheological behaviour. More commonly, empirical models have been used to describe the characteristics of the flow behaviour. Several empirical equations relating shear stress to the shear rate have been developed to model the basic flow curve shapes. The criteria for the suitability of the model are:

i. It should be simple with minimum number of independent constants
ii. Its’ constants should be easily determined and

iii. The constants should have some physical significance

Regression methods can be used to fit the equations to the rheological data. The fit of these equations can be then compared using statistical methods. Models with smallest number of coefficients are preferred because they are usually simple to use. The following sections describe some of the models commonly used to represent the rheological properties of the suspensions. Included are Newton’s law of viscosity, three models can be used to describe the shear thinning behaviour and three models that describe plastic behaviour. The shear thinning properties are often characterized by the power law model, the Cross model and the Cerreau model. Models that describe plastic flow properties must have yield stress term such as the Bingham plastic model, the Herschel Bulkley model and the Casson Model.

2.4.1 Newton’s law of viscosity

The flow behaviour of Newtonian fluids can be described by Newton’s Law of Viscosity (Equation 2.28)

\[ \tau = \mu \left( \frac{du}{dr} \right) \]  

(2.28)

Where, \( \mu \) is the Newtonian viscosity of the fluid. It has been used to describe the rheology of non-interacting spheres in Newtonian liquids \[18, 31\]. Above a critical solids concentration, suspensions exhibit non-Newtonian properties.

2.4.2 The power law model

\[ \tau = k \left( \frac{du}{dr} \right)^n \]  

(2.29)

This model (Equation 2.29) is also referred as Ostwld-De Waele model. This equation can be used to model all three basic viscous flow behaviours. The coefficients k and n are referred to as the fluid consistency index and flow index, respectively \[32\]. A high k value implies that the suspensions have a high viscosity. The deviation of n from unity is a measure of the non-Newtonian behaviour. Clearly, at n equal to unity, the equation becomes Newton’s law of viscosity and k becomes \( \mu \). For n is less than unity, the model describes pseudoplastic flow and, for n greater than one it describes the dilatant flow. This model has been used to describe many solid-liquid suspensions \[6\].
2.4.3 The Cross model

The Cross model (Equation 2.30) was developed based on the assumption that the pseudoplastic flow is due to the process of formation and rupture of linkages in chains of particles. This process is believed to occur in aggregated systems where links between particles result in the formation of chain-like aggregates which rupture due to Brownian motion and shearing \[^{33-34}\].

\[
\mu = \mu_\infty + \alpha \left( \frac{du}{dr} \right)^m \quad (2.30)
\]

Where, \( \mu_\infty \) is the apparent viscosity at infinite shear rate. A value of \( m \) equal to 2/3 was found to be suitable for many pseudoplastic systems which reduce the number of coefficients in the equation to 2 \[^{33}\]. The Cross model has been applied to suspensions that exhibit pseudoplastic behaviour \[^{35}\].

2.4.4 The Carreau model

The Carreau model (Equation 2.31) was developed to describe the pseudoplastic properties of polymer solutions and melts.

\[
\mu = \mu_\infty \left( 1 + A \left( \frac{du}{dr} \right)^2 \right)^{-\frac{n}{\gamma}} \quad (2.31)
\]

It is based on molecular network theory which describes the non-Newtonian flow with respect to the creation and loss of segments. The rate of creation and loss of these segments is function of shear rate which results in the non-Newtonian response to shear \[^{36-38}\].

2.4.5 The Bingham plastic model

The Bingham plastic model (Equation 2.32) describes the simplest type of viscoplastic flow. The model characterizes the ideal case in which a complete structure breakdown occurs once a yield stress has been exceeded. Once the yield stress is exceeded a linear relationship exists between the shear stress and shear rate \[^{39}\].

\[
\tau = \tau_0 + \eta \left( \frac{du}{dr} \right) \quad (2.32)
\]

Where, \( \tau_0 \) is the yield stress and \( \eta \) is the plastic viscosity. The Bingham model is widely used because of its simplicity. Many suspensions including red mud, kaolinite, clays, drilling mud and yeast suspensions have been modeled with equation \[^{6,35,40-41}\].
2.4.6 The Herschel Bulkley Model

The Herschel Bulkley model (Equation 2.33) is the combination of the Oswald-De Waele power law model and the Bingham plastic model. As the yield stress becomes small the equation takes form of the power law model and as the index, $n$, approaches unity, it becomes Bingham plastic model.

$$\tau = \tau_0 + k \left( \frac{du}{dr} \right)^n$$  \hspace{1cm} (2.33)

This simple, versatile and practical model has been widely used to characterize both dilute and concentrated suspensions $[6, 35]$.

2.4.7 The Casson model

$$\tau = \tau_0^{1/2} + \left( \eta \frac{du}{dr} \right)^{1/2}$$  \hspace{1cm} (2.34)

The Casson model (Equation 2.34) is a simple two parameter model that has a physical basis and is derived from structural arguments. This model is capable of fitting low shear rate curvature and therefore it estimates the yield stress compare well to the values obtained experimentally $[6]$.

2.5 Effect of Solids Concentration on Suspension Viscosity

Several studies have shown that the viscosity of suspensions increase in an exponential manner with solids concentration and becomes infinite at the maximum packing fraction. It is well known that the earliest theoretical work on the effective viscosity was due to Einstein $[18]$ whose derivation led to the effective viscosity to be linearly related to the particle concentration as follows (Equation 2.35):

$$\eta_r = 1 + 2.5\phi$$  \hspace{1cm} (2.35)

Where,

$\eta_r = \mu_r/\mu = \text{relative viscosity of the suspension}$

$\mu_m = \text{effective viscosity of the particle-fluid mixture}$

$\mu_r = \text{viscosity of fluid, and}$

$\phi = \text{solid volume fraction of the suspension}$

This expression is exact when the viscous effect is dominant so that the creeping flow equations can be applied at the particle level. This equation is based on the viscous
energy dissipated by the flow around the sphere, relating the relative viscosity of the solids content. With increasing solids concentration, there are a greater number of particle interactions: these are responsible for the exponential increase in viscosity. These interactions result in energy dissipation that can be described in terms of hydrodynamic, and aggregation effects. These effects are also responsible for the increase in non-Newtonian nature of the suspensions that is found at higher concentrations.

Following Einstein’s work, numerous expressions have been proposed to extend the range of validity to higher concentrations. They are either theoretical expansions of Einstein formula to higher order in \( \phi \), or empirical expressions that were obtained based on experimental data \([42-44]\). The theoretical expansions are usually expressed in the form of a power series as given below (Equation 2.36):

\[
\eta_r = 1 + k_1\phi + k_2\phi^2 + k_3\phi^3 + \cdots
\]  

(2.36)

Where \( k_1, k_2, k_3 \) are the coefficients. Evaluation of these coefficients other than \( k_1 \), requires the formulation of particle interactions, which is rather difficult. This is one of the reasons that only the theoretical values of the coefficients related to lower order in \( \phi \), like \( k_2 \) and \( k_3 \), can be found in the literature so far. Even for these lower order coefficients, the calculations are available only for idealized cases when the particle arrangement or its statistic properties are simple.

An example for determining the \( k_2 \)-value was presented by Batchelor and Green \([45]\) who reported that the coefficient \( k_2 \) was equal to 7.6 for a suspension undergoing a pure straining motion. It should be noted however that Batchelor and Green introduced a rough interpolation in their derivation, which led to an inaccuracy in the \( k_2 \)-value obtained. A re-calculation under the same condition was conducted by Kim and Karrila \([46]\), leading to the \( k_2 \)-value to be changed to 6.95. Other \( k_2 \)-values have been reviewed by Elic and Phan-Thien \([47]\). Furthermore, Thomas and Muthukumar \([48]\) found the third order coefficient, \( k_3 \), to be 6.40 by applying the multiple scattering theories to the evaluation of the hydrodynamic interaction of three spheres.

Following these theoretical attempts, it can be expected that more complicated configurations of the particle arrangements will lead to further difficulties in the mathematical formulation. On the other hand, particles suspended in fluid should in fact be distributed in a random manner so that their arrangement cannot be fitted to a
simple configuration or described by a simple distribution function. Therefore, although an idealized particle arrangement may enable a theoretical evaluation of the k-coefficients in higher order, the applicability of the effective viscosity so derived is limited in practice.

When aggregation effects exist, the aggregates may be considered to be the suspended units in the suspension. This result in an increase in effective solids content as the aggregate will trap liquid. The size of aggregates depends on the type of attractive forces, the shearing conditions and solids concentration in the suspensions. Many of these effects have been considered in the evaluation of the coefficients of the general power formula. This model, which is extension of the Einstein’s equation applies only to moderately concentrated suspensions and does not cover the entire solids concentration range.

In addition to the theoretical expressions, various empirical relationships have been proposed to evaluate the effective viscosity of a suspension with higher concentration. An early survey of such relationships made by Rutgers showed that large discrepancies exist among the different relationships. Table 2.2 shows some typical examples of the existing empirical formulas, where $\phi_m$ is the maximum particle concentration and $\eta$ is the intrinsic viscosity. It is noted that for those relationships where the so-called maximum concentration is included as a parameter, the effective viscosity approaches infinity when the concentration is equal to the maximum value. This may not be physically reasonable. Strictly speaking, there are only two extreme conditions that are meaningful for the effective viscosity. The first condition is a suspension without particles, implying that the effective viscosity is the same as the fluid viscosity. The other is a suspension without fluid, which would then theoretically behave as a solid with infinite viscosity. On the other hand, only for particles with regular shapes can the maximum concentration be mathematically determined for a given packing arrangement. For example, the maximum concentration can reach 1.0 for cubes packed face-to-face and 0.74 for spheres with the closest hexagonal packing arrangement. With irregular particles, the maximum value varies markedly even though the particle size is uniform. As a matter of fact, the maximum concentration and intrinsic viscosity were often used as two empirical parameters in fitting experimental data to an empirical formula.
Eilers \cite{18} used the maximum solids content as an upper limit in relating relative viscosity to the solids fraction for bitumen emulsions. Chong et al \cite{54} modified Eiler’s equation to correlate the relative viscosity to the solids content for poly-disperse suspensions. Mooney \cite{55} considered the crowding effects of mono-disperse sphere to obtain his equation. He also showed how his approach could be extended to bimodal and poly-disperse suspensions. Krieger and Dougherty \cite{56} also used particle crowding, in similar manner to derive their equation. Ting and Luebber\cite{57} considered the effects of liquid-solid density ratio, particle size distribution and particle shape on packing arrangement to derive an empirical relation between the relative viscosity and some function of these parameters for varying solids contents.
Table 2.2: Formulas for effective viscosity

<table>
<thead>
<tr>
<th>Equation</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Einstein’s Equation</td>
<td>$\eta_e = (1 + 2.5\phi)$</td>
</tr>
<tr>
<td>Eiler Equation</td>
<td>$\eta_e = \left(1 - \frac{\phi}{\phi_m}\right)^2$</td>
</tr>
<tr>
<td>Leighton and Acrivos’s Equation</td>
<td>$\eta_e = \left(1 + \frac{0.5\eta\phi}{1 - \frac{\phi}{\phi_m}}\right)^2$</td>
</tr>
<tr>
<td>Chong et al’s Equation</td>
<td>$\eta_e = \left(1 + 0.75\frac{\phi}{1 - \frac{\phi}{\phi_m}}\right)^2$</td>
</tr>
<tr>
<td>Mooney’s Equation</td>
<td>$\eta_e = \exp\left(\frac{2.5\phi}{1 - \frac{\phi}{\phi_m}}\right)$</td>
</tr>
<tr>
<td>Krieger Dowarty’s Equation</td>
<td>$\eta_e = \left[1 - \frac{\phi}{\phi_m}\right]^{-\eta\phi_n}$</td>
</tr>
<tr>
<td>Liu’s Equation</td>
<td>$\eta_e = [a(\phi_m - \phi)]^n$</td>
</tr>
<tr>
<td>Metzner’s Equation</td>
<td>$\eta_e = \left(1 + 1.25\frac{\phi}{1 - \frac{\phi}{\phi_m}}\right)^2$</td>
</tr>
<tr>
<td>Quemada’s Equation</td>
<td>$\eta_e = \left[1 - \frac{\phi}{\phi_m}\right]^{-2\phi_n}$</td>
</tr>
</tbody>
</table>

2.6 Effect of Temperature on Suspension Viscosity

Liquid viscosity decreases with the increase in temperature. The trend of decreasing viscosity at elevated temperatures occurs due to increased kinetic energy of the particles promoting the breakage of intermolecular bond between adjacent layers which results in decrease in viscosity of the suspension. For the limestone slurry of 70 weight % solids concentration, He et al [61] investigated influence of temperature on the viscosity. They found that, the slurry viscosity decreases with increase in the temperature from 13 to 55 °C. Reduction in the viscosity from the range of 0.0215 to
0.0365 Pas to the range of 0.01 to 0.019 Pas as for the increase in temperature from 13 to 55 °C was reported.

Mikulasek et al [62] shown the decrease in viscosity of the titanium oxide suspensions of 10 to 30 volume % concentration over the range of temperature from 20 to 50 °C. Similar results were observed by Roh et al [63] for coal water suspensions for the temperature range of 15 to 50 °C and Senapati et al [64] for limestone suspensions in the temperature range of 30 to 60 °C. The decrease in the suspension viscosity may be the result of decrease in the interparticle forces between particles with temperature. The decreased value of the viscosity with increased temperature confirms to fundamental properties of any viscous materials as for all normal liquids [65]. Work of different researchers [66-69] shown that the suspension viscosity also decreases with the increase in temperature.

Senapati et al [64] presented a simple Arhenious type equation for the temperature dependence of suspension viscosity. An identical relationship was confirmed by Li et al [70] for temperature dependence of silicon carbide suspensions, they observed increase in viscosity as temperature of the slurries increased from 20 to 60 °C. The increase in the viscosity was observed because the suspensions induce flocculation trend, which might be enhanced at higher temperature.

2.7 Effect of Particle Size on Suspension Viscosity

It is expected that the effects of particle size on suspension viscosity varies depending on the range of particle size where these effects are studied because the actual interparticle forces will vary with the size of particles. Renehan et al [71] found that suspensions of coal particles in a non-Newtonian suspension of bentonite clay exhibited both increase and decrease in viscosity with increasing particle size. Experimental results from Hasan et al [72] show that for high concentrations there is a decrease in viscosity with increasing particle size, whereas at lower concentrations there is an increase with increasing particle size. The size of particles in a suspension has been shown strong influence on its rheological properties. In particular it has been shown that suspensions with very large (higher than 100 µm) or very small (less than 10 µm) particles have a higher viscosity than suspension containing immediately sized particles i.e.-100+10 µm [71]. These differences in the viscosities can be
explained by contributions of the hydrodynamic and aggregation effects. These effects contribute to the viscosity to a greater or lesser degree depending on the size of the particles.

It is difficult to determine the effect of particle size on the viscosity of suspensions since parameters like shape and charge, which characterize the system may vary with particle size. Most authors do not specify precisely the accuracy of the size distribution, or provide much detail as to experimental conditions. Therefore the disparity in observed results is not unexpected.

Some investigators have reported an increase in viscosity with the decrease in the particle size [73-76]. This increase in viscosity has been attributed to the aggregation effects that dominate fine particle behaviour [43, 70, 76-79]. Inter-particle forces of attraction and repulsion are responsible for these effects which dominate the movement of colloidal size particles [43, 71]. The magnitude of these factors may be large enough to influence the movement of course particles. Rehenan et al [71] showed that the movement of coal particles as course as 200 μm were influenced by surface charge related effects, that were large enough to contribute to the rheological properties. Sweeney and Geckler [74] suggested that the contribution from electroviscous effects to the rheological properties of fine particle suspension can be very significant. They showed that the viscosity of a suspension of glass spheres in anionic suspending fluid, where electroviscous effects should be large, was greater than the viscosity of the same sphere suspension in non-ionic fluid, in which electroviscous effects should be small.

For suspensions of glass and polystyrene spheres, Parkinson et al [73] Saunders [78] explained the increase in non-Newtonian behaviour with decreasing particle size, by contribution of aggregation effects. Inter-particle forces of attraction dominate over inertial forces for small particles which make them more susceptible to aggregation than course ones. With increasing shear rate the attachments break resulting in more dispersed suspension. As the particle becomes dispersed the apparent viscosity decreased resulting in the shear thinning behaviour [75]. Since this dispersion with shearing is not instantaneous, at a given shear rate the apparent viscosity will decay with time resulting in the thixotropic characteristics of such suspensions. The structure that forms as a result of the aggregation of the fine particle contributes to the yield stress. Below a critical solids concentration the structure may not be continuous
and therefore no yield stress may be present \[80\]. The effects of particle size are more pronounced at high solids concentrations than at low ones.

The magnitude of the hydrodynamic effects is proportional to specific surface area of the particles. Since the specific surface area increases with decreasing particle size, the contribution of the hydrodynamic effects also increases. Therefore the increase in viscosity of suspensions resulting from decreasing particle sizes can be explained by the greater amount of hydrodynamic energy dissipation \[78\]. According to Hasan et al. \[72\] increased particle interaction for smaller particles leads to increase in the suspension viscosity. The formation of aggregates or flocks with decreasing particle size due to the increasing significance of inter-particle forces at smaller particle sizes increases the suspension viscosity. Kawatra and Eisele \[81\] observed an increase in viscosity with reduction in particle size for constant solids concentration. This is due to increased surface area, which binds up water molecules and thus increases the effective solids concentration.

The results of several investigations \[71, 75, 77\] have shown that the increase in viscosity occurs as particle size increases. This result has been explained by the energy dissipated due to physical particle interactions. Since coarse particles have greater inertia than fine ones they will collide rather than slip past each other. The physical collisions dissipate energy through friction and loss of translational and rotational momentum. Thomas \[43\] and Hasan et al. \[72\] suggest that at low concentrations particle interactions are minimal and viscosity may increase with particle size. Mangesana at al \[82\] shown that viscosity increases with particle size at a fixed concentration for any given shear rate. The reason suggested for this increase was that particles of greater size possess greater inertia such that on interaction, the particles are momentarily retarded and then accelerated. In both these stages their inertia affects the amount of energy required. This dissipation of energy is what may appear as extra viscosity. The contributions of the hydrodynamic, aggregation and electroviscous effects clearly depend on the size of the particles. In order to minimize the viscosity, it seems that an optimum particle size exists that is not too large and not too small.
2.8 Dispersing Agents

Dispersing agent can be classified into: (i) Inorganic compounds, and (ii) polymeric dispersants \[^{83}\]. While inorganic dispersants are primarily to control the charge density at the solid/liquid interface, the polymeric compounds also provide steric hindrance. Suspensions of aggregated particles can exhibit yield stress and typically have shear thinning flow properties \[^{84}\]. Dispersing the particles results in more Newtonian flow behaviour and reduced apparent viscosity. It should be noted, however, that by increasing electrostatic repulsion, electroviscous forces can also become important resulting in greater apparent viscosities. Organic reagents typically used in iron ore processing are polysaccharides such as starch, guar gum and carboxyl methyl cellulose which are used to flocculate iron ore \[^{85-86}\].

The effectiveness of these regents depends on the nature of the reagents, the particle surface chemistry, the type of dissolved ions and the pH. Typical inorganic dispersing agents are sodium polyphosphates and sodium silicates \[^{84}\]. These dispersing agents are widely used in processing of iron ores.

2.9 Artificial Neural Networks (ANN)

ANNs are massively connected parallel distributed processor made up of simple processing units, which has a ability for storing experimental knowledge and making it available for use. ANNs are well known for their utility as universal approximator of complex nonlinear relationships between process variables and product quality properties. ANNs have the ability to learn from input data and are very useful for the prediction of complex high-dimensional data. ANN methods have a broad range of applications in agriculture \[^{87}\], weather forecasting \[^{88}\], water resources engineering \[^{89-90}\] finance, medicine, robotics, etc. As in other fields of science, ANN methods have become popular in chemical engineering too, soon after their development.

Artificial neural networks have been successfully used for dynamic modeling and control of chemical processes and fault diagnosis \[^{91}\], in the catalytic reactor modeling and design of solid catalysts \[^{92}\] and for modeling the kinetics of a chemical reaction \[^{93}\]. The applicability of ANN modeling in emulsion liquid membranes \[^{94}\] and in the prediction/estimation of the vapor–liquid equilibrium data \[^{95-96}\] has also been investigated. Melkon Tatlier et al \[^{97}\] used ANN model for the prediction of the molar
composition of the zeolites. They estimated the quantities of Si and H$_2$O in the zeolites by using ANN. They reported the amounts of deviation from the experimental results for the Si and H$_2$O contents to be equal to about 10 and 20%, respectively.

Bhat and McAvoy \cite{98} applied ANN to dynamic modelling for a pH-controlled CSTR. The predicted pH values when compared to two other approaches demonstrated that ANN could predict more accurately than conventional method. Willis et al \cite{99} discussed the application of ANN as both inferential estimator and predictive controller. The results demonstrated that ANN could accurately predict the process output and significantly improved the control scheme. Pollard et al \cite{100} utilized backpropagation trained ANN for process identification. They concluded that ANN was particularly useful when the input-output mapping was unknown since ANN was able to accurately represent nonlinear behaviour in a black box manner. Rajasimman et al \cite{101} used ANN for modeling of an inverse fluidized bed bioreactor. Initial substrate concentration and the chemical oxygen demand (COD) values for a given retention time were predicted by using ANN. Predictions of these parameters were obtained with desired accuracy.

Shaikh and Al-Dahhan \cite{102} compared the selected literature correlation and the ANN model for the prediction of gas holdup in bubble column reactor. The standard deviation of the ANN results for different column parameters was within 14% which was still less than the other models. They further suggested the use of ANN for scale up of he bubble column reactors. Sablani et al \cite{103} developed ANN model capable of explicitly calculating the friction factor for a non Newtonian fluid over a wide range of Reynolds number. Among different architectures studies they found the maximum average mean error to be about 3% for the laminar and turbulent regions. Fadare and Ofidhe \cite{104} developed artificial neural network for the prediction of friction factor in pipe flow. The proposed model found accurate for prediction of factor using relative roughness and Reynolds number as input parameters.

ANNs are good at pattern recognition, trend prediction, modeling, control, signal filtering, noise reduction, image analysis, classification, and evaluation. In fact, the uses for neural networks are so numerous and diverse that these applications may seem to have nothing in common. However, they all share the ability to make associations between known inputs and outputs by observing a large number of examples. ANNs have been successfully applied in various fields of chemical
engineering which include, modeling of multiphase reactors \(^{105}\), Process control \(^{91, 106-109}\), Catalysis \(^{110}\), Estimation of concentration of components in a mixture \(^{108-113}\), calculation of heat transfer coefficient \(^{114}\), modeling of pump-mixer characteristics \(^{114}\), Membrane separations \(^{110}\), Reactor modeling \(^{105, 116}\), waste water treatment \(^{117}\).

The success of the ANN depends on the selection of the process variables, the quality of the data and the type of algorithm used. The backpropagation \(^{118-120}\) is the most well known and widely among all types of ANN algorithms. The backpropagation is the multilayer feed forward network with different transfer functions in an artificial neuron and a powerful learning rule. The learning rule is known as backpropagation, which is kind of error minimization technique with backward error propagation.

ANN is attractive due to its information processing characteristic such as nonlinearity, high parallelism, fault tolerance as well as capability to generalize and handle imprecise information \(^{118, 120}\). These characteristics have made ANN suitable for solving a variety of problems.

### 2.10 Critical Appraisal

The two phase systems of solid-liquid suspensions are inherently unstable. Sufficient amount of literature is available on settling behaviours of different types of suspensions for different ranges of particle Reynolds numbers. Among the relationships for the single particle terminal settling velocity in concentrated suspensions, Richardson and Zaki’s equation is confirmed and recommended by several theoretical and experimental studies. From the available literature it is well understood that, the hindered settling coefficient ‘n’ varies from suspension to suspension. No unique value of ‘n’ is available for all suspensions. It varies from 3 to 5.1 for the suspensions of different types of solids. In the literature various drag coefficient- particle Reynolds number \((C_D-Re_p)\) relationships are available. The available relationships contain two to five constants.

No settling data for the hindered settling coefficient ‘n’ of magnetite ore suspensions is available in the literature. Because of the unstable nature of these two phase systems the, available \(C_D-Re_p\) relationships are erroneous if applied to all types of suspensions. Hence there is good scope for studying settling behaviour of magnetite ore suspensions and to obtain the useful data on the ‘n’ values. From the experimental
data it would be possible to develop experimental $C_D$-$Re_p$ relationship for magnetite ore suspensions.

Rheological characteristics of the suspensions depend on various parameters. Particle size and concentration of solids significantly influence the shear stress-shear rate relationship and the relative viscosity of the suspensions. The effect of solids concentration on the suspension rheology is not the same for all types of suspensions. It depends on the nature and type of the solid particles and their surface characteristics. Studies by Some suspensions show the increase in non Newtonian behaviour with the increase in solids concentration, whereas the decrease in non Newtonian behaviour of the suspensions also has been presented in some studies. Same suspension also can show different rheological behaviours at different solids concentrations and shear rates. Particle size effect on suspension rheology and relative viscosity was studies by many workers. The earlier studies show increase in non Newtonian behaviour and relative viscosity with the increase in particle size of the solids in suspensions as well as decrease in non Newtonian behaviour and relative viscosity with increased particle size.

Well established empirical equations are available in the literature to describe the flow curves of the suspensions, but the suspensions of different types of solids particles can not be presented by a same equation. Even for same suspensions different equations were suggested by a few workers. For magnetite ore suspensions, to describe the shear stress- shear rate relationship, equations such as power law, Bingham plastic and Casson equation were proposed by earlier workers. For very dilute suspensions, the relative viscosity can be predicted by Einstein’s formula. For concentrated suspensions, numbers of equation are available in literature, but any of these equations is not suitable for all types of suspensions. Different equations are suitable for different type of suspensions. No such relationship for magnetite ore suspensions was found in the literature.

Hence there is good scope studying rheological behaviour of the magnetite ore suspensions, to investigate the effects of particle size and solids concentration on the suspensions rheology and to find out a suitable equation to describe the flow curves. It would be worth to know the effect of particle size on relative viscosity of magnetite ore suspensions and to find out a suitable better fitting equation for the dependence of relative viscosity on the solids concentration in the suspension. Further there is good
potential to develop artificial neural network models to describe the flow curves for magnetite ore suspensions and the relative viscosity-solids volume fraction relationships for the same.

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